- **Deadline: March 2015**
- **Send a pdf file with your answers to B.Kleijn@uva.nl.**
- **Your name and student number should be on your submission.**

1. Consider Hodges' estimators S_n of example 13.4. Show that, for any rate sequence (ϵ_n), $\epsilon_n \downarrow 0$, $\epsilon_n(S_n - 0) \overset{0}{\rightsquigarrow} 0$.

2. Let $\mathcal{P} = \{P_\theta : \theta > 0\}$ be the model of Poisson distributions P_θ with means θ. Show that this model is LAN for all $\theta > 0$.

3. Let $\Theta = \mathbb{R}$ and let $\mathcal{P} = \{P_\theta : \theta \in \Theta\}$ be the model of normal distributions $N(\theta, 1)$ of unit variance with means θ. Show that this model is LAN for all θ.

4. Let f be a Lebesgue density on $\mathbb{R}$ that is symmetric around the origin. Define the model $\mathcal{P} = \{P_{\mu,\sigma} : \mu \in \mathbb{R}, \sigma \in (0,\infty)\}$ by densities $f_{\mu,\sigma}(x) = \sigma^{-1} f((x-\mu)/\sigma)$. Show that the Fisher information matrix is diagonal.

5. Let $\Theta = (0,\infty)$ and $\mathcal{P} = \{N(0,\theta^2) : \theta \in \Theta\}$. Let Π be a thick prior on Θ. Show that this model satisfies the conditions of the Bernstein-von Mises theorem 14.1. Find the problematic range of parameter values in this model. *(Hint: calculate the Fisher information, find a problematic limit for it and describe the effect on the limiting sequence of normal distributions for parameter values close to the problematic limit.)*

6. Prove the following: for $\theta \in \Theta = \mathbb{R}$, let $F_\theta(x) = (1 - e^{-(x-\theta)}) \vee 0$ be the standard exponential distribution function located at θ. Assume that $X_1, X_2, \ldots$ form an *i.i.d.* sample from F_{θ_0}, for some θ_0. Let Π be a thick prior on Θ. Then the associated posterior distribution satisfies, with $h = n(\theta - \theta_0)$,

$$\sup_A \Big| \Pi_n\big(h \in A \mid X_1, \ldots, X_n \big) - \mathrm{Exp}^-_{n(\hat{\theta}_n - \theta_0)}(A) \Big| \xrightarrow{\theta_0} 0,$$

where $\hat{\theta}_n = X_{(1)}$ is the maximum likelihood estimate for θ_0 and Exp^-_a denotes the standard negative exponential distribution located at a.