# Probabilistic Robotics

## Chapter 2: Recursive State Estimation

BSc course Kunstmatige Intelligentie 2010

http://www.science.uva.nl/~arnoud/education/ProbabilisticRobotics/

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# Trends in Robotics Research

**Classical Robotics (mid-70's)**
- exact models
- no sensing necessary

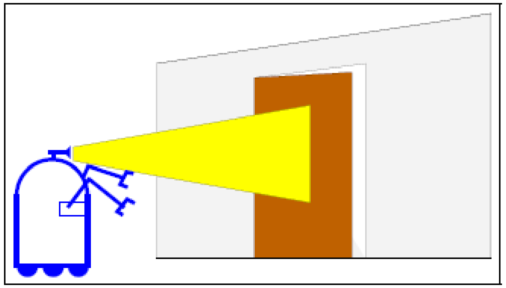
**Reactive Paradigm (mid-80's)**
- no models
- relies heavily on good sensing

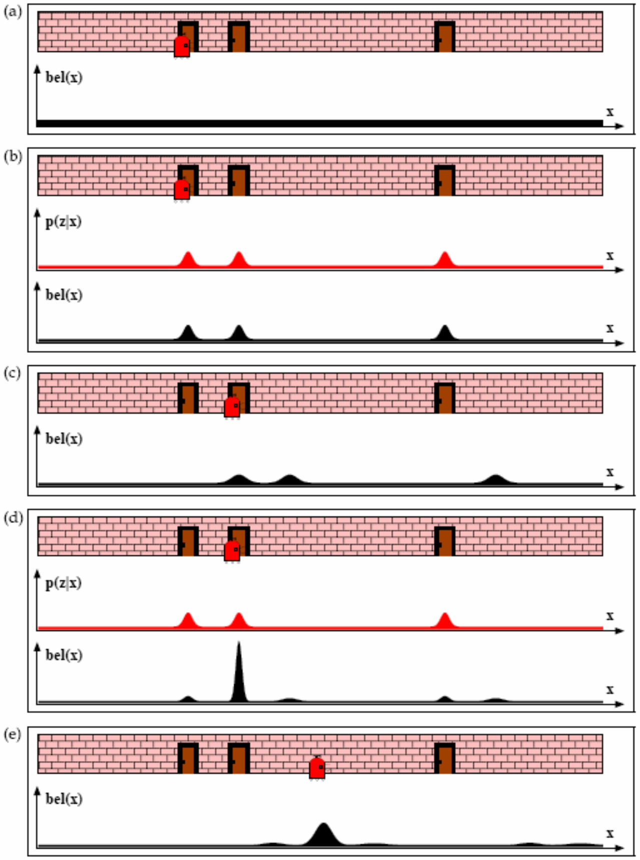
**Hybrids (since 90's)**
- model-based at higher levels
- reactive at lower levels

**Probabilistic Robotics (since mid-90's)**
- seamless integration of models and sensing
- inaccurate models, inaccurate sensors

# The big picture of Probabilistic Robotics

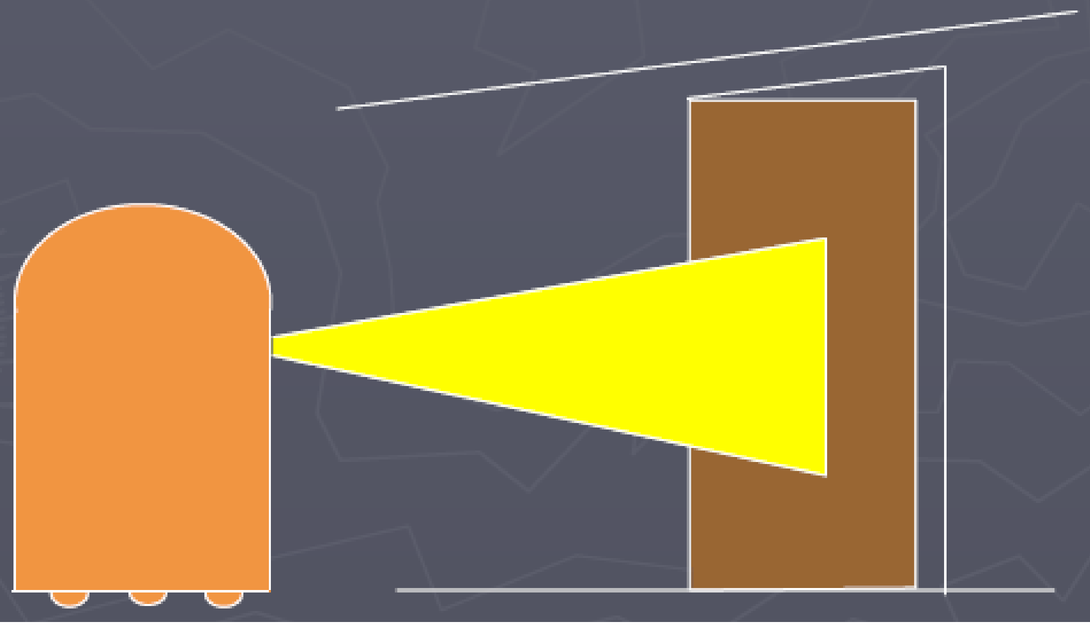
- Instead of relying on a single 'best guess', maintain information of probability distributions over a whole space of guesses.





# Simple example of conditonial probability

- Suppose a robot obtains measurement z
- What is *P(doorOpen|z)?*



# Reverse the evidence: Bayesian Reasoning

- $P(open|z)$ is diagnostic.
- $P(z|open)$ is causal.
- Often causal knowledge is easier to obtain.
- Bayes rule allows us to use causal knowledge for diagnostics:

$$P(open \mid z) = \frac{P(z \mid open)P(open)}{P(z)}$$

# Bayes Formula

$$P(x,y) = P(x \mid y)P(y) = P(y \mid x)P(x)$$

$$\Rightarrow$$

$$\boxed{P(x \mid y) = \frac{P(y \mid x)\; P(x)}{P(y)}}$$

$$\sim$$

$$\boxed{\text{posterior} = \frac{\text{likelihood} \cdot \text{prior}}{\text{evidence}}}$$

# Thomas Bayes (1702 –1761)

“Now that the errors arising from the imperfection of the instruments & the organs of sense shou’d be reduced to nothing or next to nothing only by multiplying the number of observations seems to me extremely incredible.

On the contrary the more observations you make with an imperfect instrument the more certain it seems to be that the error in your conclusion will be proportional to the imperfection of the instrument made use of. “

# Bayes’ imperfect instrument

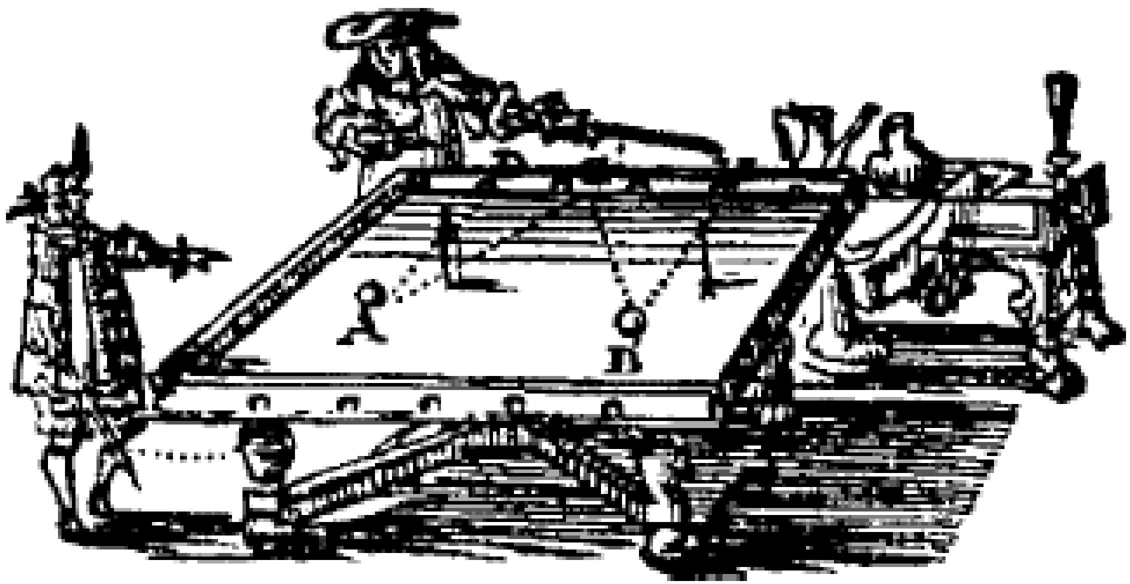


FIG. 15. *An early billiard table without pockets.*

# Modern Example

- $P(z|open) = 0.6$ $\qquad$ $P(z|\neg open) = 0.3$
- $P(open) = P(\neg open) = 0.5$

$$P(open \mid z) = \frac{P(z \mid open)P(open)}{P(z \mid open)\,p(open) + P(z \mid \neg open)\,p(\neg open)}$$

$$P(open \mid z) = \frac{0.6 \cdot 0.5}{0.6 \cdot 0.5 + 0.3 \cdot 0.5} = \frac{2}{3} = 0.67$$

- observation *z* raises the probability that the door is open.

# Combining Evidence

- Suppose our robot obtains another observation $z_2$.
- How can we integrate this new information?
- More generally, how can we estimate $P(x| z_1...z_n)$?

# Example: Second Measurement

- $P(z_2|open) = 0.5$ $\quad$ $P(z_2|\neg open) = 0.6$
- $P(open|z_1)=2/3$

$$P(open \mid z_2, z_1) = \frac{P(z_2 \mid open)\, P(open \mid z_1)}{P(z_2 \mid open)\, P(open \mid z_1) + P(z_2 \mid \neg open)\, P(\neg open \mid z_1)}$$

$$= \frac{\frac{1}{2}\cdot\frac{2}{3}}{\frac{1}{2}\cdot\frac{2}{3}+\frac{3}{5}\cdot\frac{1}{3}} = \frac{5}{8} = 0.625$$

- observation $z_2$ lowers the probability that the door is open.

# Recursive Bayesian Updating

$$P(x \mid z_1,\ldots,z_n) = \frac{P(z_n \mid x, z_1,\ldots,z_{n-1})\, P(x \mid z_1,\ldots,z_{n-1})}{P(z_n \mid z_1,\ldots,z_{n-1})}$$

**Markov assumption**: $z_n$ is independent of $z_1,\ldots,z_{n-1}$ if we know $x$.

$$P(x \mid z_1,\ldots,z_n) = \frac{P(z_n \mid x)\, P(x \mid z_1,\ldots,z_{n-1})}{P(z_n \mid z_1,\ldots,z_{n-1})}$$

$$= \eta\, P(z_n \mid x)\, P(x \mid z_1,\ldots,z_{n-1})$$

$$= \eta_{1\ldots n} \prod_{i=1\ldots n} P(z_i \mid x)\, P(x)$$

# Complete state

- State $x_t$ will be called *complete* if it is the best predictor of the future
- I.e., no additional information would help us predict the future more accurately

# Typical Actions

- The robot **turns its wheels** to move
- The robot **uses its manipulator** to grasp an object
- Plants grow over **time**…

- Actions are **never carried out with absolute certainty**.
- In contrast to measurements, **actions generally increase the uncertainty**.

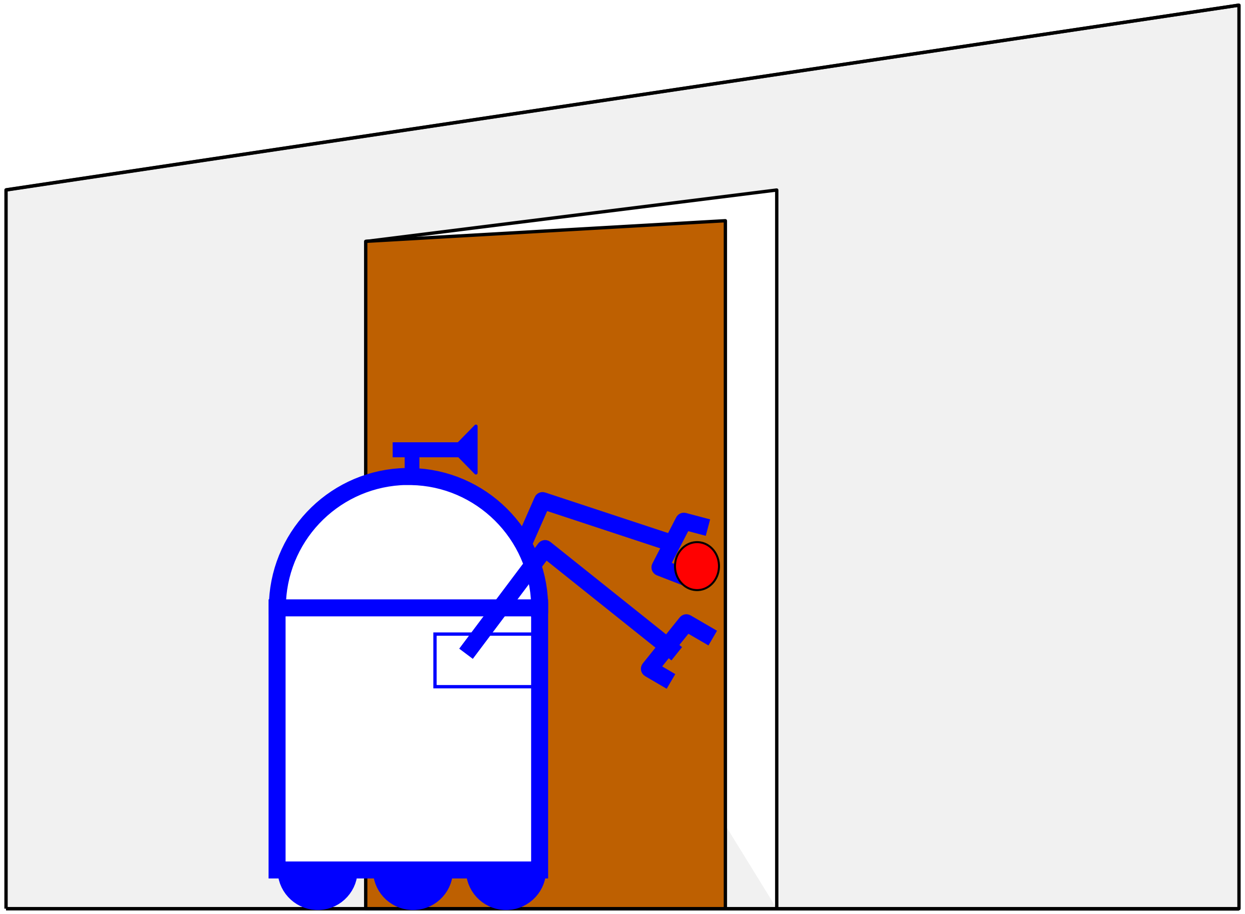
# Modeling Actions

- To incorporate the outcome of an action *u* into the current "belief", we use the state transition function

$$P(x|u,x')$$

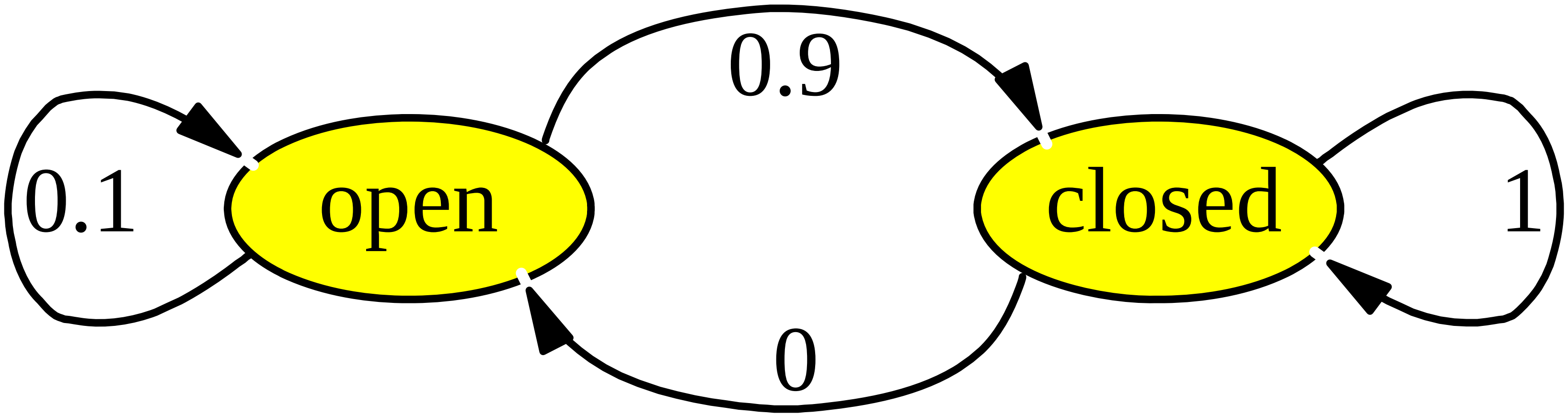
- This term specifies the probability distribution function that **executing *u* changes the state from *x'* to *x***.

# Example: Closing the door



# State Transitions

$P(x|u,x')$ for $u$ = "close door":




If the door is open, the action "close door" succeeds in 90% of all cases.

# Integrating the Outcome of Actions

Continuous case:

$$P(x \mid u) = \int P(x \mid u, x') P(x') dx'$$

Discrete case:

$$P(x \mid u) = \sum P(x \mid u, x') P(x')$$

# Example: The Resulting Belief

$$P(closed \mid u) = \sum P(closed \mid u, x')P(x')$$

$$= P(closed \mid u, open)P(open)$$

$$+ P(closed \mid u, closed)P(closed)$$

$$= \frac{9}{10} * \frac{5}{8} + \frac{1}{1} * \frac{3}{8} = \frac{15}{16}$$

$$P(open \mid u) = \sum P(open \mid u, x')P(x')$$

$$= P(open \mid u, open)P(open)$$

$$+ P(open \mid u, closed)P(closed)$$

$$= \frac{1}{10} * \frac{5}{8} + \frac{0}{1} * \frac{3}{8} = \frac{1}{16}$$

$$= 1 - P(closed \mid u)$$

# Bayes Filters: Framework

- **Given:**
  - Stream of observations *z* and action data *u:*

$$d_t = \{u_1, z_1 \ldots, u_t, z_t\}$$

  - Sensor model $P(z|x)$.
  - Action model $P(x|u,x')$.
  - Prior probability of the system state $P(x)$.
- **Wanted:**
  - Estimate of the state *X* of a dynamical system.
  - The posterior of the state is also called **Belief**:

$$Bel(x_t) = P(x_t \mid u_1, z_1 \ldots, u_t, z_t)$$

# Summary

- Bayes rule allows us to compute probabilities that are hard to assess otherwise.
- Under the Markov assumption, recursive Bayesian updating can be used to efficiently combine evidence.
- Bayes filters are a probabilistic tool for estimating the state of dynamic systems.

# Homework

- This Thursday: Assignment 2.8.1