Probabilistic Robotics, BAIPR6, Spring 2012

Reexam, Monday February 27th, 15:00 - 17:00, room D1.115

Arnoud Visser

February 27, 2012

Question 1

A student takes a multiple-choice examination where each question has exactly m possible answers. Assume that a student knows the correct answer to a proportion p of all the questions; if he does not know the correct answer, he makes a random guess.

You, as teaching assistant, have to grade this exam. You observe that question two is correctly answered ($z_2 = true$) by this student. What was the probability that the student was guessing based on this observation? Derive the formula for the conditional probability and calculate the actual percentage for $p = 0.5$ and $m = 4$.

Question 2

The *extended Kalman filter localization* algorithm in section 7.4 of the textbook depends on a multivariate Gaussian representation of uncertainty in the motion and measurement model. Noise enters the equations through the addition of a (hopefully) small factor. It is important to understand that this is always an approximation: real systems never experience zero-mean white Gaussian noise. For an *extended Kalman filter* this approximation is made with an first-order Taylor expansion. For each of the noise sources below, briefly describe how the zero-mean white Gaussian noise assumption fails for a *classical Kalman filter*, and to what extend the EKF approximation solves this problem.

- odometry error in a differentially steered wheeled robot due to a mismatch in wheel size
- odometry error in a wheeled robot due to wheel slippage
- sonar errors due to multipath reflections
- temperature dependent drift in a rate gyro

Question 3

Consider a world with only four possible robot locations: $X = x_1, x_2, x_3, x_4$. Consider a Monte Carlo localization algorithm which may use N samples among this locations. Initially, the samples are uniformly distributed over those locations. As usual, it is perfectly acceptable if there are less particles as locations. Let Z be a Boolean sensor variable characterized by the following probabilities:

$$\begin{array}{ll} p(z|x_1) = 0.8 & p(\neg z|x_1) = 0.2 \\ p(z|x_2) = 0.4 & p(\neg z|x_2) = 0.6 \\ p(z|x_3) = 0.1 & p(\neg z|x_3) = 0.9 \\ p(z|x_4) = 0.1 & p(\neg z|x_4) = 0.9 \end{array}$$

Monte Carlo uses these probabilities to generate particle weights, which are subsequently normalized and used in the resampling process. For simplicity, let us assume we only generate one new sample in the resampling process, regardless of N. This sample might correspond to any of the four locations in X. Thus, the sampling process defines a probability distribution over X. With $N = \infty$ this distribution is equal to true posterior.

a) Calculate the evidence probability.

Calculate both $P(Z = z)$ and $P(Z = \neg z)$ based on the prior uniform distribution.

b) Calculate the true posterior.

Based on the prior uniform distribution, calculate the conditional probabilty $p(X|z)$ for each of the locations $X = x_1, x_2, x_3, x_4$ and normalize this to the true posterior.

c) Calculate the propability that a particle is resampled

Assume that you use only two particles $N = 2$. There are $2^4 = 16$ possible combinations possible. The following table contains values which could be used to calculate the resampling probability.

number	samples	probability of sample set	$p(z\|s)$ for each sample s	weights for each sample s	probability of resampling for each location x_i
1	$x_1\ x_1$	$\frac{1}{16}$	$\frac{4}{5}\ \ \frac{4}{5}$	$\frac{1}{2}\ \ \frac{1}{2}$	$\frac{1}{16}\ \ 0\ \ 0\ \ 0$
2	$x_1\ x_2$	$\frac{1}{16}$	$..\ \ \frac{2}{5}$	$\frac{6}{9}\ \ \frac{3}{9}$	$\frac{1}{24}\ \ \frac{1}{48}\ \ 0\ \ 0$
3	$x_1\ x_3$	$\frac{1}{16}$	$..\ \ \frac{1}{10}$	$\frac{8}{9}\ \ ..$	$\ldots\ \ 0\ \ \frac{1}{144}\ \ 0$
4	$x_1\ x_4$	$\frac{1}{16}$	$..\ \ \frac{1}{10}$	$..\ \ ..$	$\ldots\ \ 0\ \ 0\ \ \ldots$
5	$x_2\ x_1$	$\frac{1}{16}$	$\frac{2}{5}\ \ ..$	$..\ \ ..$	$\ldots\ \ \ldots\ \ \ldots\ \ 0$
6	$x_2\ x_2$	$\frac{1}{16}$	$..\ \ ..$	$..\ \ ..$	$\ldots\ \ \frac{1}{16}\ \ \ldots\ \ \ldots$
7	$x_2\ x_3$	$\frac{1}{16}$	$..\ \ ..$	$\frac{4}{5}\ \ ..$	$\ldots\ \ \ldots\ \ \frac{1}{80}\ \ \ldots$
8	$x_2\ x_4$	$\frac{1}{16}$	$..\ \ ..$	$..\ \ ..$	$\ldots\ \ \ldots\ \ \ldots\ \ \ldots$
9	$x_3\ x_1$	..	$\frac{1}{10}\ \ ..$	$\frac{1}{9}\ \ ..$	$\ldots\ \ 0\ \ \ldots\ \ 0$
10	$x_3\ x_2$	..	$..\ \ ..$	$..\ \ ..$	$\ldots\ \ \ldots\ \ \ldots\ \ \ldots$
11	$x_3\ x_3$	..	$..\ \ ..$	$..\ \ ..$	$\ldots\ \ \ldots\ \ \ldots\ \ \ldots$
12	$x_3\ x_4$	..	$\frac{1}{10}\ \ ..$	$..\ \ \frac{1}{2}$	$\ldots\ \ \ldots\ \ \ldots\ \ \frac{1}{32}$
13	$x_4\ x_1$	..	$..\ \ ..$	$..\ \ ..$	$\ldots\ \ \ldots\ \ \ldots\ \ \ldots$
14	$x_4\ x_2$	..	$..\ \ ..$	$\frac{1}{5}\ \ ..$	$\ldots\ \ \frac{1}{20}\ \ \ldots\ \ \ldots$
15	$x_4\ x_3$	..	$..\ \ \frac{1}{10}$	$..\ \ ..$	$\ldots\ \ \ldots\ \ \ldots\ \ \ldots$
16	$x_4\ x_4$	..	$\frac{1}{10}\ \ \frac{1}{10}$	$..\ \ \frac{1}{2}$	$0\ \ 0\ \ 0\ \ \frac{1}{16}$
					$\ldots\ \ \frac{73}{240}\ \ \ldots\ \ \ldots$

d) Is a particle filter *biased*?

Compare the answer of **b)** and **c)**. Are the two probability distributions the same? In which direction a particle filter with 2 particles is *biased*? Explain this difference.

Question 4

Consider a robot that operates in a triangular environment with three types of landmarks, as illustrated in Figure 1:

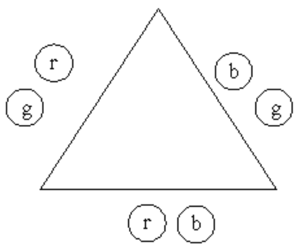

Figure 1: A triangular environment

Each arc is a location and each location has two different landmarks, each with a different color. Let us assume that in every round the robot can only inquire about the presence of one landmark type: either the one labeled ”r”, the one labeled ”g” or the one labeled ”b”.

a) Clockwise

Suppose that robot first fires the detector for ”b” landmarks and moves clockwise to the next arc. What would be the optimal landmark detector to use next?

b) Counterclockwise

How would the answer change if the robot moved counterclockwise to the next arc?

Success!

Acknowledgements

One question is based on an exercise from the Probability Theory refresher in Falko Bause’s book[1]. Another question is based on an exercise from the Robotics chapter of Russell and Norvig [2].

References

[1] F. Bause and P. S. Kritzinger, *Stochastic Petri nets - an introduction to the theory (2nd ed.)*, 2002.

[2] S. J. Russell and P. Norvig, *Artificial Intelligence: A Modern Approach*, Prentice Hall, 3rd edition, 2009.