Probabilistic Robotics (BAIPR6, Autumn 2008)
Examination: Basics & Markov localization

Assigned: Week 43, Due: Week 46 (Monday November 10th, 15:00)

Arnoud Visser

October 21, 2008

The solutions have to be mailed individually to Arnoud Visser `<A.Visser@uva.nl>`.

Question 1

Assume the following 1-D lineair dynamic system, with a simple motion model:

$$x_t = x_{t-1} + u_t + \epsilon_t \tag{1}$$

and a simple measurement model:

$$y_t = x_t + \delta_t \tag{2}$$

The terms ϵ_t and δ_t represent respectively the control and measurement error, a random number from a Gaussian distribution $\mathcal{N}(x; 0, Q_t)$ and $\mathcal{N}(y; 0, R_t)$. For the moment you can assume that the variance $Q_t = 0$ and $R_t = 1$, which means that you have perfect control over the dynamic system (ϵ_t can be ignored). For all timesteps, the same input is given ($u_t = 0.5$). The initial estimate is represented with a Gaussian distribution $\mathcal{N}(x; \mu_0, \Sigma_0)$ with $\mu_0 = 5$ and $\Sigma_0 = 10$.

You receive the following measurements ($y_1 = 0.0, y_2 = 2.1, y_3 = 5.6$).

(a) Use the measurements (y_1, y_2, y_3) to estimate (μ_1, μ_2, μ_3). For this lineair system you can use a traditional Kalman Filter, as described in section 3.2 of the book. This will be a two step approach, a prediction and an update step. The result of the prediction step will be a Gaussian distribution $\mathcal{N}(x; \overline{\mu}_0, \overline{\Sigma}_t)$. In the update step you can shift and narrow this distribution to $\mathcal{N}(x; \mu_t, \Sigma_t)$ making use of the measurements and the following precalculated Kalman gain ($K_1 = \frac{10}{11}, K_2 = \frac{10}{21}, K_3 = \frac{10}{31}, K_4 = \frac{10}{41}, K_5 = \frac{10}{51}$).

(b) Explain why the Kalman Gain decreases for every time step.

(c) Lets drop the assumption of perfect control, and reintroduce the control noise ϵ_t modelled with a Gaussian distribution $\mathcal{N}(x; 0, 1)$. Recalculate $(K_1, K_2, K_3, K_4, K_5)$ for the given variance $Q_t = 1$. Explain the observed pattern in the time series of the Kalman Gain K_t.

(d) Make a new estimate of (μ_1, μ_2, μ_3) based on the recalculated Kalman Gain K_t.

Question 2

Consider a wheeled robot which moves over a flat surface. Each wheel has an y-axis. When a rolling motion occurs, all y-axis overlap at a point; the instantaneous Center of Curvature. Consider the case that the Instantaneous Center of Curvature is outside the robot, and the robot moves from point $(0, 0)$ to (x, y) (see figure 1).

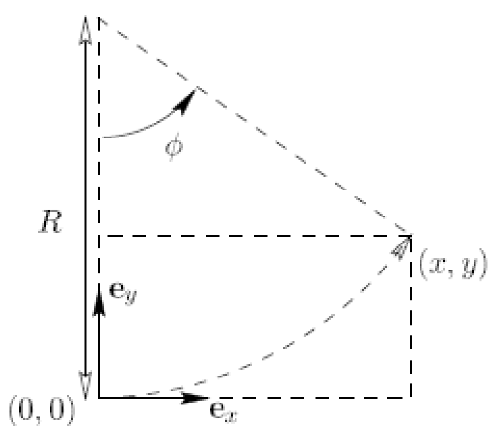

Figure 1: Turning to point (x, y)

A natural way to represent the movement as an circular movement with a radius R an the sector angle ϕ. (x, y) is a point on the circle, which means

$$\begin{cases} x = R\sin\phi \\ y = R(1-\cos\phi) \end{cases} \iff \begin{cases} R = \dfrac{x^2+y^2}{2y} \\ \phi = atan(\dfrac{2xy}{x^2-y^2}) \end{cases}$$

This representation has disadvantage that for small y (straight ahead!), a small change in (x, y) may cause a big change in parameter R. You can verify this with by computing the Jacobian; you should get:

$$\begin{pmatrix} dR \\ d\phi \end{pmatrix} = \begin{pmatrix} \frac{x}{y} & \frac{y^2-x^2}{2y^2} \\ \frac{-2y}{x^2+y^2} & \frac{2x}{x^2+y^2} \end{pmatrix} \begin{pmatrix} dx \\ dy \end{pmatrix}$$

For this reason it is much better to characterize the path by the *curvature* $\kappa \equiv 1/R$, which changes smoothly around the forward direction.

Now, compute the Jacobian for the (κ, ϕ) representation and show that it changes more smoothly.

Question 3

Solve exercise 6.10.1 from the Probabilistic Robotics book.