Probabilistic Robotics
PRRO6, Fall 2017
Book Assignment 3.8.1
Assigned: Monday September 11;
Due: Tuesday September 12, 13:00 in the afternoon

September 11, 2017

In this and the following exercise, you are asked to design a Kalman filter for a simple dynamical system: a car with linear dynamics moving in a linear environment. Assume $\Delta t = 1$ for simplicity. The position of the car at time t is given by x_t. Its velocity is $\dot{x}_t$, and its its acceleration is $\ddot{x}_t$. Suppose the acceleration is set randomly at each point in time, according to a Gaussian with zero mean and covariance $\sigma^2 = 1$.

(a) What is a minimal state vector for the Kalman filter (so that the resulting system is Markovian.

(b) For your state vector, design the state transition probability $p(x_t|u_t, x_{x-1})$. Hint: this transition function will possess linear matrices A and B and a noise covariance R (c.f., Equation (3.4) and Table 3.1).

(c) Implement the state prediction step of the Kalman filter. Assuming we know at time $t = 0, x_0 = \dot{x}_0 = \ddot{x}_0 = 0$. Compute the state distributions for times $t = 1, 2, \ldots, 5$.

(d) For each value of t, plot the joint posterior over x and $\dot{x}$ in a diagram, where x is the horizontal and $\dot{x}$ is the vertical axis. For each posterior, you are asked to plot an *uncertainty ellips*, which is the ellipse of points that are one standard deviation away from the mean. Hint: If you do not have access to a mathematics library, you can create those ellipses by analyzing the eigenvalues of the covariance matrix.

(e) What will happen to the correlation between x_t and $\dot{x}_t$ as $t \uparrow \infty$?

Hand-In

This assignment doesn't have to be handin, it will be discussed in class.