Probabilistic Robotics
PRRO6Y, Fall 2017
Book Assignment 2.8.1
Assigned: Tuesday September 5;
Due: Thursday September 7, 13:00 in the afternoon

September 6, 2017

A robot uses a range sensor that can measure ranges from $0m$ to $3m$. For simplicity, assume that actual ranges are distributed uniformly in this interval. Unfortunatelly, the sensor can be faulty. When the sensor is faulty, it constantly outputs a range below $1m$, regardless of the actual range in the sensor's measurement cone. We know that the prior probability for a sensor to be faulty is $p = 0.01$.

Suppose the robot queried its sensor N times, and every single time the measurement value is below $1m$. What is the posterior probability of a sensor fault, for $N = 1, 2, \ldots, 10$? Formulate the corresponding probabilistic model.

Hint: Evidence is build up when the sensor is queried, so the normalizer in Bayes rule can't be ignored.

Answer

Let's call the prior probability that the sensor is faulty on the first timestep $P_{t=1}(f)$, which has the given value 0.01. The probability that the next time ($t = 2$) the observation $Z < 1m$ is measured, is depended on two cases. The first case is that the sensor is faulty ($P(f)$), with the consequence that with a probability of 1.0 that $Z < 1m$ is measured. The other case is that the sensor is not faulty (which occurs $P(\bar{f}) = (1 - P(f))$. The chance that then $Z < 1m$ is measured is $1/3$, because the actual ranges are distributed uniformly in the interval $0m$ to $3m$. The result is $P_{t=2}(Z < 1m) = P_{t=1}(f) * 1.0 + P_{t=1}(\bar{f}) * 0.333$. This is the change that $P(Z < 1m)$. Yet, we are interested in $P(f|z)$. We could now apply Bayes rule:

$$P(f|z) = \frac{P(z|f)P(f)}{P(z)} \quad (1)$$

Initially the chance that the sensor is faulty is the prior $P_0(f) = 0.01$, there after it is the result of the previous calculation $P_{i-1}(f) = P(f|z_1, \cdots, z_{i-1})$:

$$P_i(f|z) = \frac{P(z|f)P_{i-1}(f)}{P(z)} \quad (2)$$

$$= \frac{P(z|f)P_{i-1}(f)}{P(z|f)P_{i-1}(f) + P(z|\bar{f})P_{i-1}(\bar{f})} \quad (3)$$

$$= \frac{1.0 \cdot P_{i-1}(f)}{1.0 \cdot P_{i-1}(f) + 0.333 \cdot (1 - P_{i-1}(f))} \quad (4)$$

Now we can calulate $P(f|z)$ for every timestep:

```
Pf(1) = 0.01;

for i = 2:11
    Pz(i) = Pf(i-1) + 0.333 * (1-Pf(i-1))
    Pf(i) = Pf(i-1) / Pz(i);
end
```

The result is that after 10 observations $Z < 1m$, one is quite sure that the sensor is faulty. (with a convidence of 99.5%), as can be seen in the following plot (generated with the following Matlab commands):

```
plot(1:11,[Pf(:)])

title( 'Posterior probability sensor fault P(f|z_{t ... 1}<1|)')
xlabel ('measurement z_t')
ylabel ('Conditional probability')
```

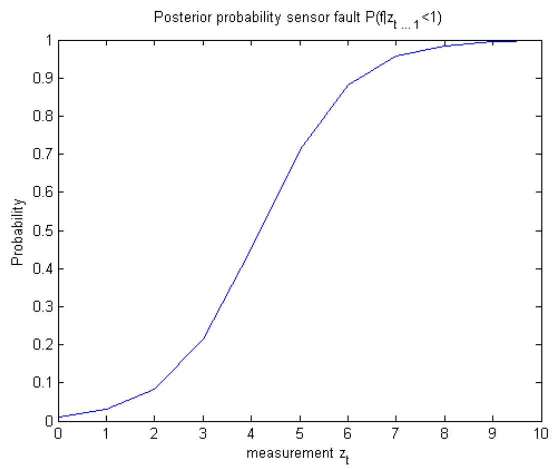

Figure 1: The probability of a sensor fault as function a repeated observation $Z < 1m$