University of Amsterdam

Faculty of Science

Exam

Probabilistic Robotics

Master Artifical Intelligence

Final Exam
Date: October 24, 2018
Time: 13.00-16.00

Number of pages: 6 (including front page)
Number of questions: 6
Maximum number of points to earn: 10

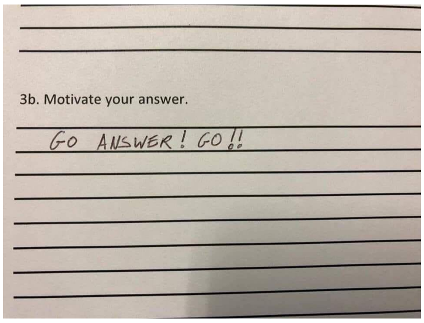
BEFORE YOU START

- Please **wait** until you are instructed to open the booklet.
- Check if your version of the exam is complete.
- Write down **your name, student ID number**, and if applicable the **version number** on **each sheet** that you hand in. Also **number the pages**.
- Your **mobile phone** has to be switched off and in the coat or bag. Your **computer or e-reader** has to be in flight-mode. Your **coat and bag** must be under your table.
- **Tools allowed**: scrap paper, calculator, ruler, notes on a single A4, (electronic) book.

PRACTICAL MATTERS

- The first 30 minutes and the last 15 minutes you are not allowed to leave the room, not even to visit the toilet.
- You are obliged to identify yourself at the request of the examiner (or his representative) with a proof of your enrollment or a valid ID.
- During the examination it is not permitted to visit the toilet, unless the proctor gives permission to do so.
- 15 minutes before the end, you will be warned that the time to hand in is approaching.
- Please fill out the evaluation form at the end of the exam.

Good luck!


3b. Motivate your answer.
GO ANSWER! GO!!

Probabilistic Robotics, PRR06, Fall 2018

Retake Exam, Wednesday October 24th, 13:00 - 16:00, room G3.02

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November 18, 2018

Question 1

Explain in a few sentences all of the components of the EKF, i. e., μ_t, Σ_t, g, G_t, h, H_t, Q_t, R_t, K_t and why they are needed.

Answer

The Extended Kalman Filter depends on the definition of the following components:

μ_t: This is the estimate for the mean of the distribution of the state x at time t computed by the EKF. In mobile robotics applications the state x typically includes the pose of the robot, the path of the robot and/or the location of the landmarks.

Σ_t: This is the estimate for the covariance matrix of the distribution of the state x at time t computed by the EKF.

$g(u, x)$: This function estimates the current state x_t from the control u and previous state variables x_{t-1}. In mobile robotics applications this function often defines the motion model of the system.

G_t: The Jacobian matrix of g evaluated at the previous estimate of the state namely:

$$G_t = \frac{\partial g}{\partial x}\Big|_{x=\mu_{t-1}}$$

$h(x)$: This function estimates the measurement z_t from the current state variables x_t. In mobile robotics applications this function defines the measurement model of the system, such as range and bearing of a landmark.

H_t: The Jacobian matrix of h evaluated at the current estimate of the state from prediction only, namely:

$$H_t = \frac{\partial h}{\partial x}\Big|_{x=\bar{\mu}_t}$$

Q_t: This is the covariance matrix (assumed to be known) of the zero-mean Gaussian error that corrupts the prediction of the state, i.e., the present state variables are given by the prediction of the function g plus some noise. This is necessary as in reality every prediction model is subject to some additional noise (e.g., imperfections in the terrain, wheel slippage, etc.).

R_t: This is the covariance matrix (assumed to be known) of the zero-mean Gaussian error that corrupts the measurements, i.e., the measurement is given by the function h plus some noise. This is necessary as any measurement is subject to some noise (e.g., accuracy of the sensor, systematic errors, etc.).

K_t: This is the Kalman Gain, which indicates the balance between the predictions from the motion model g and the corrections from the measurement model h, which together gives the new estimate on the distribution of the state x. The Kalman Gain is time dependent, because it is a function of the predicted covariance at time t.

Question 2

A robot is moving along the middle of a corridor with a given accurate map, as depicted in figure 1.

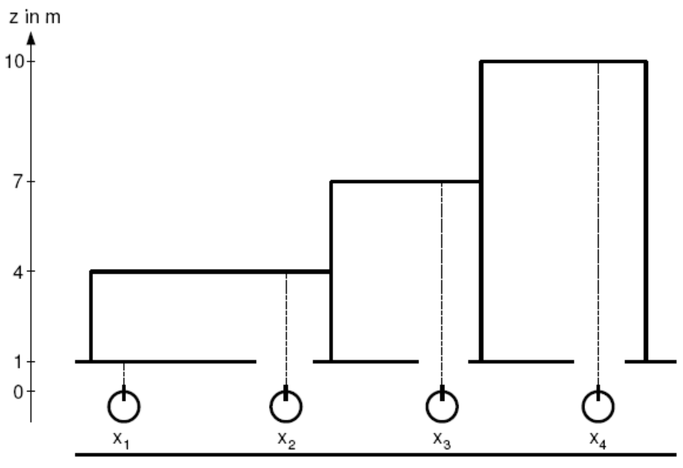


Figure 1: Accurate map of a corridor with three rooms

At some of the given locations x_i the robot takes a measurement of the distance z_k, using a laser beam. Every measurement is corrupted only with additive Gaussian noise $\mathcal{N}(\mu, \sigma)$ with $\mu = 0m$ and $\sigma = 1m$. The scanner range is $80m$. The measured distances are $z_1 = 1.1m$, $z_2 = 2.1m$, $z_3 = 8.6m$, $z_4 = 9.4m$. The correspondence between z_k and x_i is unknown.

(a) For each measurement, determine the most likely robot pose by calculating the probabilities for each position given the measurement using Bayes'rule. Assume an uniform distributed *prior*. The *evidence* term (denominator) can be neglected, but the probabilities should be scaled such that $\sum_{i=1}^{4} P(x_i|z) = 1$.

(b) The robot believes that taking measurements at the positions x_2 and x_3 is in general three times as likely as doing so at x_1 and x_4. Use this prior information to recalculate the probabilities of (a).

(c) Because people are present in the corridor, a faulty measurement of $z = 1m$ can occur in 33% of the cases, no matter the actual distance. How does this change the results of (a) and (b).

Answer

(a) The most likely position for a given measured distance z, can be calculated by

$$\underset{i}{\operatorname{argmax}}(P(x_i|z) = \underset{i}{\operatorname{argmax}}(\frac{P(z|x_i)P(x_i)}{P(z)})$$

because we are calculating the maximum value, and $P(z)$ is constant, we can ignore this value. Because there is an uniform distributed prior, $P(x_i)$ will be $\frac{1}{4}$ for all i. So, the most likely position can be calculated with

$$\underset{i}{\operatorname{argmax}}(\tfrac{1}{4}P(z|x_i))$$

With the given map, one would expect at the given positions $(x_1, \cdots, x_4)$ the observations $h(x_i) = (\bar{z}_1 = 1m, \bar{z}_2 = 4m, \bar{z}_2 = 7m, \bar{z}_4 = 10m)$. $P(z|x_i)$ can be calculated from the difference between the observed measurement z_k and the expected measurement z_i at location z_i:

$$P(z_k|x_i) = \mathcal{N}(z_k - h(x_i)|\sigma) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(z_k - \bar{z}_i)^2}{2}}$$

taking into account the Gaussian noise $\mathcal{N}(\mu, \sigma)$ with $\sigma = 1m$. So, for each combination of z_k and x_i the following calculation can be made:

$$\begin{aligned}
p(x_1|z_1) &= \tfrac{1}{4}P(z_1|x_1) = \tfrac{1}{4}\tfrac{1}{\sqrt{2\pi}}e^{-\frac{(1.1-1)^2}{2}} = 0.0992 && \rightarrow && P(x_1|z_1) \simeq 0.99 \quad \leftarrow \\
p(x_2|z_1) &= \tfrac{1}{4}P(z_1|x_2) = \tfrac{1}{4}\tfrac{1}{\sqrt{2\pi}}e^{-\frac{(1.1-4)^2}{2}} = 0.0015 && \rightarrow && P(x_2|z_1) \simeq 0.01 \\
p(x_3|z_1) &= \tfrac{1}{4}P(z_1|x_3) = \tfrac{1}{4}\tfrac{1}{\sqrt{2\pi}}e^{-\frac{(1.1-7)^2}{2}} = 2.8 \times 10^{-9} && \rightarrow && P(x_3|z_1) \simeq 0.00 \\
p(x_4|z_1) &= \tfrac{1}{4}P(z_1|x_4) = \tfrac{1}{4}\tfrac{1}{\sqrt{2\pi}}e^{-\frac{(1.1-10)^2}{2}} = 6.3 \times 10^{-19} && \rightarrow && P(x_4|z_1) \simeq 0.00
\end{aligned}$$

$$\begin{aligned}
p(x_1|z_2) &= \tfrac{1}{4}P(z_2|x_1) = \tfrac{1}{4}\tfrac{1}{\sqrt{2\pi}}e^{-\frac{(2.1-1)^2}{2}} = 0.0545 && \rightarrow && P(x_1|z_2) \simeq 0.77 \quad \leftarrow \\
p(x_2|z_2) &= \tfrac{1}{4}P(z_2|x_2) = \tfrac{1}{4}\tfrac{1}{\sqrt{2\pi}}e^{-\frac{(2.1-4)^2}{2}} = 0.0164 && \rightarrow && P(x_2|z_2) \simeq 0.23 \\
p(x_3|z_2) &= \tfrac{1}{4}P(z_2|x_3) = \tfrac{1}{4}\tfrac{1}{\sqrt{2\pi}}e^{-\frac{(2.1-7)^2}{2}} = 6.1 \times 10^{-7} && \rightarrow && P(x_3|z_2) \simeq 0.00 \\
p(x_4|z_2) &= \tfrac{1}{4}P(z_2|x_4) = \tfrac{1}{4}\tfrac{1}{\sqrt{2\pi}}e^{-\frac{(2.1-10)^2}{2}} = 2.8 \times 10^{-15} && \rightarrow && P(x_4|z_2) \simeq 0.00
\end{aligned}$$

$$\begin{aligned}
p(x_1|z_3) &= \tfrac{1}{4}P(z_3|x_1) = \tfrac{1}{4}\tfrac{1}{\sqrt{2\pi}}e^{-\frac{(8.6-1)^2}{2}} = 2.9 \times 10^{-14} && \rightarrow && P(x_1|z_3) \simeq 0.00 \\
p(x_2|z_3) &= \tfrac{1}{4}P(z_3|x_2) = \tfrac{1}{4}\tfrac{1}{\sqrt{2\pi}}e^{-\frac{(8.6-4)^2}{2}} = 2.5 \times 10^{-6} && \rightarrow && P(x_2|z_3) \simeq 0.00 \\
p(x_3|z_3) &= \tfrac{1}{4}P(z_3|x_3) = \tfrac{1}{4}\tfrac{1}{\sqrt{2\pi}}e^{-\frac{(8.6-7)^2}{2}} = 0.0277 && \rightarrow && P(x_3|z_3) \simeq 0.43 \\
p(x_4|z_3) &= \tfrac{1}{4}P(z_3|x_4) = \tfrac{1}{4}\tfrac{1}{\sqrt{2\pi}}e^{-\frac{(8.6-10)^2}{2}} = 0.0374 && \rightarrow && P(x_4|z_3) \simeq 0.58 \quad \leftarrow
\end{aligned}$$

$$\begin{aligned}
p(x_1|z_4) &= \tfrac{1}{4}P(z_4|x_1) = \tfrac{1}{4}\tfrac{1}{\sqrt{2\pi}}e^{-\frac{(9.4-1)^2}{2}} = 4.8 \times 10^{-17} && \rightarrow && P(x_1|z_4) \simeq 0.00 \\
p(x_2|z_4) &= \tfrac{1}{4}P(z_4|x_2) = \tfrac{1}{4}\tfrac{1}{\sqrt{2\pi}}e^{-\frac{(9.4-4)^2}{2}} = 4.6 \times 10^{-8} && \rightarrow && P(x_2|z_4) \simeq 0.00 \\
p(x_3|z_4) &= \tfrac{1}{4}P(z_4|x_3) = \tfrac{1}{4}\tfrac{1}{\sqrt{2\pi}}e^{-\frac{(9.4-7)^2}{2}} = 0.0056 && \rightarrow && P(x_3|z_4) \simeq 0.06 \\
p(x_4|z_4) &= \tfrac{1}{4}P(z_4|x_4) = \tfrac{1}{4}\tfrac{1}{\sqrt{2\pi}}e^{-\frac{(9.4-10)^2}{2}} = 0.0833 && \rightarrow && P(x_4|z_4) \simeq 0.94 \quad \leftarrow
\end{aligned}$$

The indication $\leftarrow$ in the calculation above indicates the maximum for each location, so the most likely location for observations ($z_1 = 1.1m, z_2 = 2.1m$) is identified as x_1 and for ($z_3 = 8.6m, z_4 = 9.4m$) is identified as x_4.

(b) The robot modifies its prior belief, which results in different values for $P(x_i)$. This new priors are respectively $P(x_1) = P(x_4) = \frac{1}{8}$ and $P(x_2) = P(x_3) = \frac{3}{8}$. Although $P(z_k|x_i)$ are the same numbers as calculated in **(a)**, the likelihoods $p(x_i|z_k)$ and the normalization have to be recalculated:

$$\begin{aligned}
p(x_1|z_1) &= \tfrac{1}{8}P(z_1|x_1) = \tfrac{1}{8}\tfrac{1}{\sqrt{2\pi}}e^{-\frac{(1.1-1)^2}{2}} = 0.0496 && \rightarrow && P(x_1|z_1) \simeq 0.96 \quad \leftarrow \\
p(x_2|z_1) &= \tfrac{3}{8}P(z_1|x_2) = \tfrac{3}{8}\tfrac{1}{\sqrt{2\pi}}e^{-\frac{(1.1-4)^2}{2}} = 0.0022 && \rightarrow && P(x_2|z_1) \simeq 0.04 \\
p(x_3|z_1) &= \tfrac{3}{8}P(z_1|x_3) = \tfrac{3}{8}\tfrac{1}{\sqrt{2\pi}}e^{-\frac{(1.1-7)^2}{2}} = 4.1 \times 10^{-9} && \rightarrow && P(x_3|z_1) \simeq 0.00 \\
p(x_4|z_1) &= \tfrac{1}{8}P(z_1|x_4) = \tfrac{1}{8}\tfrac{1}{\sqrt{2\pi}}e^{-\frac{(1.1-10)^2}{2}} = 3.1 \times 10^{-19} && \rightarrow && P(x_4|z_1) \simeq 0.00
\end{aligned}$$

$$\begin{aligned}
p(x_1|z_2) &= \tfrac{1}{8}P(z_2|x_1) = \tfrac{1}{8}\tfrac{1}{\sqrt{2\pi}}e^{-\frac{(2.1-1)^2}{2}} = 0.0272 && \rightarrow && P(x_1|z_2) \simeq 0.53 \quad \leftarrow \\
p(x_2|z_2) &= \tfrac{3}{8}P(z_2|x_2) = \tfrac{3}{8}\tfrac{1}{\sqrt{2\pi}}e^{-\frac{(2.1-4)^2}{2}} = 0.0264 && \rightarrow && P(x_2|z_2) \simeq 0.47 \\
p(x_3|z_2) &= \tfrac{3}{8}P(z_2|x_3) = \tfrac{3}{8}\tfrac{1}{\sqrt{2\pi}}e^{-\frac{(2.1-7)^2}{2}} = 9.1 \times 10^{-7} && \rightarrow && P(x_3|z_2) \simeq 0.00 \\
p(x_4|z_2) &= \tfrac{1}{8}P(z_2|x_4) = \tfrac{1}{8}\tfrac{1}{\sqrt{2\pi}}e^{-\frac{(2.1-10)^2}{2}} = 1.4 \times 10^{-15} && \rightarrow && P(x_4|z_2) \simeq 0.00
\end{aligned}$$

$$\begin{aligned}
p(x_1|z_3) &= \tfrac{1}{8}P(z_3|x_1) = \tfrac{1}{8}\tfrac{1}{\sqrt{2\pi}}e^{-\frac{(8.6-1)^2}{2}} = 1.4 \times 10^{-14} && \rightarrow && P(x_1|z_3) \simeq 0.00 \\
p(x_2|z_3) &= \tfrac{3}{8}P(z_3|x_2) = \tfrac{3}{8}\tfrac{1}{\sqrt{2\pi}}e^{-\frac{(8.6-4)^2}{2}} = 3.8 \times 10^{-6} && \rightarrow && P(x_2|z_3) \simeq 0.00 \\
p(x_3|z_3) &= \tfrac{3}{8}P(z_3|x_3) = \tfrac{3}{8}\tfrac{1}{\sqrt{2\pi}}e^{-\frac{(8.6-7)^2}{2}} = 0.0416 && \rightarrow && P(x_3|z_3) \simeq 0.69 \quad \leftarrow \\
p(x_4|z_3) &= \tfrac{1}{8}P(z_3|x_4) = \tfrac{1}{8}\tfrac{1}{\sqrt{2\pi}}e^{-\frac{(8.6-10)^2}{2}} = 0.0187 && \rightarrow && P(x_4|z_3) \simeq 0.31
\end{aligned}$$

$$
\begin{aligned}
p(x_1|z_4) &= \tfrac{1}{8}P(z_4|x_1) = \tfrac{1}{8}\tfrac{1}{\sqrt{2\pi}}e^{-\frac{(9.4-1)^2}{2}} = .48\times 10^{-17} &\rightarrow\quad P(x_1|z_4)\simeq 0.00\\
p(x_2|z_4) &= \tfrac{3}{8}P(z_4|x_2) = \tfrac{3}{8}\tfrac{1}{\sqrt{2\pi}}e^{-\frac{(9.4-4)^2}{2}} = 6.7\times 10^{-8} &\rightarrow\quad P(x_2|z_4)\simeq 0.00\\
p(x_3|z_4) &= \tfrac{3}{8}P(z_4|x_3) = \tfrac{3}{8}\tfrac{1}{\sqrt{2\pi}}e^{-\frac{(9.4-7)^2}{2}} = 0.0084 &\rightarrow\quad P(x_3|z_4)\simeq 0.17\\
p(x_4|z_4) &= \tfrac{1}{8}P(z_4|x_4) = \tfrac{1}{8}\tfrac{1}{\sqrt{2\pi}}e^{-\frac{(9.4-10)^2}{2}} = 0.0417 &\rightarrow\quad P(x_4|z_4)\simeq 0.83 \quad\leftarrow
\end{aligned}
$$

The indication $\leftarrow$ in the calculation above indicates the maximum for each location, so the best estimate has only changed for z_3: the most likely location for observations $(z_1 = 1.1m, z_2 = 2.1m)$ is still x_1, for $(z_3 = 8.6m)$ now x_3 and for $(z_4 = 9.4m)$ still x_4.

(c) In section 6.3 this type of measurement error was described as **Unexpected objects**. The measurements should now be modelled with the weighted average of two distributions: $p_{hit}(z|x_i)$ and $p_{short}(z|x_i)$. The weight z_short is given (0.33), so the weight z_{hit} is $(1 - z_{short}) = 0.67$. Together, the estimate of the posterior becomes:

$$
P(z|x_i) = \begin{pmatrix} z_{hit} \\ z_{short} \end{pmatrix}^T \cdot \begin{pmatrix} p_{hit}(z|x_i) \\ p_{short}(z|x_i) \end{pmatrix}
$$

In the case of a faulty measurement the observation $z = 1.0m$ is independent of the location. So $p_{short}(z|x_i) = \frac{1}{\sqrt{2\pi}}e^{-\frac{(z_k-1.0)^2}{2}}$. For the distribution $p_{hit}(z|x_i)$ the results of **(a)** and **(b)** can be used. So for the first case ($p(x_1|z_1)$ in question **2a**) the calculation is as follows (with for both p_{hit} and p_{short} an expected observation $\bar{z} = 1m$:

$$
\begin{aligned}
p(x_1|z_1) &= \tfrac{1}{4}P(z_1|x_1) = \tfrac{1}{4}(z_{hit}\cdot p_{hit}(z|x_1) + z_{short}\cdot p_{short}(z|x_1))\\
&= \tfrac{1}{4}(0.67\cdot \tfrac{1}{\sqrt{2\pi}}e^{-\frac{(z_i-1)^2}{2}} + 0.33\cdot \tfrac{1}{\sqrt{2\pi}}e^{-\frac{(z_i-1.0)^2}{2}})
\end{aligned}
$$

After normalization, the probabilities for each of the combination is:

$$
\begin{aligned}
&P(x_1|z_1)\simeq 0.50 \quad\leftarrow\\
&P(x_2|z_1)\simeq 0.17\\
&P(x_3|z_1)\simeq 0.16\\
&P(x_4|z_1)\simeq 0.16\\[1em]
&P(x_1|z_2)\simeq 0.39 \quad\leftarrow\\
&P(x_2|z_2)\simeq 0.25\\
&P(x_3|z_2)\simeq 0.18\\
&P(x_4|z_2)\simeq 0.18\\[1em]
&P(x_1|z_3)\simeq 0.19\\
&P(x_2|z_3)\simeq 0.19\\
&P(x_3|z_3)\simeq 0.29\\
&P(x_4|z_3)\simeq 0.33 \quad\leftarrow\\[1em]
&P(x_1|z_4)\simeq 0.19\\
&P(x_2|z_4)\simeq 0.17\\
&P(x_3|z_4)\simeq 0.19\\
&P(x_4|z_4)\simeq 0.46 \quad\leftarrow
\end{aligned}
$$

The indication $\leftarrow$ in the calculation above indicates the maximum for each location, so even with a faulty sensor the estimate for question **2a** is not changed; the most likely location for observations $(z_1 = 1.1m, z_2 = 2.1m)$ is still x_1 and for $(z_3 = 8.6m, z_4 = 9.4m)$ still x_4.

This is not the case with the priors of question **2b**, they dominate the answer now that the measurement is less certain:

$$
\begin{aligned}
P(x_1|z_1) &\simeq 0.30 \\
P(x_2|z_1) &\simeq 0.31 \quad \leftarrow \\
P(x_3|z_1) &\simeq 0.30 \\
P(x_4|z_1) &\simeq 0.10
\end{aligned}
$$

$$
\begin{aligned}
P(x_1|z_2) &\simeq 0.21 \\
P(x_2|z_2) &\simeq 0.40 \quad \leftarrow \\
P(x_3|z_2) &\simeq 0.30 \\
P(x_4|z_2) &\simeq 0.10
\end{aligned}
$$

$$
\begin{aligned}
P(x_1|z_3) &\simeq 0.10 \\
P(x_2|z_3) &\simeq 0.29 \\
P(x_3|z_3) &\simeq 0.45 \quad \leftarrow \\
P(x_4|z_3) &\simeq 0.17
\end{aligned}
$$

$$
\begin{aligned}
P(x_1|z_4) &\simeq 0.10 \\
P(x_2|z_4) &\simeq 0.30 \\
P(x_3|z_4) &\simeq 0.33 \quad \leftarrow \\
P(x_4|z_4) &\simeq 0.27
\end{aligned}
$$

The indication $\leftarrow$ in the calculation above indicates the maximum for each location, so with a faulty sensor and a $3\times$ more likely belief for x_2 and x_3 the most likely location for observations ($z_1 = 1.1m, z_2 = 2.1m$) becomes x_2 and for ($z_3 = 8.6m, z_4 = 9.4m$) the most likely location becomes x_3.

Question 3

Assume you have a robot equipped with a sensor capable of measuring the distance and bearings to landmarks. The sensor furthermore provides you with the identity of the observed landmarks. A sensor measurement $z = (z_r, z_\theta)^T$ is composed of the measured distance z_r and the measured bearing z_θ to the landmark with signature l. Both the range and the bearing measurements are subject to zero-mean Gaussian noise with variances σ_r^2 and σ_θ^2. The range and the bearing measurements are independent from each other. A sensor model models the probability of a measurement z of landmark l observed by the robot from pose x. Design a sensor model $p(z|x,l)$ for this type of sensor.

Answer

The likelihood model for the sensor has the following form assuming that the range and the bearing measurements are independent given the pose

$$p(z|x,l) = p(z_r, z_\theta|x,l) = \mathcal{N}(z_r - \hat{z}_r, \sigma_r^2)\mathcal{N}(z_\theta - \hat{z}_\theta, \sigma_\theta^2)$$

where $\hat{z}_r$ is the expected range measurement, $\hat{z}_\theta$ is the expected bearing measurement and $\mathcal{N}(\cdot, \sigma^2)$ is the zero mean Gaussian distribution.

Question 4

Suppose an indoor robot uses sonar sensors with a 15 degree opening cone, mounted on a fixed height so that they point out horizontally and parallel to the ground. This is a common configuration for an indoor robot. Discuss what happens when the robot faces an obstacle whose height is just below the height of the sensor (for example, 15 cm below). Specially, answer the following questions:

(a) Under what conditions will the robot detect the obstacle? Under what conditions will it fail to detect it? Be concise.

(b) What implications does this all have for the binary Bayes filter and the underlying Markov assumption? How can you make the occupancy grid algorithm fail?

(c) Based on your answer to the previous question, can you provide an improved occupancy grid mapping algorithm that will detect the obstacle more reliably than the plain occupancy grid mapping algorithm?

Answer:

(a) The robot will detect the obstacle when it is further away, and will fail to detect to obstacle when it is close to the robot. Most of the signal in the main cone of an sonar sensor will pass over the obstacle, although reflections from side lobs (with a much lower signal) still have a chance to be detected (because nearby reflections give a stronger signal). Yet, the main cone will only hit the obstacle at the cone is expanded to 15cm, which is at slightly more than a meter (1.14m = $0.15/tan(15^o/2)$).

(b) This will result in many faulty updates of the Bayes Filter, resulting that the occupancy grid converges to free space, even when obstacles have been seen before. This is a violation of the Markov assumption, which assumes that the state (occupancy grid) aggregates all previous information, so that no history has to be maintained.

(c) Aggregating the number of times the number of times a hit or a miss is observed for each grid cell (a likelihood field), would provide an improved occupancy grid mapping algorithm for this situation.

Question 5

GraphSLAM and the Sparse Extended Information Filter both model the state with an information matrix Ω and information vector ξ, and both see off-diagonal elements in the information matrix Ω as constraints. Yet, both algorithms reduce the number of constraints in a completely different way.

(a) Explain in your own words the difference between the reduction of constraints between GraphSLAM and the Sparse Extended Information Filter.

(b) The Sparse Extended Information Filter conditions away all passive features m^-, by assuming $m^- = 0$. Why is this done? What would be the update equation if these features would not be conditioned away? Would the result be more accurate or less accurate? Would the computation be more or less efficient? Be consise.

Answer:

(a) GraphSLAM reduces constraints by removing links between robot poses and observed features, by adding the observed constraints to the corresponding link between the involved robot poses. In contrary, SEIF reduces constraints by removing contraints from the links between robot poses, instead adding those constraints to links between nearby (active) features.

(b) For an Information Filter, assuming $m^- = 0$ is equivalent with the observation that no information is available, which is a valid for features not nearby (passive landmarks). So this approximation allows to ignore dependencies between passive landmarks and robot poses. This makes the information matrix Ω sparse. Without this sparsification step is SEIF is equivalent with an Extended Information Filter, as described in chapter 3; an approach which is only useful for localization on a limited number of landmarks but doesn't scale to full online SLAM. The sparsification step makes the result less accurate but faster to compute. Most computation gain originates from the calculation of the inverse of the Information Matrix Ω, which can be calculated much faster for a sparse matrix.

Question 6

You have a robot which received the following map:

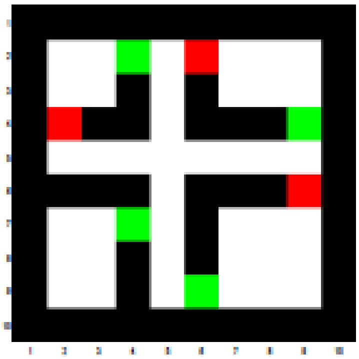
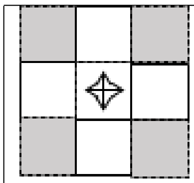
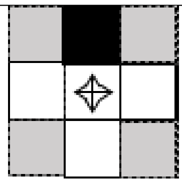
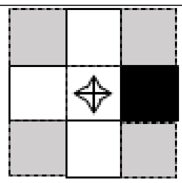
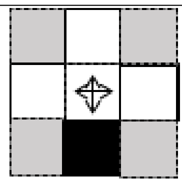
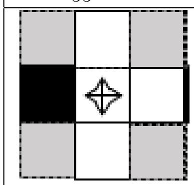
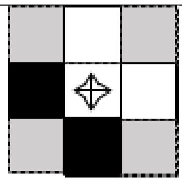
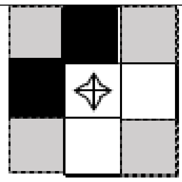
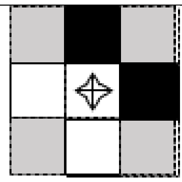
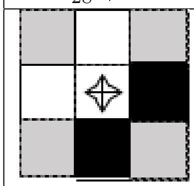
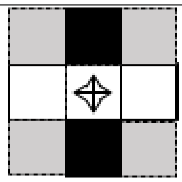
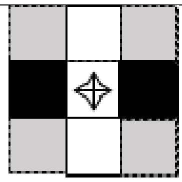
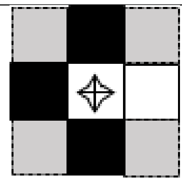
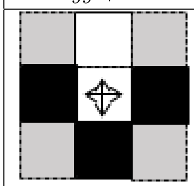
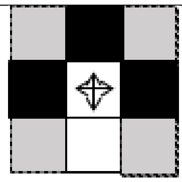
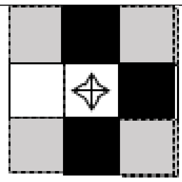
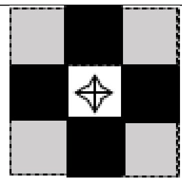


Figure 2: A map of environment with 4 rooms. Courtesy Kevin P. Murphy [1]

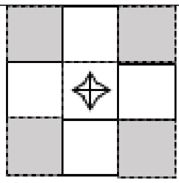
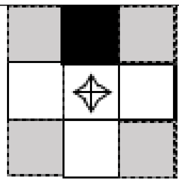
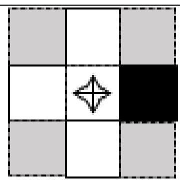
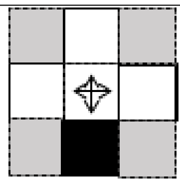
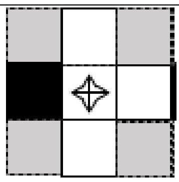
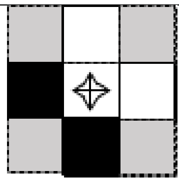
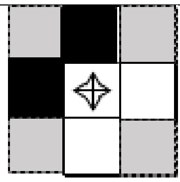
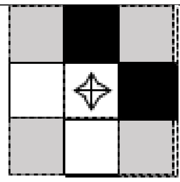
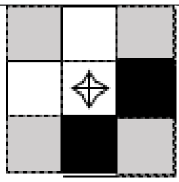
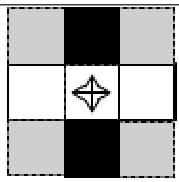
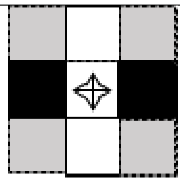
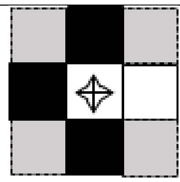
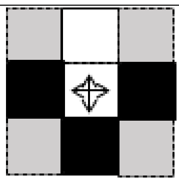
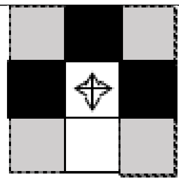
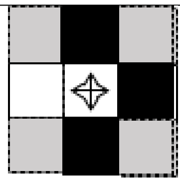
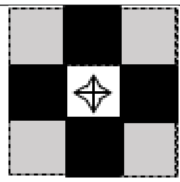
This map is a occupancy map, with four possible labels for each grid cell (closed doors - red, open doors - green, walls- black, free space - white). The robot has a sensor on board, which can only detect if there is an obstacle to the nearby horizontal and vertical grid cells. This means that there are $4^2 = 16$ different observations z_t possible. In the following table those observations are indicated, with as example one location where that observation could be made.

			
$m_{55} + \cdots$	$m_{32} + \cdots$	$m_{98} + \cdots$	$m_{83} + \cdots$
			
$m_{28} + \cdots$	$m_{29} + \cdots$	$m_{77} + \cdots$	$m_{92} + \cdots$
			
$m_{99} + \cdots$	$m_{75} + \cdots$	$m_{53} + \cdots$	$m_{25} + \cdots$
			
$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$

Note that the four later observations do not occur, because an open door is also perceived as no obstacle.

(a) Calculate the probability for each of the observations $z^{1\cdots16}$ to be measured on this map.

Answer:

			
$m_{55} + m_{88}$ $\rightarrow P(z^1) = \frac{2}{44}$	$m_{32} + m_{82} + m_{73} + m_{78}$ $\rightarrow P(z^2) = \frac{4}{44}$	$m_{98} + m_{93} + m_{38} + m_{57}$ $\rightarrow P(z^3) = \frac{4}{44}$	$m_{79} + m_{89} + m_{38}$ $\rightarrow P(z^4) = \frac{3}{44}$
			
$m_{28} + m_{78}$ $\rightarrow P(z^5) = \frac{2}{44}$	$m_{23} + m_{29} + m_{73} + m_{59}$ $\rightarrow P(z^6) = \frac{4}{44}$	$m_{22} + m_{27} + m_{72} + m_{77}$ $\rightarrow P(z^7) = \frac{4}{44}$	$m_{92} + m_{79} + m_{52}$ $\rightarrow P(z^8) = \frac{3}{44}$
			
$m_{99} + m_{33} + m_{39} + m_{95}$ $\rightarrow P(z^9) = \frac{4}{44}$	$m_{75} + m_{35} + m_{65} + m_{85} +$ $m_{42} + m_{47} + m_{69} + m_{45}$ $\rightarrow P(z^{10}) = \frac{8}{44}$	$m_{53} + m_{54} + m_{56} + m_{58} + m_{94}$ $\rightarrow P(z^{11}) = \frac{5}{44}$	m_{25} $\rightarrow P(z^{12}) = \frac{1}{44}$
			
$\varnothing$ $\rightarrow P(z^{13}) = \frac{0}{44}$	$\varnothing$ $\rightarrow P(z^{14}) = \frac{0}{44}$	$\varnothing$ $\rightarrow P(z^{15}) = \frac{0}{44}$	$\varnothing$ $\rightarrow P(z^{16}) = \frac{0}{44}$

As a final check, we apply the rule of Total Probability $\sum_{k=1}^{16} P(z^k) = 1$ to see if no location m_{ij} is forgotten. Fortunatelly $2 + 4 + 4 + 3 + 2 + 4 + 4 + 3 + 4 + 8 + 5 + 1 = 44$.

(b) There is a small draft, with manifest itself for this environment in a small chance P_c that an open door closes. How should such a state-change be included in the description of the world, in respectively a Kalman Filter, an Extended Kalman Filter and a Particle Filter?

Answer:

This small draft is a state transition $x_{t_1} \rightarrow x_t$, independent from any control u_t. For a Kalman Filter this can be modelled in the state transition matrix A_t, because the small chance P_c effects the first two elements of the tuple (closed doors - red, open doors - green, walls- black, free space - white): $red_t = P_c * green_{t-1}$ and $green_t = (1 - P_c) * green_{t-1}$. For the Extended Kalman Filter the state transition can be included in the function $g()$. For the Kalman filter it should be included in the state transition probability $P(x_t|u_t, x_{t-1})$ on line 4 of Table 4.3 of the textbook.

The robot is not only equipped with a range sensor which can detect nearby obstacles, but in addition also with a microphone. The robot hears a door bang, and knows for sure that one of the four open doors is now closed. So, each of the four new maps are as likely:

(c) Design a policy to determine the robot's location by executing an minimum of combinations of (u_i, z_i) when the first observation is z_0 is equal to observation that can be made at m_{29}.

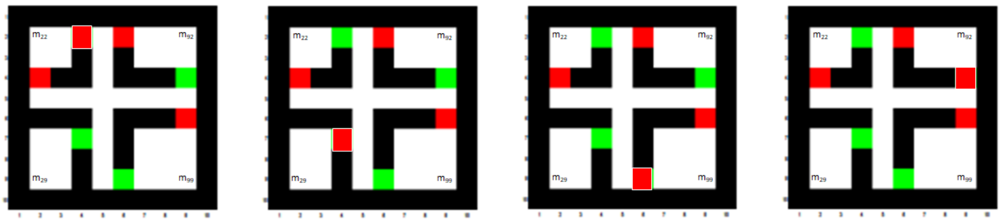


Figure 3: Four possible maps of environment with an additional closed door

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Answer:

The first observation $z_0 = z^{13}$ can for each of the four worlds made on four different locations. In addition, in the third world w_c of Fig. 3 the observation can also be made at fifth location m_{78}. So, the robot can be at 17 different world-location combinations. For all 17 locations the robot can move without danger into two directions: $u_1 = \uparrow$ or $u_1 = \rightarrow$. When $u_1 = \uparrow$ is chosen the next observation at $t = 1$ is already unique for one location $(m)58$ in all possible worlds. For the other 13 possible world-location combinations additional moves u and observations z are needed. The same is true for the other choice $u_1 = \rightarrow$, but we will work out the branch for $u_1 = \uparrow$, because for this branch a combination of moves and observations can be created which is safe to be executed in all cases:

$$w_a - m_{23} : (z_0 = z^{13}, u_1 = \uparrow, z_1 = z^7, u_2 = \rightarrow, z_2 = z^8) \therefore m_{32} \text{ at } t = 2 \tag{1}$$

$$w_a - m_{29} : (z_0 = z^{13}, u_1 = \uparrow, z_1 = z^5, u_2 = \rightarrow, z_2 = z^3) \therefore m_{38} \text{ at } t = 2 \tag{2}$$

$$w_a - m_{59} : (z_0 = z^{13}, u_1 = \uparrow, z_1 = z^{11}) \therefore m_{58} \text{ at } t = 1 \tag{3}$$

$$w_a - m_{73} : (z_0 = z^{13}, u_1 = \uparrow, z_1 = z^7, u_2 = \rightarrow, z_2 = z^2, u_3 = \rightarrow, z_3 = z^8) \therefore m_{92} \text{ at } t = 3 \tag{4}$$

$$w_b - m_{23} : (z_0 = z^{13}, u_1 = \uparrow, z_1 = z^7, u_2 = \rightarrow, z_2 = z^2, u_3 = \rightarrow, z_3 = z^{10}) \therefore m_{42} \text{ at } t = 3 \tag{5}$$

$$w_b - m_{29} : (z_0 = z^{13}, u_1 = \uparrow, z_1 = z^5, u_2 = \rightarrow, z_2 = z^3) \therefore m_{38} \text{ at } t = 2 \tag{6}$$

$$w_b - m_{59} : (z_0 = z^{13}, u_1 = \uparrow, z_1 = z^{11}) \therefore m_{58} \text{ at } t = 1 \tag{7}$$

$$w_b - m_{73} : (z_0 = z^{13}, u_1 = \uparrow, z_1 = z^7, u_2 = \rightarrow, z_2 = z^2, u_3 = \rightarrow, z_3 = z^8) \therefore m_{92} \text{ at } t = 3 \tag{8}$$

$$w_c - m_{23} : (z_0 = z^{13}, u_1 = \uparrow, z_1 = z^7, u_2 = \rightarrow, z_2 = z^2, u_3 = \rightarrow, z_3 = z^{10}) \therefore m_{42} \text{ at } t = 3 \tag{9}$$

$$w_c - m_{29} : (z_0 = z^{13}, u_1 = \uparrow, z_1 = z^5, u_2 = \rightarrow, z_2 = z^3) \therefore m_{38} \text{ at } t = 2 \tag{10}$$

$$w_c - m_{59} : (z_0 = z^{13}, u_1 = \uparrow, z_1 = z^{11}) \therefore m_{58} \text{ at } t = 1 \tag{11}$$

$$w_c - m_{73} : (z_0 = z^{13}, u_1 = \uparrow, z_1 = z^7, u_2 = \rightarrow, z_2 = z^2, u_3 = \rightarrow, z_3 = z^8) \therefore m_{92} \text{ at } t = 3 \tag{12}$$

$$w_c - m_{78} : (z_0 = z^{13}, u_1 = \uparrow, z_1 = z^5, u_2 = \rightarrow, z_2 = z^1) \therefore m_{86} \text{ at } t = 2 \tag{13}$$

$$w_d - m_{23} : (z_0 = z^{13}, u_1 = \uparrow, z_1 = z^7, u_2 = \rightarrow, z_2 = z^2, u_3 = \rightarrow, z_3 = z^{10}) \therefore m_{42} \text{ at } t = 3 \tag{14}$$

$$w_d - m_{29} : (z_0 = z^{13}, u_1 = \uparrow, z_1 = z^5, u_2 = \rightarrow, z_2 = z^3) \therefore m_{38} \text{ at } t = 2 \tag{15}$$

$$w_d - m_{59} : (z_0 = z^{13}, u_1 = \uparrow, z_1 = z^{11}) \therefore m_{58} \text{ at } t = 1 \tag{16}$$

$$w_d - m_{73} : (z_0 = z^{13}, u_1 = \uparrow, z_1 = z^7, u_2 = \rightarrow, z_2 = z^2, u_3 = \rightarrow, z_3 = z^8) \therefore m_{92} \text{ at } t = 3 \tag{17}$$

So, the minimum policy to localize in this possible worlds is a sequence $(z_0, u_1 = \uparrow, z_1, u_2 = \rightarrow, z_2, u_3 = \rightarrow, z_3)$ combined with the the following decision tree:

procedure LOCALIZE(sequence)
  $s \leftarrow 0$
  while s $<=$length(sequence) **do**
    u_t = sequence(++s)
    execute(u_t)
    z_t = sequence(++s)
    if $z_t == z^1$ **then return** m_{86}
    if $z_t == z^3$ **then return** m_{38}
    if $z_t == z^8$ **and** $s > 5$ **then return** m_{92}
    if $z_t == z^8$ **then return** m_{32}
    if $z_t == z^{10}$ **then return** m_{42}
    if $z_t == z^{11}$ **then return** m_{58}

Success!

Acknowledgements

The first question is based on an assignment from the Albert-Ludwigs-Universität Freiburg, written by Wolfram Burgard. The third question is originating from Jana Košecká course ”Autonomous Robots”. The fifth question is directly from the ”Probabilistic Robotics” book [2]. The last question is inspired by an article of Kevin P. Murphy [1].

References

[1] K. P. Murphy, “Bayesian map learning in dynamic environments”, in “Advances in Neural Information Processing Systems”, pp. 1015–1021, 2000.

[2] S. Thrun, W. Burgard and D. Fox, *Probabilistic Robotics (Intelligent Robotics and Autonomous Agents)*, The MIT Press, September 2005, ISBN 0-262-20162-3.