Autonomous Mobile Robots (AUMR6Y, Fall 2012)
Examination: Localization, Planning & Navigation

Thursday December December 20th, 13:00-15:00, A1.04

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Question 1

Solve exercise 5.9.2 from the Autonomous Mobile Robots book[1].

Question 2

Assume the following 1-D lineair dynamic system, with a simple probabilistic motion model:

$$\hat{x}_t = x_{t-1} + u_t + \epsilon_t \tag{1}$$

and a simple probabilistic measurement model:

$$\hat{z}_t = \hat{x}_t + \delta_t \tag{2}$$

The terms ϵ_t and δ_t represent respectively the control and measurement error, a random number from a Gaussian distribution $\mathcal{N}(x; 0, R_t)$ and $\mathcal{N}(z; 0, Q_t)$. For the moment you can assume that the variance $R_t = 0$ and $Q_t = 1$, which means that you have perfect control over the dynamic system (ϵ_t can be ignored). For all timesteps, the same input is given ($u_t = 0.5$). The initial estimate is represented with a Gaussian distribution $\mathcal{N}(x; \mu_0, \Sigma_0)$ with $\mu_0 = 5$ and $\Sigma_0 = 10$.

You receive the following measurements ($z_1 = 0.0, z_2 = 2.1, z_3 = 5.6$).

(a) Is in this case the assumption of white noise made? Explain your answer.

[1]Question 2 originates from the course 'Design and Organization of Autonomous Systems' from the Universiteit van Amsterdam. Question 3 originates from the book 'Principles of Robot Motion - Theory, Algorithms and Implementations'. Question 4 originates from the book 'Artificial Intelligence - A Modern Approach'.

(b) Use the measurements (z_1, z_2, z_3) to estimate (μ_1, μ_2, μ_3). For this lineair system you can use a traditional Kalman Filter, as described in section 5.6.8 of the book. This will be a two step approach, a prediction and an update step. The result of the prediction step will be a Gaussian distribution $\mathcal{N}(x; \hat{\mu}_0, \hat{\Sigma}_t)$. In the update step you can shift and narrow this distribution to $\mathcal{N}(x; \mu_t, \Sigma_t)$ making use of the measurements and the following precalculated Kalman gain $(K_1 = \frac{10}{11}, K_2 = \frac{10}{21}, K_3 = \frac{10}{31}, K_4 = \frac{10}{41}, K_5 = \frac{10}{51})$.

(c) Explain why the Kalman Gain decreases for every time step.

(d) Lets drop the assumption of perfect control, and reintroduce the control noise ϵ_t modelled with a Gaussian distribution $\mathcal{N}(x; 0, 1)$. Recalculate $(K_1, K_2, K_3, K_4, K_5)$ for the given variance $Q_t = 1$. Explain the observed pattern in the Kalman Gain K_t.

(e) Make a new estimate of (μ_1, μ_2, μ_3) based on the recalculated Kalman Gain K_t.

Question 3

What happens if you apply the particle filter SLAM algorithm to a robot whose sensor is almost perfect? For example, what happens when the robot uses (almost) noise-free range sensors? Hint: For near-perfect sensors, the likelihood-function $P(z|x)$ will be extremely peaked, i.e., it will be almost zero for all measurements that are slightly off the correct nois-free value. How does the accuracy of the sensor affect the number of particles needed?

Question 4

Which of the following statements are true and which are false? Explain your answers.

(a) Depth-first search always expands at least as many nodes as A* with an admissible heuristic[2].

(b) A* is of no use in robotics because observations, states and movements are continuous.

(c) Breath-first search is complete[3] even if zero step costs are allowed.

(d) Assume a rook on a chessboard; the piece can move any number of squares in a straight line, horizontally or vertically, but cannot jump over other pieces. Manhattan distance[4] is an admissible heuristic for moving the rook from square A to square B in the smallest number of moves.

[2] An **admissible heuristic** is a measure that *never overestimates* the cost to reach a goal.
[3] **Completeness** indicates that the algorithm is *guaranteed* to find a solution when there is one.
[4] **City block** or **Manhattan distance** is the the sum of *horizontal* and *vertical* distances between grid cells