

# Translating a Process Algebra with Symbolic Data Values to Linear Format

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## Abstract

Historically, process algebras have been studied mostly *without* data. In this paper the transformation is described of the valued process algebra  $\mu\text{CRL}$  [GP95] to a symbolic transition system in the spirit of [Sch94]. The data oriented specifications thus obtained, seem to be in a better format for checking modal properties.

## 1. Introduction

Historically, much effort has been put into understanding the theories of pure process algebra calculi. Also, process algebra tools concentrate on process calculi with explicit data values as an input language. A few front ends have been developed which translate a calculus with symbolic values into a pure calculus, such as the value passer [Bru91], which translates value-passing CCS [Mil89] into pure CCS. In the LOTOS community several tools have been constructed which can translate (valued) LOTOS to C programs or labeled transitions system, e.g. [FGM<sup>+</sup>92, KBG93].

We aim at building a tool which can do model checking for  $\mu\text{CRL}$  [GP95].  $\mu\text{CRL}$  is a specification language for a process algebra with symbolic data values, where the process part is based on ACP [BW90] and the data part on algebraic specifications as in [EM85]. Until now  $\mu\text{CRL}$  has been mainly used for manual verification proving equivalences between processes (e.g. [BG94a]), but we are thinking of checking properties in a suitable logic. Sometimes we know only part of the desired behavior, as in the case of safety criteria for railroads [GKvV94].

In this respect [Sch94] is highly interesting, which treats a calculus with symbolic data values as a first class citizen. In [Sch94] value-passing CCS is mapped onto a data structure called *parametrized graphs*, which are essentially symbolic transition systems. This has two advantages over the conventional procedure of translating the valued calculus to the pure calculus and then perform modal checking. First, the structure of the processes is still visible in the parametrized graphs. Second, part of the state explosion is avoided because data is not expanded.

Whereas [Sch94] is mainly focused on checking various equivalences, we are interested in checking modal formulas as in [GvV94] or [HL93]. We think that translating the language  $\mu\text{CRL}$  to parametrized graphs is an interesting experiment in itself and a signal for model checking. We refer to the experience that Hennessy-Milner Logic seems to be checked more efficiently on a restricted form of pure CCS [Hol89]. A second point of interest is that the parameterized graphs are a kind of *data oriented* specifications. In several case studies verification starts by transforming specifications to such a form by hand and then performing further analysis, see e.g. [Bru95, GS95].

We describe the transformation of  $\mu\text{CRL}$  to parametrized graphs, which we will define syntactically.

## 2. Translating a Fragment of $\mu\text{CRL}$ to a Single-Linear Specification

We start with a translation of a fragment of  $\mu\text{CRL}$  to single-linear format by means of typical examples<sup>1</sup>. This format is a direct translation of a graph grammar and can be seen also as a fragment of value-passing CCS. Next we explain how a larger part of the  $\mu\text{CRL}$  specifications in the full calculus can be translated to this format. We end with some conclusions on the implementation of the transformation in the ASF+SDF system [Kli93].

### 2. Translating a Fragment of $\mu\text{CRL}$ to a Single-Linear Specification

The specification language  $\mu\text{CRL}$  has come out of the SPECS project, as the essence of the language CRL [BDE<sup>+</sup>93]. It has been developed under the assumption that a study of the basic concepts of specification languages will yield more fundamental insights than studying the complete language.

The data part contains equational specifications. The process part contains processes described in the style of CCS, CSP or ACP, where the syntax has been taken from the last. It basically consists of a set of uninterpreted actions that may be parametrized by data. These actions represent various activities, depending on the usage of the language. There are sequential composition, alternative and parallel composition operators. Furthermore, recursive processes are specified in a simple way. See for a complete definition of jargon, syntax and semantics [PVvV95].

In this section we describe the translation of a basic fragment of  $\mu\text{CRL}$  to linear format. It is similar to BPA, in that it contains only alternative and sequential composition<sup>2</sup> It extends BPA by the presence of data and the if-then-else and sum construct. First we define this fragment and linear specifications in a precise way. Next we describe the translation by means of examples.

#### 2.1 Specifications in BPS Format

A well-formed  $\mu\text{CRL}$  *specification*  $E$  is a *specification* in Basic Process Syntax, BPS for short, iff all *process-declarations* occurring in  $E$  have in their right-hand sides *process-expressions* that are in BPS:

**Definition 2.1** The syntactical category BPS that constitutes the class of processes in BPS has the following BNF syntax.

$$\begin{array}{l}
 \textit{process-expression} ::= \textit{process-expression} + \textit{process-expression} \\
 \quad | \textit{process-expression} \triangleleft \textit{data-term} \triangleright \textit{process-expression} \\
 \quad | \textit{process-expression} \cdot \textit{process-expression} \\
 \quad | \sum(\textit{single-variable-declaration}, \textit{process-expression}) \\
 \quad | \delta \\
 \quad | \textit{name} \\
 \quad | \textit{name}(\textit{data-term-list}) \\
 \quad | (\textit{process-expression}).
 \end{array}$$

In the above definition  $+$  is the choice operator,  $\cdot$  sequential composition and  $\triangleleft \triangleright$  is the notation for the if-then-else construct in  $\mu\text{CRL}$ .  $\sum$  is the notation for a summation over data.  $\delta$  is the deadlocked process. The precedence is in the order  $\cdot, \triangleleft \triangleright, +$  (as can be seen from definition above).

**Example 2.2** Consider the following well-formed  $\mu\text{CRL}$  *specification* in BPS,  $E$  of the sender in the Alternating Bit Protocol of [GP95]:

<sup>1</sup>Formal approaches to  $\mu\text{CRL}$  proof theory are e.g., [GP93],[BG94b].

<sup>2</sup>In linear formats, sequential composition can actually be replaced by action prefixing.

## 2. Translating a Fragment of $\mu\text{CRL}$ to a Single-Linear Specification

$$\begin{aligned}
E &\equiv \text{sort } bit, D, error, \mathbf{Bool} \\
&\text{func } T, F : \rightarrow \mathbf{Bool} \\
&\quad 0, 1 : \rightarrow bit \\
&\quad e : \rightarrow error \\
&\quad invert : bit \rightarrow bit \\
&\quad d_1, d_2, d_3 : \rightarrow D \\
&\text{act } r1 : D \\
&\quad r6 : error \\
&\quad r6 : bit \\
&\quad s2 : D \times bit \\
&\quad c : \mathbf{Bool} \\
&\text{rew } invert(0) = 1 \\
&\quad invert(1) = 0 \\
&\text{proc } S = S(0) \cdot S(1) \cdot S \\
&\quad S(n : bit) = sum(d : D, r1(d) \cdot S(d, n)) \\
&\quad S(d : D, n : bit) = s2(d, n) \cdot ((r6(invert(n)) + r6(e)) \cdot S(d, n) + r6(n))
\end{aligned}$$

We will use the terms *process* and *action* as follows: let  $E$  be a  $\mu\text{CRL}$  *specification* and  $q$  a *process-expression* that is Statically and Semantically Correct (SSC, [GP95]) with respect to  $E$  and has no free data variables, then  $p$  **from**  $E$  is called a process from  $E$ . We will use the term *parametrized process name* for the name in the left-hand side of a process specification, which has a type given by the parametrization<sup>3</sup>. Furthermore an *action* is a process that refers directly to an *action-specification* in  $E$  and has no free data variables. So in the example above  $S(0)$  is a process from  $E$ , and  $r6(invert(0)), r6(e)$  are actions from  $E$ . If  $E$  is fixed, we just speak of “the process  $p$ ”.

We will restrict our attention to a decidable class of guarded specifications in BPS. We will admit only those specifications where the defining right hand side of every process name is such that the process name occurs only guarded, i.e. either directly or indirectly in the scope of an action.

**Definition 2.3** Let  $P$  be the set of process names occurring in the specification  $E$  and  $p, p_1, \dots, p_n, q \in P$  (parametrized) process names. Let  $UG(p, E)$  be a set of tuples of the form  $\langle p, q \rangle$  where  $q$  is a (parametrized) process name occurring unguarded in the declaration of  $p$ , i.e. not in the scope of a preceding action.  $E$  is *syntactically guarded* iff  $\bigcup_{p \in P} UG(p, E)$  contains no cycle, i.e. a subset of the form  $\{\langle p_1, p_2 \rangle, \langle p_2, p_3 \rangle \dots \langle p_{n-1}, p_n \rangle\}$  so that  $p_1 \equiv p_n$ .

Given a  $\mu\text{CRL}$  *specification*  $E$ , we associate with each process from  $E$  a (referential) transition system that describes its meaning. The intended semantics of a process  $p$  from a  $\mu\text{CRL}$  *specification*  $E$  is a transition system  $\mathcal{A}(\mathbf{A}_{N_E}, p \text{ from } E)$  where  $\mathbf{A}_{N_E}$  is the canonical term algebra of  $E$ , and where the labels of transitions may be parameterized by the fixed representations of the elements of  $\mathbf{A}_{N_E}$ . These transition systems are considered modulo bisimulation equivalence, notation  $\simeq_{\mathbf{A}_{N_E}}$ , as this is the coarsest congruence that respects operational behaviour.

Now processes from syntactically guarded  $\mu\text{CRL}$  *specifications* in BPS constitute the **source language** for the translation described in the sequel.

*Conventions.* For readability we adopt the following conventions.

- Binary operations associate to the right, brackets are omitted if possible.

<sup>3</sup>So in Example 2.2 the three process declarations have a *different* parametrized process name, although their name is the same.

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- Instead of repeatedly denoting  $\mu\text{CRL}$  *specifications* in a syntactically correct way (as was done in the example above), we often only write down a *process-specification* without the keyword **proc**, and assume that it is part of some well-defined  $\mu\text{CRL}$  *specification*. In doing so we use  $a, b, c, \dots$  as syntactic variables for action *names* and  $X, Y, Z, \dots$  as syntactic variables for process *names*.
- Whenever convenient, we assume that any  $\mu\text{CRL}$  *specification* under consideration contains the (standard) functions  $\neg$  and  $\wedge$  on the standard sort **Bool**. Applications of the function  $\wedge$  will always be written in an infix manner. Note that from the point of view of describing *processes* this convention causes no loss of generality, as we can always extend *specifications* with these functions.  $\square$

### 2.2 Single-Linear Process Specifications

In this section we define the syntax of “single-linear” *process-specifications* that play a crucial role in our canonical translation.

We start by introducing the following two archetypes of  $\mu\text{CRL}$  *process-specifications* in BPS. In their definition we use the symbol  $\Sigma$  also as a shorthand to denote **finite** sums (not to be confused with the *sum operator* of  $\mu\text{CRL}$ ): let  $p_1, p_2, \dots$  be *process-expressions*, then the expression

$$\sum_{i=1}^k p_i$$

abbreviates  $\delta$  in case  $k = 0$ , and  $p_1 + p_2 + \dots + p_k$  otherwise.

**Definition 2.4** A *process-specification* of the form  $pd_1 \dots pd_m$  with  $m \geq 1$  from some  $\mu\text{CRL}$  *specification*  $E$  is in *normal form* iff for all  $1 \leq i \leq m$  the declaration  $pd_i$  has a right-hand side of the form

$$\sum_{j=1}^{k_i} p_{ij}$$

where each of the *process-expressions*  $p_{ij}$ <sup>4</sup> is of the form

$$\begin{aligned} & (\sum_{k=1}^{k_{ij}} \Sigma(d_{ijk} : D_{ijk}, a_{ijk} \cdot X_{ijk}^1 \cdot X_{ijk}^2) + \\ & \sum_{l=1}^{l_{ij}} \Sigma(d_{ijl} : D_{ijk}, b_{ijl} \cdot X_{ijl}^3) + \\ & \sum_{m=1}^{m_{ij}} \Sigma(d_{ijm} : D_{ijm}, c_{ijm})) \triangleleft t_{ij} \triangleright \delta \end{aligned}$$

with the  $d_{ijk}$  single variables over data types  $D_{ijk}, a_{ijk}, b_{ijk}, c_{ijk}$  (possibly parameterized) *process-expressions* over the *names* in the *action-specifications* from  $E$ , and the  $X_{ijk}^1, X_{ijk}^2, X_{ijk}^3$  (possibly parameterized) *process-expressions* over the *names* in the left-hand sides of the declarations  $pd_1, \dots, pd_m$ .

In the special case that  $k_{ij} = 0$  for all appropriate  $i, j$  we say that the *process-specification*  $pd_1 \dots pd_m$  is in *linear form*.

Now we can define what is meant by a “single-linear” *process-specification*.

**Definition 2.5** Let  $E$  be a  $\mu\text{CRL}$  *specification*. A *process-specification* occurring in  $E$  is *single-linear* iff it is in linear form and contains exactly one *process-declaration*.

<sup>4</sup>We use of course the axiom  $\sum(d : D, p) = p$ ,  $d$  not free in  $p$ , to remove summations.

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**Example 2.6** Consider the following specification:

$$\begin{aligned}
E \equiv & \text{ sort } \mathbf{Bool}, S \\
& \text{ func } T, F : \rightarrow \mathbf{Bool} \\
& \quad C : \rightarrow S \\
& \quad f : \mathbf{Bool} \rightarrow S \\
& \quad g : S \rightarrow \mathbf{Bool} \\
& \text{ act } a, d \\
& \quad b : \mathbf{Bool} \\
& \quad c : S \times \mathbf{Bool} \\
& \text{ proc } X(x : \mathbf{Bool}, y : S) = (a \cdot X(x, f(x)) + b(x)) \triangleleft x \triangleright \delta \\
& \quad + (c(y, g(y)) \cdot X(g(y), f(x)) + d) \triangleleft g(y) \triangleright \delta
\end{aligned}$$

that has a single-linear *process-specification*.

### 2.3 The Translation

Given a syntactically guarded well-formed  $\mu\text{CRL}$  *specification*  $E$  in BPS and a process  $p$  **from**  $E$ , we describe in this section the construction of a syntactically guarded  $\mu\text{CRL}$  *specification*  $E'$  such that

- $E'$  is a  $\mu\text{CRL}$  *specification*, obtained from  $E$  by the (possible) addition of *sort*-, *function*-, *rewrite*- and *process-specifications* in such a way that  $p$  **from**  $E \simeq_{\mathbf{A}_{N_{E'}}} p$  **from**  $E'$ .
- there is a process  $p'$  from  $E'$  such that
  - $p'$  satisfies  $p'$  **from**  $E' \simeq_{\mathbf{A}_{N_{E'}}} p$  **from**  $E'$ , i.e.  $p$  and  $p'$  behave the same,
  - $p'$  is a process that is specified in a single-linear way, i.e. the *name* of  $p'$  is declared in a single-linear *process-specification* contained in  $E'$ .

We just describe the construction of  $E'$  by means of examples, and refrain from formal descriptions which are required for a correctness proof. We hope that the suggestion of provability is sufficiently clear.

We distinguish six consecutive steps in this type of construction, each of which should be applied in case its conditions hold. Application of such a step extends the *specification* with at least a *process-specification*. We assume that these extensions always yield a  $\mu\text{CRL}$  *specification*, so in particular we assume that the newly added *sort*-, *function*- and *process-specifications* have fresh *names*.

*1. Introducing a process expression as a new declaration.* Let  $p$  **from**  $E$  be the object for translation. This step applies whenever  $p$  is not of the form  $n$  or  $n(t_1, \dots, t_k)$  for some *process name*  $n$ . In this case we extend  $E$  to  $E_1$  by adding a *process-specification* that specifies a process  $p_1$  of the form  $n$  or  $n(t_1, \dots, t_k)$  that behaves the same as  $p$  **from**  $E_1$ .

*Example of step 1.* Let  $p \equiv X(t) + b(u)$  where  $X(x : S)$  is specified as follows:

$$X(x : S) = a(x) \cdot X(x) + a(x)$$

and the *action-specification*  $\text{act } b : S'$  is also contained in  $E$ . We extend  $E$  to  $E_1$  by adding the *process-specification*

$$X'(x : S, y : S') = X(x) + b(y)$$

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Note that

$$X(t) + b(u) \text{ from } E_1 \Leftrightarrow_{A_{N_{E_1}}} X'(t, u) \text{ from } E_1.$$

(End example.)

2. *Translating the process declarations to normal form.* Let  $p_1$  **from**  $E_1$  satisfy  $p_1 \equiv n$  or  $p_1 \equiv n(t_1, \dots, t_k)$ . This step applies whenever the *process-specification* of  $p_1$  is not in normal form. In this case we extend  $E_1$  to  $E_2$  by adding a *process-specification* in normal form of a process  $p_2$  that behaves the same as  $p_1$  **from**  $E_2$ .

*Example of step 2.* Let  $p_1 \equiv X(t)$  where  $X(d : D)$ , is specified as follows, with  $d_0 \in D$  a constant:

$$X(d : D) = \sum(e : D, a(d) \cdot X(d_0) \cdot X(e) \cdot X(d)) + b$$

We sketch the technique to obtain a *process-specification* in normal form that defines the same process(es) as  $X(d : D)$ . The main problem here is the summand  $\sum(e : D, a(d) \cdot X(d_0) \cdot X(e) \cdot X(d))$ , as it is essentially different from the ‘normal form syntax’. We start by replacing this subterm by the term  $\sum(e : D, a(d) \cdot X_1(d, e))$ . We add the new process declaration

$$X_1(d : D, e : D) = X(d_0) \cdot X(e) \cdot X(d)$$

and thus obtain the specification

$$\begin{aligned} X(d : D) &= \sum(e : D, a(d) \cdot X_1(d, e)) + b \\ X_1(d : D, e : D) &= X(d_0) \cdot X(e) \cdot X(d). \end{aligned}$$

The process declaration for  $X$  is now essentially in normal form. We repeat the same step for the process declaration for  $X_1$ . The new specification becomes

$$\begin{aligned} X(d : D) &= \sum(e : D, a(d) \cdot X_1(d, e)) + b \\ X_1(d : D, e : D) &= X(d_0) \cdot X_2(d, e) \\ X_2(d : D, e : D) &= X(e) \cdot X(d). \end{aligned}$$

Having done this, we can replace the specification using the *new* declaration for  $X$ , i.e.,

$$\begin{aligned} X(d : D) &= \sum(e : D, a(d) \cdot X_1(d, e)) + b \\ X_1(d : D, e : D) &= (\sum(e : D, a(d_0) \cdot X_1(d_0, e)) + b) \cdot X_2(d, e) \\ X_2(d : D, e : D) &= (\sum(e' : D, a(e) \cdot X_1(e, e')) + b) \cdot X(d). \end{aligned}$$

Using the axioms for the sum operator distributivity and the conditional construct this gives a specification which is in normal form. From this sketch it follows in what we can extend  $E_1$  to  $E_2$  with a *process-specification* in *normal form* that defines a process behaving like  $X(t)$ :

$$\begin{aligned} X'(d : D) &= \sum(e : D, a(d) \cdot X'_1(d, e)) + b \triangleleft T \triangleright \delta \\ X'_1(d : D, e : D) &= \sum(e : D, a(d_0) \cdot X'_1(d_0, e) \cdot X'_2(d, e)) + b \cdot X'_2(d, e) \triangleleft T \triangleright \delta \\ X'_2(d : D, e : D) &= \sum(e' : D, a(e) \cdot X'_1(e, e') \cdot X'(d)) + b \cdot X'(d) \triangleleft T \triangleright \delta. \end{aligned}$$

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We remark that a *process-specification* in normal form has a syntax comparable to the *restricted Greibach Normal form* (rGNF) as defined in [BBK87]. They do not give an explicit method to obtain this form but give a sketch in the proof. We believe that their method is more difficult to implement than the method presented above, as we restrict ourselves to syntactically guarded specifications.

(End example.)

*3. Disambiguate the formal parameters.* Let  $p_2$  **from**  $E_2$  be specified in a *process-specification* that is in normal form. This step applies whenever it is the case that the *process-specification* of  $p_2$  has overloading of variable *names*. By definition of  $E_2$  being Statically Semantically Correct (SSC), this can only be the case if the *process-specification* of  $p_2$  contains more than one declaration. In this case we extend  $E_2$  to  $E_3$  by adding a *process-specification* in normal form that has uniquely typed variable *names*, and that defines a process  $p_3$  that behaves like  $p_2$  **from**  $E_3$ .

*Example of step 3.* Let  $p_2 \equiv X(t)$  where  $X(x : S)$  is specified as follows:

$$\begin{aligned} X(x : S) &= (a \cdot Y(f(x)) + b) \triangleleft t \triangleright \delta \\ Y(x : S') &= (c \cdot X(g(x)) + d(x)) \triangleleft h(x) \triangleright \delta \end{aligned}$$

We extend  $E_2$  to  $E_3$  by adding the *process-specification*

$$\begin{aligned} X'(x : S) &= (a \cdot Y'(f(x)) + b) \triangleleft t \triangleright \delta \\ Y'(y : S') &= (c \cdot X'(g(y)) + d(y)) \triangleleft h(y) \triangleright \delta \end{aligned}$$

Note that

$$X(t) \text{ **from** } E_3 \triangleq_{A_{N_{E_3}}} X'(t) \text{ **from** } E_3.$$

(End example.)

*4. Globalize formal parameters.* Let  $p_3$  **from**  $E_3$  be specified in a *process-specification* that is in normal form and that has uniquely typed variable *names*. This step applies whenever it is not the case that the *process-specification* of  $p_3$  has *global parameterization*:

**Definition 2.7** A *process-specification* in normal form with uniquely typed variable *names* has *global parameterization* iff each occurring variable *name* is declared in *all* of its declarations, that is in all occurring process parameter lists.

Note that a single-linear *process-specification* has by definition global parameterization. If step 4 applies, we extend  $E_3$  to  $E_4$  by adding a *process-specification* in normal form and with uniquely typed variables that has global parameterization, and that defines a process  $p_4$  that behaves like  $p_3$  **from**  $E_4$ . The next step will show the purpose of this extension.

*Example of step 4.* Let  $p_3 \equiv X(t)$  and let  $X(x : S)$  be specified as follows:

$$\begin{aligned} X(x : S) &= (a \cdot Y(f(x)) \cdot X(g(x)) + b(x)) \triangleleft t_1 \triangleright \delta \\ Y(y : S') &= (c \cdot Y(h(y)) + d(y)) \triangleleft t_2 \triangleright \delta \end{aligned}$$

We extend  $E_3$  to  $E_4$  by adding the *process-specification*

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$$\begin{aligned} X'(x : S, y : S') &= (a \cdot Y'(x, f(x)) \cdot X'(g(x), y) + b(x)) \triangleleft t_1 \triangleright \delta \\ Y'(x : S, y : S') &= (c \cdot Y'(x, h(y)) + d(y)) \triangleleft t_2 \triangleright \delta \end{aligned}$$

Note that  $x$  and  $y$  being different *names* is essential for application of this step. This extension has the following property:

$$X(t) \text{ from } E_4 \Leftrightarrow_{\mathbf{A}_{N_{E_4}}} X'(t, u) \text{ from } E_4$$

for any closed *data-term*  $u$  of sort  $S'$ .  
(End example.)

5. *Form single declaration.* Let  $p_4$  **from**  $E_4$  be specified in a *process-specification* in normal form that has uniquely typed variable *names* and global parameterization. This step applies whenever the *process-specification* of  $p_4$  contains more than one *process-declaration*. In this case we extend  $E_4$  to  $E_5$  by adding a *sort-specification*, a *function-specification* and a *process-specification* containing only one declaration that defines a process  $p_5$  which behaves the same as  $p_4$  **from**  $E_5$ . The following example also shows how the data-part of  $\mu\text{CRL}$  may be used, and the purpose of global parameterization (step 4).

*Example of step 5.* Let  $p_4 \equiv X'(t, u)$  where  $X'(x : S, y : S')$  is specified as in the example of step 4:

$$\begin{aligned} X'(x : S, y : S') &= (a \cdot Y'(x, f(x)) \cdot X'(g(x), y) + b(x)) \triangleleft t_1 \triangleright \delta \\ Y'(x : S, y : S') &= (c \cdot Y'(x, h(y)) + d(y)) \triangleleft t_2 \triangleright \delta \end{aligned}$$

We extend  $E_4$  to  $E_5$  by adding a new sort *Sort* with constants  $X', Y'$ , an equality function on *Sort* (we use infix notation) and the *process-specification*

$$\begin{aligned} Z(n : \text{Sort}, x : S, y : S') &= (a \cdot Z(Y', x, f(x)) \cdot Z(X', g(x), y) + b(x)) \triangleleft t_1 \wedge n = X' \triangleright \delta \\ &\quad + (c \cdot Z(Y', x, h(y)) + d(y)) \triangleleft t_2 \wedge n = Y' \triangleright \delta \end{aligned}$$

The summands  $b(x)$  and  $d(y)$  show the purpose of global parameterization: the process  $Z$  has to be parameterized with both the sorts  $S$  and  $S'$  in order to have the *specification*  $E_5$  SSC. Note that indeed

$$X'(t, u) \text{ from } E_5 \Leftrightarrow_{\mathbf{A}_{N_{E_5}}} Z(X', t, u) \text{ from } E_5.$$

(End example.)

6. *Linearize the process declaration.* Let  $p_5$  **from**  $E_5$  be specified in a *process-specification* in normal form containing one *process-declaration*. This step applies whenever the *process-specification* of  $p_5$  is not linear. In this case we extend  $E_5$  to  $E_6$  by adding *sort-*, *function-* and *rewrite-specifications*, and a single-linear *process-specification* that defines a process  $p_6$  that behaves the same as  $p_5$  **from**  $E_6$ .

*Example of step 6.* Let  $p_5 \equiv Z(X', t, u)$  where  $Z(n : \text{Sort}, x : S, y : S')$  is specified as in the example of step 5:

$$\begin{aligned} Z(n : \text{Sort}, x : S, y : S') &= (a \cdot Z(Y', x, f(x)) \cdot Z(X', g(x), y) + b(x)) \triangleleft t_1 \wedge n = X' \triangleright \delta \\ &\quad + (c \cdot Z(Y', x, h(y)) + d(y)) \triangleleft t_2 \wedge n = Y' \triangleright \delta \end{aligned}$$

### 3. From $\mu\text{CRL}$ to Single-Linear Specifications

We add two sorts to  $E_5$ . First a sort *Unproper* over which the *data-terms* are of the form  $X', t', u'$  and  $Y', t', u'$ , for all *data-terms*  $t', u'$  over the sorts  $S$  and  $S'$ , respectively. Note that this cannot be proper  $\mu\text{CRL}$  syntax, as *names* may not contain commas. However, for the purpose of readability we do not care for the moment and underline the elements of the unproper sort.

Next we add a sort *Stack* defined over *Unproper* and the constant  $\lambda$  for the empty stack, and the functions *pop*, *push*, *rest* and *is-empty* with rewrite rules as expected. We extend  $E_5$  to  $E_6$  by also adding the *process-specification*

$$\begin{aligned} Z'(n : S, x : S, y : S', s : \text{Stack}) = & \\ & (a \cdot Z'(Y', x, f(x), \text{push}(\underline{X'}, g(x), y, s)) + b(x)) \quad \triangleleft t_1 \wedge n = X' \wedge \text{is-empty}(s) \triangleright \delta \\ & + (a \cdot Z'(Y', x, f(x), \text{push}(\underline{X'}, g(x), y, s)) + b(x)) \cdot Z'(\text{pop}(s), \text{rest}(s)) \quad \triangleleft t_1 \wedge n = X' \wedge \neg(\text{is-empty}(s)) \triangleright \delta \\ & + (c \cdot Z'(Y', x, h(y), s) + d(y)) \quad \triangleleft t_2 \wedge n = Y' \wedge \text{is-empty}(s) \triangleright \delta \\ & + (c \cdot Z'(Y', x, h(y), s) + d(y)) \cdot Z'(\text{pop}(s), \text{rest}(s)) \quad \triangleleft t_2 \wedge n = Y' \wedge \neg(\text{is-empty}(s)) \triangleright \delta \end{aligned}$$

Note that

$$Z(X', t, u) \text{ from } E_6 \Leftrightarrow_{\mathbf{A}_{N_{E_6}}} Z'(X', t, u, \lambda) \text{ from } E_6.$$

(End example.)

The general idea behind step 6 is that we can define a sort that has a class of (properly encoded) *process-expressions* as its closed *data-terms*, and a sort *Stack* of stacks over this sort. Upon a summand of the form  $a \cdot X \cdot Y$  we stack the subprocess  $Y$ , and upon a non-recursive summand of the form  $a$  and a non-empty stack, we pop the first subprocess for execution.

### 3. From $\mu\text{CRL}$ to Single-Linear Specifications

The Basic Process Syntax of the previous section is a concise way to specify processes with data, but somewhat inconvenient to specify protocols. Usually protocols are specified as a parallel composition of processes. Therefore we reintroduce more involved operators (merge, encapsulation etc.) into the syntax. This will make specifying easier, but at the same time we have to be attentive that the specifications we allow can be translated to a linear format.

It is well-known that regularity (and hence linearity) is undecidable when the occurrence of parallelism in the syntax is unrestricted [BK89]. Moreover finiteness conditions as in the case of process algebra *without* data such as in [MV90] become undecidable if processes and data interact.

It will be sufficient for our purposes to exclude specifications like

$$X(n : \text{Int}) = a(n) \parallel a(n+1) \cdot X(n+2)$$

where a merge operator is used in the scope of the recursion. For convenience the above mentioned operators will only be used to compose processes which are in BPS, or can be translated to it. Such a strategy is straightforward and is used in e.g. the AUTO tool [SR91] to specify processes. In [Sch94] syntactic conditions similar to ours are formulated and motivated with examples.

We formalize the restriction to a specification with a safe use of parallel operators with the aid of syntactic guardedness.

**Definition 3.1** Let  $E$  be a well-formed  $\mu\text{CRL}$  specification.  $E$  is *safely linearizable* iff

#### 4. Conclusions and Future Work

1.  $E$  is the extension of a syntactically guarded  $\mu\text{CRL}$  specification  $E_{\text{syng}}$  with (parametrized) process names  $N_{\text{syng}}$  and,
2. All right hand sides of process declarations in the extension  $E - E_{\text{syng}}$  are process expressions in which only (parametrized) process names in  $N_{\text{syng}}$  occur.

Without proof we state that well-formed  $\mu\text{CRL}$  specifications, which are safely linearizable are bisimilar to linear process specifications (see e.g. [BP94]). The receipt to obtain such a specification is obvious. We translate in an innermost-outermost fashion all process declarations to single-linear format, starting with the declarations in BPS. The other operators are eliminated in the usual way, by expansion and straight forward data parametric substitution, using the recursive specification principle RSP [GP93].

Of course the conditions of Definition 3.1 can be relaxed to allow more nesting. For this an iteration à la syntactic guardedness suffices.

#### 4. Conclusions and Future Work

In this paper we aim at arriving at a single-linear format. We believe that this is a natural format for a parametrized graph or a symbolic transition system. Of course other formats are possible. The use of steps 3–5 can be avoided if we had aimed at a *linear format*, i.e. several coexisting linear declarations. One could say that this is a matter of taste, but we feel that Step 6 becomes more difficult and the resulting specification is less insightful. If several (mutually dependent) process declarations remain, process calls are not uniform and explicit list access has to be introduced, instead of implicit bindings. Also extra control information has to be supplied to process calls, to allow correct selection of the called process. Also in some way or another, process calls have to be stacked with varying types of parameters. The data structure needed will be a list of lists of varying types, and hence be complicated.

At the moment the first author is implementing the above described translation in the ASF+SDF system [Kli93]. This general purpose term rewriting system has several built-in possibilities, among them the possibility to compile rewriting systems to C code. We can make ample use of the fact that  $\mu\text{CRL}$  data and process specification is ASF like. We are aiming to integrate this “linearizer” with the well-formedness checker [HK95] developed for  $\mu\text{CRL}$ .

We see several next steps. A first (conservative) next step is to build an “instantiator”, a front end which can translate single-linear specifications to labeled transition systems. These can then be interfaced with the tools in the Concur 2 project, which offer various model checking facilities for pure calculi. Of course it will then be essential that all data types are finitary.

A second, more ambitious step is to implement a part of the logic of [GvV94], which is tailored to the syntax of  $\mu\text{CRL}$ . An obvious strategy would be to expand modal formulas, to instantiate data and check the pure formulas on a labeled transition system.

Third, we can make a detailed investigation of the complexity of the various steps and suggest optimizations. Furthermore we can look for a class of specifications for which the stacking of processes in Step 6 of Section 2 can be avoided, using the results of [MM94].

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