

Walk like a Physicist

An Example of Authentic Education

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You learn walking at the age of one. Hereafter it goes naturally. You do not give it much thought anymore until you are getting on a bit or in case you get an injury of your legs. Then you realize how many times you actually use your limbs and how many ways of walking you practice: normal walking, jogging, running, hopping, race walking, strolling, crutch walking after an accident, and so on. You may start wondering what the differences are between normal walking and running, and why you make a transition from one gait pattern to another in a natural way. The mentioned gait patterns all have in common that they are periodic motions in which one step is set after the other. The center of gravity goes up and down regularly and the arms are moving like a pendulum. Gait analysis deals with a scientific description of human locomotion. Mathematics and physics play an important role in it and video measurement is a frequently used research tool.

We have created learning materials for students in upper pre-university education to act like human movement scientists in the classroom. They record video clips of their own ways of walking with a web cam and they use the Coach computer learning environment for measuring on these movies and analyzing the collected data. They determine gait signatures and hip-knee cyclograms of their movements, and they compare their results with those of fellow students.

We interpret the authentic nature of our activities as the opportunity for students to work directly with high-quality, real-time data about human gait in much the same way movement scientists do. We lead them to use the same theoretical framework, nomenclature, research methods, and techniques as practicing professionals. In essence, we try to make their science learning resemble science practice, in which investigations can often be characterized as being challenging, complex, open-ended, and cross-disciplinary, and as requiring a strong commitment of participants plus a broad range of skills.

In our educational research we address the question how ICT and real-life contexts can contribute to the realization of authentic tasks for students. We apply the method of developmental research: we work out our ideas about mathematics and science education in learning materials and ICT tools, we try these out in practice, and we reflect on the evaluation. This provides input for both the development of the learning environment Coach and the creation of learning materials for students that stimulate and enable them to carry out investigation tasks at a rather high level. In this particular study, our main research questions are: (i) Is the computer learning environment Coach a valuable tool for the students in their practical work, in the sense that it supports them in obtaining, organizing, displaying, manipulating and analyzing data? (ii) Does the instructional strategy of first familiarizing students in a classroom setting with the way professionals work before engaging them in doing their own gait analysis work well, i.e., does the chosen setup make authentic science feasible at student level?

This paper is organized as follows. In the first four sections we present our learning material. We cite from the report of a team of two students to convey how they understood their task. We compare this with the authors' intentions as stated in their introductory text. We outline the instructional setup and the contents of the students' activities. Finally, we discuss the mathematical and biomechanical aspects of gait analysis presented in our learning material. In the fifth section we describe our objectives both from author's point of view and from researcher's point of view. In the next two sections we focus on the ICT aspects of the students' activities. We describe how the students collect gait data using the data video tool of Coach and we put our work in the perspective of previous research on video-based laboratories. Hereafter we report on our classroom experiences and we discuss possible extensions of the students' activities towards larger research projects, as we tried out in a master class on human gait. We end with a summary of our research findings.

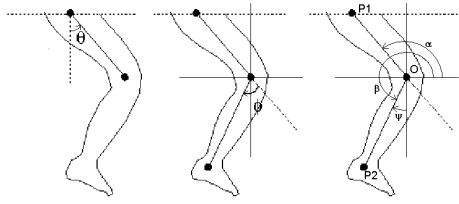
1. The Task According to Two Students

Let us first translate the introduction unabridged of the research report of Gerda and Vianne, because it gives a good impression of how students understood their task and how they described it in their own words.

“This research is about walking. A child learns walking at the age of one, approximately. Hereafter walking becomes a very usual thing. This is why most people do not give it much thought anymore, even though it is such an interesting process. The fact is, every person walks differently and can walk in various styles. One can walk, run, tiptoe, or go with a book on your head, just to mention a silly walk. The way you walk sometimes reveals the way you feel. When you go dragging your feet you might be unhappy or tired, and when you have a springy step you may be in a burst of liveliness.

Various disciplines do research about walking and running: medicine, sport science, motion-picture industry, and forensic science (identification). All these disciplines need in their own way good insight in human gait.

In the project ‘Stilstaan bij lopen’ [Pondering on Walking] we analyzed human gait by means of the data video tool of the Coach software. The ultimate aim was to compare two kinds of walking. We looked at tiptoeing and walking with a book on your head. We made ourselves a mathematical analysis of walking with a book on your head. We measured the hip and knee angle. Because the angles θ and ϕ could not be measured directly, we had to measure the angles α and β by taking the knee as origin and by taking the hip and ankle as measurement points.



With formulas we could then plot the hip angle and the knee angle against each other. The graph that we created is known as a cyclogram. Hereafter we compared our cyclogram with the one of Kah-Kih, who analyzed tiptoeing. We found it difficult to understand what the differences precisely were, but in the end we succeeded.”

2. The Task in the Authors’ Eyes

We cite from the introduction of the learning material, which is meant for students in upper pre-university education.

“In this practical investigation task you will get acquainted with kinematical aspects of gait analysis using the data video tool of Coach. One of the issues that come up is how you can describe the movements of arms and legs during walking with trigonometric functions. You will also investigate the connection between the angular displacements at the knee joint and at the hip joint; the angle-angle diagram in question is called the hip-knee cyclogram. In the final part of the investigation task you record with a web cam a gait cycle of your choice, you construct and analyze the corresponding hip-knee cyclogram, and you compare this with a cyclogram of a different gait (preferably a diagram of a fellow student). For grading purposes we expect a written report of this final part of the task only.”

We think that it was clear to Gerda and Vianne what we expected from them. In a very readable introduction of their report they wrote down what the task was according to them and what they did. We borrowed most of their wording in the abstract of this paper. It is also clear what comparing of cyclograms of two different gaits means for them, viz., finding the differences and trying to understand them. They do not look for common things in the cyclograms.

3. Outline of the Learning Material

The learning material has been designed for students who are in their penultimate year of pre-university education (age 16-17 yr.), and who have already some experience with practical investigation tasks and with Coach, including the data video tool. The material can be downloaded from the web page www.science.uva.nl/~heck/research/walking/ and it consists of the following four assignments:

Activity 1. Mathematical Analysis of Human Gait. Students are introduced into the typical normal walk cycle and the events of gait. They also practice their skills in using the graphical and video facilities of Coach and its curve fitting tool. The

students do not yet collect themselves data from the video clip. Instead, the authors have prepared these.

Activity 2. Swing Phase in Sauntering Gait. Students analyse the motion of the swing leg during a slow walk. They record themselves the coordinates of hip and the foot with respect to the knee joint and they derive from these data the hip angle and knee angle as function of time. They check how well these functions can be approximated by sums of two sine functions. Instructions are detailed and guide students through technical steps.

Activity 3. The Gait Cycle in Sauntering Gait. This activity is a continuation of the previous one, but now the complete gait cycle is considered, i.e., the interval of time or sequence of motion occurring from heel strike to heel strike of the same foot. The students do not need to record data themselves. They only investigate how the hip and knee angle can still be described by sums of two sine functions. Furthermore, they investigate the periodicity of the motion via the hip-knee cyclogram, which is a diagram in which the knee angle is plotted against the hip angle. The cyclogram is a parametric curve with respect to time and characteristic points on the curve correspond with events in the gait.

Activity 4. Investigating one’s own motion. The first three assignments are video-based laboratories that can be done during regular, fifty-minute lessons. The main purpose of these activities is to prepare the students for the fourth assignment, which is a small investigation task with an estimated workload of three to four hours. Much of the work can be done outside regular lesson hours. In the fourth activity the students choose to perform a gait of their own choice and to record it with a web cam. Hereafter they collect and analyze data on their video clip. To limit the practical work to a rather short assignment the students only need to construct the hip-knee cyclogram of their motion, to compare their result with one obtained by a fellow student, and to report their findings in a short note.

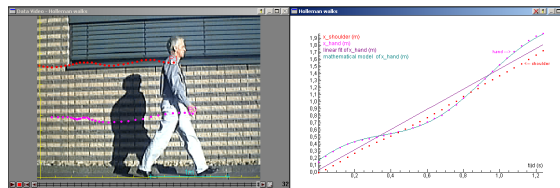
4. The Learning Material in Detail

Below we describe the four student activities in great detail. This section is mainly meant for the reader who wants to know more about the mathematical and biomechanical aspects of gait analysis that come up in the learning material.

Activity 1. Mathematical Analysis of Human Gait. The purpose of this activity is to let students familiarize themselves with the topic, with video measurement, and with sinusoidal regression. The two main themes are

- (i) the determination of temporal-spatial gait parameters such as stride length, stride rate, gait speed, the length of the stance phase and the swing phase, or the bipedal time, by means of video recordings.
- (ii) the determination of a curve that fits the collected data on the video clip in the best way.

In the first activity we concentrate on the movements of the arms during normal walking. The arm motion with respect to the shoulder joint is similar to that of a pendulum. To study this motion we use a video clip in which the teacher walks outside on a sunny afternoon. We created it with a web cam connected to a notebook computer. In the screen shot below you see the video clip with measured points marked and the diagram window that displays the measured horizontal positions of the shoulder and the hand with respect to the fixed coordinate system.



Screen shot 1: arm movement during normal walking.

At first sight, the shoulder moves horizontally with constant velocity. We model the horizontal position of the hand, $x(t)$, mathematically as a combination of a straight line and a sinusoid, i.e., by the formula $x(t) = at + b\sin(ct + d) + e$, with parameters a , b , c , d , and e that must be estimated. For this we use the 'function fit' tool of Coach: first apply a linear least-squares fit of the measured data and subsequently make a sinusoidal fit of the graph of the difference between measured data and the linear fit. In this example the period of the pendular motion of the arm turns out to be 1.1 second. This number is close to, but greater than the natural period of the arm, which was computed for the walker as 1 second, using the force-driven harmonic oscillator model of (Holt et al., 1990) and the formula

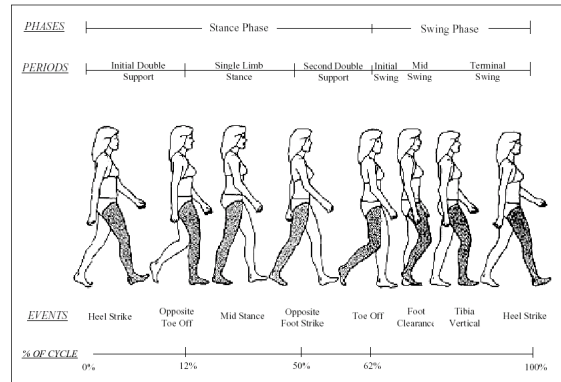
$$\text{natural period} = 2\pi \sqrt{\frac{I}{2mgd}}$$

Here I denotes the moment of inertia of the arm (including the hand), g represents the constant of gravity, m stands for the mass of the arm, and d is the distance from the center of gravity of the arm to the shoulder joint. This formula and others (see Dumont & Waltham, 1997) may be out of scope for high school students, but the least one can do is ask them to verify that the natural period is not given by the formula for the mathematical pendulum

$$\text{natural period} = 2\pi \sqrt{\frac{l}{g}}$$

where l denotes the arm length, i.e., ask them to verify that the traditional harmonic oscillator model does not work and ask them to explain why it does not work actually. As a matter of fact, the measured period of the arm motion is very close to the natural period of the leg of the walker when you apply Holt's formula to this body segment. In other words, we found that the force-driven harmonic oscillator model accurately predicts the preferred

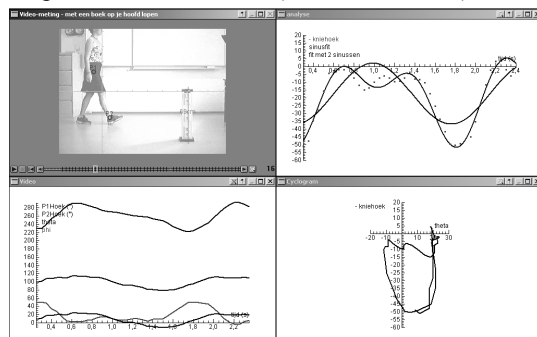
stride frequency for normal walking and that the arms are swinging in such way that they match to the leg movements. Note that this is not in agreement with the observations on the process of walking in (Bachman, 1976), where the natural arm swinging period was computed by modeling the arm as a free swinging cone and where the author stated that the casual walking period corresponds to that of the swinging arm.



Typical normal walk cycle illustrating the events of gait (Rose and Gamble, 1994)

Activity 2. Swing Phase in Sauntering Gait. Gait is a periodic movement of each foot from one position of support to the next position of support in the direction of progress. The previous picture taken from (Rose & Gamble, 1994) describes the various phases in human walking.

In this activity, students study the swing phase of sauntering gait. They measure on a video clip the angles that the thigh and the shank make with respect to the coordinate system that has its origin at the knee joint. After the data collection, the empirical work starts: they try to describe the measured quantities mathematically. A sum of two sinusoidal signals with different frequencies turns out to be a rather good description of the angles as functions of time; see the diagrams in the upper-right window of the screen shots 2 to 6. In mathematical physics terms, this means that the movement of the leg is well described by a bilateral and dynamically coupled oscillator model (Yam et al., 2002).

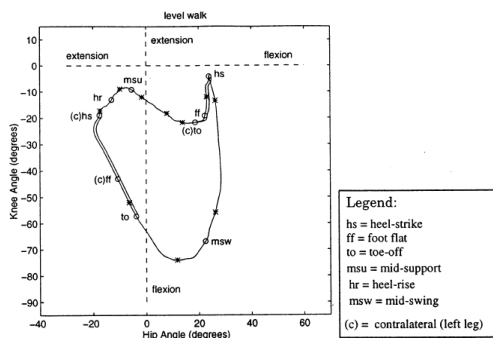


Screen shot 2: Vianne walking with a book on her head.

Activity 3. The Gait Cycle in Sauntering Gait. This activity is a continuation of the previous investigation. But now our focus is on the complete gait cycle, including the stance phase when the leg supports the upper body. Again, a sum of two sinusoi-

dal signals with different frequencies turns out to be a rather good description of the angles as functions of time during the gait cycle. The Fourier representation of the joint angles during walking is called the gait signature. But the motions of the limb segments are not completely independent from each other: for example, the further you stretch your hip, the less you can bend your knee, unless you are a ballet dancer. In order to get better insight in the way the movements of thigh and shank are coordinated during a stride we plot the joint angles of the hip and the knee against each other. From mathematical point of view this is nothing more than a parametric curve (with time as the independent variable), which is ideally closed because of the periodicity of the leg motion.

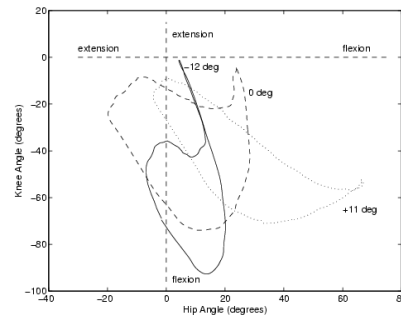
Below we show the cyclogram of a normal equal-level walk, which we took from (Goswami, 1998). The joint angle assignment convention is such that the cyclogram has a counter-clockwise direction. The complete gait cycle is divided into 10 equal temporal segments and are marked by ‘*’ on the cyclogram. The spacing of these points is directly proportional to respective joint velocities: when the joints bend and stretch slowly, the points are closely spaced. On the diagram you also see points marked with an ‘o’: they belong to important events of the stride such as heel-contact, heel-rise, and toe-off.



Let us travel along the cyclogram to see how the diagram contains information about the posture of the leg and the limb motion during a stride. An almost vertical line characterizing the rapid knee flexion and little hip movement represents the period just after the heel-strike. The inclined line connecting foot-flat and mid-support shows that the hip begins to extend along with the knee in this phase. At mid-support the knee extension reaches again a (local) maximum and the knee translates to flexion, which continues through stance into swing phase. Somewhat later in the stance phase the hyperextension of the hip reaches a maximum and gradually reverses. The toe-off occurs before the knee is fully flexed. Typically the swing phase starts at 0° hip angle. There is almost no thigh movement between the mid-swing and the heel-strike.

The geometrical features of a hip-knee cyclograms characterize human gait patterns in a quantitative and objective way, and they often change

when gait conditions alter. For example, the figure below exhibits the substantial change in the forms of the hip-knee cyclogram for natural human walks on a treadmill at different inclinations (ascent, descent, and level walking).



In (Goswami, 1998) it is also reported that the range of the hip movement has a linearly increasing trend going from -13° to $+13^\circ$, whereas the knee angle behaves in an opposite but symmetric manner, i.e., the range of the knee movement decreases linearly with increasing slope. Another example of how the hip-knee cyclogram changes its form when the gait conditions alter can be found in (Hershler & Milner, 1980). The authors reported that, for normal healthy gaits at different speeds, the perimeter P and the area A of the cyclogram are approximately linearly related to the average gait speed, whereas the quantity P/\sqrt{A} stays roughly constant.

Activity 4. Investigating one's own Motion. In the previous activities the students were introduced into the methods and techniques of gait analysis so that they would be ready for doing a small investigation task on their own. The student research question could be formulated as “What says a cyclogram about a gait cycle and can it be used to distinguish gait patterns?” Although we make suggestions for looking at movements such as jogging, walking backwards, silly walks like goose walking, and fitness movements, we do invite the students to use their imagination and to select a motion that is most appealing to them.

In this activity the students use a web cam to record their own (funny) gait. They analyze the collected data in the form of a hip-knee cyclogram: they identify the important events in their gait and they relate the geometrical features of the diagram with their motion. They also compare their cyclogram with one obtained by a fellow student, i.e., they search for differences and similarities, and they try to understand their findings in terms of the gait patterns. Finally, they write a short note about their study.

The main reason to restrict the students' investigation task to the creation and analysis of cyclograms is the limited amount of time usually given to practical work at school. But students who get infected by this investigation task with the gait virus can extend it to a larger research project.

5. Authors' and Researchers' Objectives

The developer of learning material and the educational researcher are in developmental research often one and the same person. This study is no exception. So it comes to no surprise that the objectives of the learning material are closely connected with the research questions that we want to address. Our main objectives as authors of the learning material are to let the students

- work with real data collected from video clips made by a web cam;
- carry out practical work in which they can apply much of their present knowledge of mathematics and physics in a real life context;
- practice ICT-skills, in particular making a video clip and carrying out measurements on it with a data video tool;
- experience that diagrams that are used in practice are not just pretty pictures, but contain much information about the real life phenomenon under study;
- be in contact with current research work, in our case movement science, including the nomenclature and research methods used.

These objectives are rooted in our belief that the main purpose for doing practical work is to experience authentic mathematics and science, and to enjoy and become competent in it.

We deliberately let students do the experiments themselves and let them study an ordinary real life phenomenon. We could have chosen to study, for example, body movements on a training apparatus in a fitness room instead of natural walking, but then the topic would have become more artificial and it would have come to less surprise that mathematics and physics have some bearing on real life. We also permitted the students some freedom of choice in the gait pattern that they eventually investigated. We hoped that this would motivate them and make them more strongly committed to the task. It also offers the students the opportunity to study a gait pattern in which they have personal interest or that is meaningful to them. We chose on purpose a complex phenomenon for which a complete mathematical and biomechanical description fails and certainly would be out of reach of students, but for which simplified models still yield interesting results and provide qualitative answers to research questions. Walking (gait) certainly meets these criteria; see (Messenger, 1994; Jones & Barker, 1999) for a comprehensive analysis of the human gait cycle and the parameters required to model the cycle. The attitude of pursuing unanswered questions and of strong commitment are one of the key features of authentic science practice listed in (Edelson, 1998). They mention two other features, viz., 'tools and techniques' and 'social interaction', which we will discuss below.

The main reason for letting the students do the experiments themselves, following more or less

their own route, is that they get in this way first-hand experience of mathematical and physics concepts and of techniques used in research work. We hope and expect that it will give meaning to these concepts as well as to the research methods, tools and techniques. Therefore we try to let the students' tools resemble the professional tools. This practical work was meant to contribute to students' understanding of what video analysis means, what it is good for, and how it can be applied. What they have learned in this small investigation task, they can utilize in the larger research project that they carry out in the final year at school. As an example of success we mention that two students in the class that participated in the experiment realized in their research project one year later that they could beneficially use interactive video to study bungee jumping. Two other couples decided to make human gait the subject of their practical work.

In the activity we tried to pay much attention to reading and interpreting graphs. The main reason for this is that students tend to interpret any graph shown to them in a narrow-minded way. In physics lessons it is a distance vs. time graph and in mathematics lessons it is a graph of a formula. In this way students do not experience that a single picture can convey much information about a phenomenon and that plotting a graph may help to interpret measured data. This provides a serious handicap to their understanding of many subjects of study.

Mathematics and science is not just investigation. It is a human activity in which scientists share results, concerns, ideas, plans, and questions among collaborators. To mimic this as much as possible we chose an instructional setup in which students work in pairs and in which teams are obliged to work with results of others. Furthermore, we organized that students could work not only during regular lesson hours, but also outside the classroom.

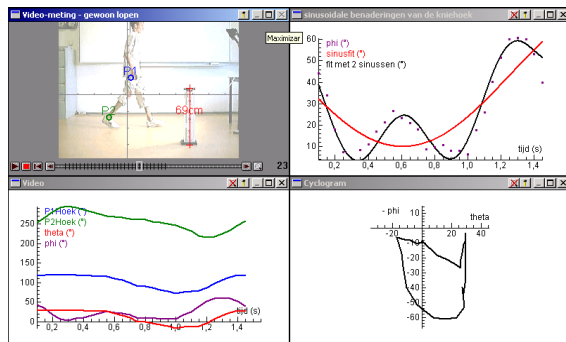
The benefits of making mathematics and science learning better resemble mathematics and science practice are clear. But how does one implement it at school? In this paper we report about such an attempt on a small scale and we describe how our instructional setup in the authentic learning model worked in practice. Our main research questions are:

- (i) Is the computer learning environment Coach (Mioduszevska et al., 2001) a valuable tool for the students in their practical work, in the sense that it supports them in obtaining, organizing, displaying, manipulating and analyzing data?
- (ii) Does the instructional strategy of first familiarizing students in a classroom setting with the way professionals work before engaging them in doing their own gait analysis work well, i.e., does it make authentic science feasible at student level?

Note that the chosen instructional strategy closely resembles authentic science practice: scientists usually assemble prior knowledge about their subject of study from expert sources and they acquaint themselves with the common methods and techniques. If necessary, they invest time to acquire the skills needed for using new and promising technology in the field of study.

6. Video Measurement

Our first research question can be specialized into the question whether students using a web cam, a computer, and Coach can obtain such a nice hip-knee cyclogram as shown before. The answer can be found in screen shot 3, which comes from two students who did the practical work.



Screen shot 3: Jordi sauntering in the physics classroom.

On the left-hand side you see the video clip and the time-graphs of the recorded angles. In the upper-right window are shown the sinusoidal fit of the knee angle and the fit consisting of a sum of two sine graphs. The latter graph was created with the ‘function fit’ tool of Coach similarly as the arm movement was modeled in the first activity. First a sinusoidal least-squares fit of the measured data was applied and subsequently a sinusoidal fit of the graph of the difference between measured data and the fit already found was made. The sum of the two fits is the final result. The cyclogram in the lower-right window looks very similar to the one taken from the scientific literature. Not only the geometry of the diagram is the same, but also the locations of the important events during the walk (like heel-strike, mid-support, heel-rise, toe-off, and so on) are almost the same.

But how do the students collect the data with Coach in the first place? How can they measure the angles of the hip joint and the knee joint on each frame? Do they use an electronic protractor for this purpose, or what? As a matter of fact, clicking on points in the video clip suffices: the students inform the system that they want to reposition the origin of the coordinate system for each frame of the video clip and also that they want to measure the positions of two points. Now look again at the picture in the introduction of Gerda and Vianne: the hip angle θ and the knee angle φ are the quantities of

interest. They can be computed with the following formulas (with angles in degrees)

$$\theta = \alpha - 90 \quad \text{and} \quad \varphi = \beta - \alpha - 180,$$

where α and β are the angles of the polar coordinates of the hip joint (P1) and the ankle joint (P2) with respect to the coordinate system with the origin O located at the knee joint. But when you click a point on the video clip, Coach automatically records these polar coordinates along with the Cartesian coordinates. So, no extra work is needed. By the way, when the students define a wrong formula for the hip angle or for the knee angle, this catches the eye immediately: it will result in graphs of body movements that are physiologically impossible.

Do not get the impression that the data video tool in Coach has been specially developed to make gait laboratory work possible. In fact, the tool has been implemented long before the real-life context of human gait came into view in our work. The data video tool has its roots even in the early nineties (Ellermeijer et al., 1996). It illustrates more that the design of educational technology to support students in performing authentic tasks can be successful, even though education puts higher demands on the user-friendliness of technology than the science workplace. Scientists are more willing to invest time in learning to use the tools that are relevant to them because they can apply them frequently, they can collaborate in this way with colleagues in the field, and they have already obtained the knowledge to understand the use of the tools. For example, professional gait analysis software allows its users to work with stick models, offers a ‘follow-mode’ for tracking body markers, and it provides various facilities to create diagrams that are dynamically connected with each other. It also works in the three dimensional world with high-speed cameras and it combines data coming from a force platform, electronic goniometers, and from angular rate sensors with data collected on video clips. All this makes the technology also too complex and too expensive for use at school.

Although the design of the data video tool in Coach is based on requirements that have been set before implementation, it is more a process of prototyping and iterative refinement that runs simultaneously with innovation of curricula, pedagogy, and technology. For example, the availability of low-cost web cams enables the recording of video clips by students. The educational reform that advocates the use laboratory work as an effective means to bring students in contact with current science research gives room for authentic use of technology. Classroom experiments like the one described in this paper give an idea of how students work with and learn from the tools in practice. These experiences are used in subsequent design and implementation of the computer learning environment Coach.

7. Research on Video-Based Activities

Research on video-based activities, or on Video-Based Laboratory (VBL) as it is also called, has mainly focused on physics education and particularly on the field of kinematics, in which concepts like position, velocity, and acceleration play a key role. We refer to (Hilscher, 2000) for an overview of the recent state of affairs on the use of video in physics education. Here we only review few of the important results of educational research.

Beichner (1996, 1999) conducted research on student understanding of kinematical graphs and the effect of VBL on the learning process. His main conclusions were that

- the students' ability to interpret graphs improved with VBL in comparison with traditional approaches;
- occasional use of VBL in a teacher-led demonstration does not bring something extra;
- a combination of demonstrations with hands-on activities has the greatest impact.

Improvements occurred through VBL with respect to all classical misconceptions on graphs and graphing. Research (see e.g., Leinhardt et al., 1990) identifies several areas of difficulties: connecting graphs with physical concepts, connecting graphs with the real world, transitioning between graphs and physical events. VBL helps in all these areas.

A major contribution of Boyd and Rubin (1996) is that they looked for recognizable situations, in their case study a series of pictures of animal movements, from which high school students could develop graphical and other mathematical representations. They investigated to what extent VBL helps students to see a graph not just as a nice picture corresponding with some mathematical function, but also as an instrument in their reasoning. The authors paid much attention to the differences between every day experiences, VBL, and mathematical representations. For example, time is in the real world a continuous, irreversible quantity that cannot be stopped. However, in VBL you can play with time: you can play the movie clip at various speeds or in reverse mode, you can step forward and backward through the clip frame-by-frame, you can pause the movie, etc. Students benefit from being conscious about such differences

Zollmann and Brungardt (1995) examined the effect of delayed-time versus simultaneous-time viewing of a motion event recorded on a videodisc and a corresponding kinematical graph. Using quantitative, qualitative and retention data, they found no significant learning difference between using simultaneous-time and delayed-time analysis for student understanding of kinematics graphs. However, videotapes of treatments and end of class interviews uncovered some possible advantages of simultaneous-time technique. Students who could view the motion graph simultaneously with the

video replay were aware of the simultaneous feature and seemed motivated by it. They also more often ignored small fluctuations in graphs and exhibited fewer eye motions between the video screen and the computer graphs than students in the delayed-time group. The interviews suggested that the simultaneous-time group of students demonstrated more discussion during graphing and displayed less confusion on velocity-time and acceleration-time graphs than those in the delayed-time group. These findings were on the whole confirmed in the work of McCullough (2000), in which the effects of VBL on a cooperative-group problem solving pedagogy in introductory physics were examined.

Students need of course more skills than being able to interpret graphs in motion studies. They must also acquire good understanding of physical concepts like force, momentum, and energy, and they must develop skills in the area of problem-solving, modeling, hypothesis-testing, interpreting data from measurements or data bases, and so on. All this not only in physics, but also in many other disciplines! For this reason, a cross-disciplinary approach that includes science and mathematics seems most suitable. Research on the possible contribution of VBL to the development of these skills and of conceptual knowledge in mathematics and science is still in its early stages. One reason may be that the technology is rather new and that in particular recording of video clips by students themselves at school on a large scale has only recently become possible through the introduction of web cams. Another reason may be that whenever data can be collected with a computer using sensors or data loggers, preference is given to such an experiment because it goes in general faster, it provides more data, and it seems to stimulate more between-student collaboration and discussion. At least, these were conclusions in (George et al., 2000), which is a study on student learning of conservation of momentum and energy in interactions (collisions) using two specific instructional technologies, viz., VBL and microcomputer-based laboratories (MBL, also referred to as 'computer based data logging').

It may well be that VBL is restricted to situations in which there exist no alternatives for measuring data because a 'real' experiment would be too expensive, too dangerous, or too difficult to carry out. However, Laws and Pfister (1998) listed the following advantages of VBL in comparison to MBL and traditional laboratory work:

- It is an easy, fast, and broadly applicable method to collect data in practical work. One may also profit from the fact that students nowadays grow up with video technology so that they can devise their own projects.
- Because simple mouse clicking replaces the tedious work of recording data, students make

less mistakes in measuring than in a traditional laboratory experiments and they can concentrate on the physical phenomena under study.

- Once video clips are available, no experimental setup is required anymore. This saves time, takes away practical issues that must be dealt with in laboratory experiments, lowers costs of equipment, and offers the possibility of investigating “real world” events such as sports events, dance performances, movie stunts.
- From the data recorded on a video clip, one can compute a new point, e.g., the center of mass of a moving body, and mark this calculated point automatically on the movie. In our experiment on human locomotion we could for instance compute the center of mass of the swinging leg and study the movement of this point with respect to the hip joint.

In our research we have noticed the following additional advantages of VBL:

- It is not necessary to determine in advance in detail what and how you are going to measure. Instead you may let yourself be driven more by curiosity after watching the video clip. You can use one and the same clip for different investigations. This promotes the idea that one can look at the same phenomenon from several points of view. In our example of gait analysis, you do not have to restrict yourself to looking at forces and other physical concepts, but you may also pay for example attention to the posture and coordination of body segments during a variety of movements.
- At any time and any place, e.g., later at home or elsewhere out of the classroom, a student can verify his/her video measurements and, if necessary, correct them. Work is not restricted to the lesson hours, but can be done over an extended period of time.
- The video clip on which you measure and the corresponding mathematical representations such as graphs and tables are always synchronized in Coach. This means that pointing at a graph or a table entry automatically shows the corresponding video frame and that selecting a particular frame highlights the corresponding points in diagrams, when scanning mode is on. This makes scrubbing, i.e., advancing or reversing a clip manually, an effective means to precisely identify and mark interesting events in the video clip and to relate them with graphical features. This supports students to transition between graphs and physical events.
- VBL clarifies the use of a variable as a symbol for a variable object, in addition to the use of a variable as a placeholder and a polyvalent name. The position of a moving object has coordinates that change in time. The angles of the hip and knee joint in our example about human locomotion are time-dependent.

8. Our Classroom Experiences

The experiment took place just before summer holidays in a class of 18 students matching the learner profile that was described at the beginning of the third section. The practical assignment was not part of the students’ examination portfolio for physics, but it was graded as a regular test in the semester. The students worked in pairs for three weeks during regular lesson hours (twice weekly), in which the physics teacher and the authors of the learning material were present as assistants. In these weeks, most students went through the introductory activities and obtained the video for their final investigation task. The students also had a home version of the software including the project files. Estimated study load was 6 to 8 hours in total.

For practical reasons we used one experimental setup for recording the video clips of students. We placed a web cam connected to a notebook computer in the physics classroom and prepared the video capturing and processing software such that one clip after another could be created without the need to set up everything again. We used for this purpose VirtualDub, which can be downloaded freely from www.virtualdub.org. You need video processing software to improve the quality of a video clip (e.g., the brightness), to remove superfluous frames from a clip, and to compress it in a format that Coach recognizes. All this would be too time-consuming for students to do in this practical work.

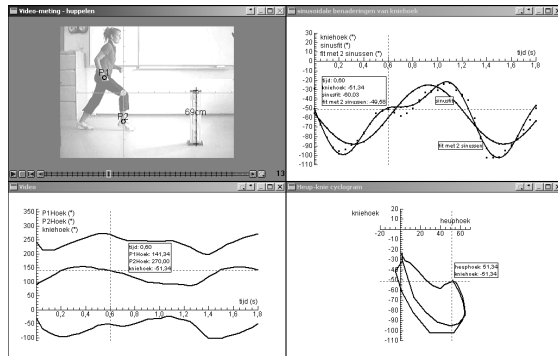
Deliberately we spoke in the first sentence of this section about an experiment in the classroom. The following two questions could only be answered in practice:

1. Will the free choice of a gait pattern by the students in the final activity result in useful cyclograms?
2. Are the mathematical analyses that the students practiced in the first part of the project be applicable to their own body motions.

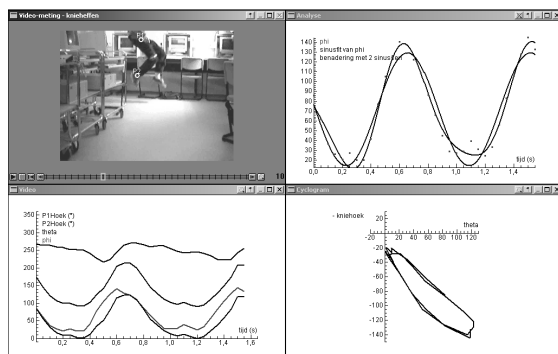
All things considered, the body movements during locomotion are complex and we only apply simple mathematical and physical models.

One of the most important messages coming from this experiment is that it goes surprisingly well, also when you use simple recording apparatus like a web cam. The screen shots in this paper speak for themselves. Below you see two screen shots with results of skipping and knee raising. The hip-knee cyclogram of the last example may look different from the ones shown before, but it is still in agree with scientific literature reports: its shape is similar to the cyclograms of the running A and B drills with sprinting (Kivi & Alexander, 1998). No matter what the motion actually is – normal walking, hopping, jogging, running, walking backwards, walking on your toes, or stair climbing – you will almost always get useful cyclograms. And we are not the only ones who were surprised. The students

Dries and Edwin commented on the project in the following words: “It is cool to see that your legs make such a nice mathematical curve, the stroke when your foot makes contact with the ground, and so on.” They add to this: “Furthermore, it is something different from the usual standard lesson. This was nice.”



Screen shot 4: Manon skipping.



Screen shot 5: Remco raising his knees.

In this practical investigation task it is true that we focus more on applying knowledge than on learning new mathematics and physics. But it is in any case a beautiful example for students to experience how a complex situation can be well described by a rather simple mathematical and physical model, and how such a model helps to better understand the situation. We abandon here the classical approach in mathematics lessons to start with a formula and draw a graph of it. Instead, a student first makes a video clip of an every day movement and does measurements on the movie. Hereafter (s)he represents the collected data graphically and uses the graphs as a starting point for modeling. We expect from the students doing the practical work the following things: He or she can

- check in what sense a mathematical and physical model adequately approximates the real world and what are the weak points (this includes formulating the meaning of “best fit”);
- demonstrate connections between graphs originating from measured data, simulations of mathematical models, and between important events in the scene on a video clip.

But what happens in school practice? We observed that very few students had difficulties with using

the data video tool, with creating the graphs, and with applying a given regression model. They built on their prior experiences with Coach, which can also be seen from a number of students who filtered their collected data unsolicited in order to get smoother graphs. The introductory activities worked well in the sense that they made the students more proficient with the software.

But interpreting graphs in the context of body movements turned out to be a different story. For many a student, comparing cyclograms boiled down to writing down the differences, without coupling them to the motions in the video clips. A possible explanation is that most teams only provided their fellow students the required diagram and not all their Coach results, including the video clip. Another reason may be that many a student still did not quite grasp the meaning and purpose of a cyclogram. In retrospect, we should have paid more attention to this in the learning material and during the lessons. After all, it is not so easy to imagine the exact body motion that belongs to a given cyclogram. All this resulted in comparisons of the following kind:

“We compared our cyclogram with the diagram of Manon and Marleen. You can clearly see that it is a different gait. You can see this from the movie clip. Manon skipped and Sietske jogged. Manon has a different hip and knee angle during skipping. In the movie clips, skipping is a more constant motion than jogging.”

Luckily there were also reports in which better attempts were made to find the differences between cyclograms and to explain them in terms of body motion. For example, Remco and Niek compared their knee raising (screen shot 5) with Manon’s skipping (screen shot 4) and wrote down:

“Clear differences show up immediately. The cyclogram of skipping is much more round, in contrast with the angular cyclogram of the knee-raising motion. It is easy to explain that because the dynamics of the knee raising movement is a much tauter and less fluid than skipping. Besides, the hip angle and the knee angle in the knee-raising motion get higher values than in the skipping motion. This is of course logical because in knee-raising the aim is to raise your knees as high as possible. Therefore, the hip angle gets large of course, twice as large as the highest value in skipping. You also try to bring your lower leg as close as possible to the upper leg and because of this the knee angle gets very large, too. The angles in our motion also get smaller values. The reason is that the stance leg must be kept as straight as possible in knee-raising; in skipping you do this less.”

Gerda and Vianne still did better in their report on “walking with a book on your head” (screen shot 2). They compared their cyclogram with the one of Gijs and Kah-Kih on ‘marching on your toes’; the diagram of the latter movement is not in-

cluded in this paper, but it is qualitatively similar to the diagram of knee raising in screen shot 5. They wrote two separate paragraphs about the differences between the diagrams and the reasons for these differences, using the nomenclature of gait analysis that they had learned in previous activities. We will not withhold presenting you the second paragraph.

“The cause of the differences in the range and the relation of the angles is in the manner of walking. Gijs walked on his toes; so, his heels did not touch the ground. In this way it is not really possible to flex your hip without flexing you knee. The knee angle therefore must increase as soon as the hip angle increases. Besides, Gijs also raised his legs very high so that the hip angle and the knee angle got extra large.

When you walk with a book on your head, you just must walk extra ‘carefully’ and make as few changes in your movements as possible. The fact is, when you flex your legs, the height of your head changes and this is just what you must avoid. Moreover, Vianne could place her heel on the ground so that she did not have to flex her knee much, while her leg was moving in the direction of motion. Because she wanted to make as few movements as possible, she also lifted her feet less. Therefore her hip angle and her knee angle were much smaller.

While tiptoeing you flex your legs to a great extent, while walking with a book on your head you do not. This explains the differences between the two cyclograms.”

It is clear that for Gerda and Vianne the graphs are about something and that many things read in the diagrams can be connected with the gait process. This was precisely one of our learning goals!

By the way, students wrote reports in various styles. Although we asked deliberately for a short, say one A4-page, report only, we also received reports with more pages. Apparently, these students had liked the task so much or had set such an ambitious level in their work that they felt that they could not do with less. On the other hand, the same students complained about shortage of time; we quote the opinion of Sanne and Sander about the research task:

“The subject of our research was according to us interesting, but there was too little time to do the investigation quite well and the explanation was on some points too complicated or too summary. This was a pity of course. We think that the practical work would be nicer if there was more time available. Coach is a very convenient program for this kind of work. We had used the software before, so that we more or less knew beforehand how to work with it. And with the explanation in the guide we could easily do the measurements. Coach could be used for many other sorts of research, for example,

in sports research (think of running, ball games, and gymnastics), in traffic research (speed, brake path, etc.), and so on.”

Also during the lessons we noticed the good work climate and the eagerness of students: they worked more intensely in the video-based activities than normally with pencil and paper. Apparently they are more attracted by the multimedia component. We were pleasantly surprised to see that the students kept their minds on the lesson in the very first class after a school trip abroad. At first, we thought that they were just still tired from the journey. But also during the next lesson hours they were at work with great concentration and diligence, more than usual in the last three weeks before summer holidays.

9. Extensions towards Student Research

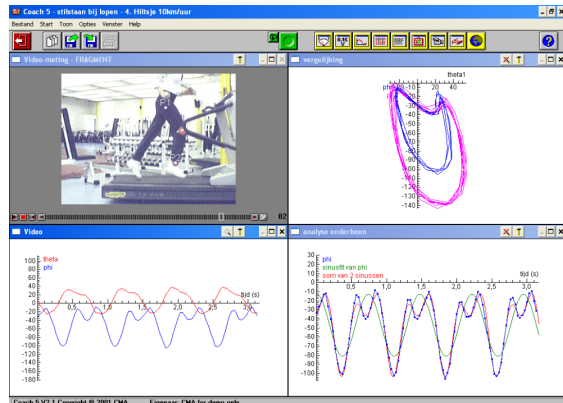
In recent years, the Dutch Ministry of Education has introduced a new concept for education in the upper level of secondary education (called ‘Studiehuis’ [study house]) which emphasizes inquiry skills and self-responsible learning, and it has added new ICT skills to the curriculum. A new examination program has been implemented, in which students are required to choose from four fixed combinations of subjects. In this program, the students are required to carry out some smaller practical investigation tasks and one rather large, cross-disciplinary research or design assignment. Students do this work mostly in the last two years of their secondary education. They usually practice this kind of work one year before in order to get familiar with investigation tasks. Our classroom experiment serves this purpose.

Stimulated by the results of the classroom experiment we held a one-day master class on gait analysis, in which we made use of the facilities of a fitness center. The students could use treadmills with adjustable speed and inclination, which is ideal for doing walking experiments. Most of the students participated in the master class as part of their research project. Below, we describe four extensions for student research that we considered.

1. Running on a Treadmill

Let us first look at the results of a video-based activity in which Hiltse is running at a speed of 10 kilometers per hour and compare these with the results of running at a speed of 15 kilometers per hour. In the screen shot below you see in the upper-left window a video clip of Hiltse running. Notice the homemade marker on her knee: that is where the origin of the coordinate system is placed in each frame. On the hip joint and ankle joint you may recognize the measured point P1 and P2. In the lower-left diagram the hip angle and knee angle are plotted against time. Using a treadmill we can easily record more than one gait cycle. This has the advantage that we can verify periodicity of the

movements and can filter irregularities if we wish. In the lower-right diagram you see how well the knee angle is modeled by a sum of two sinoids. Two cyclograms are visible in the upper-right window: one belongs to a speed of 10 km/hr, the other belongs to a speed of 15 km/hr. Which one belongs to which curve and why?



Screen shot 6: Hiltje running on a treadmill.

In the video-based experiments in de classroom we could record only one gait cycle because of geometrical constraints. When you use a treadmill, the runner stays in a fixed area so that you can easily record more than one gait cycle. This brings limitations of the current data video tool to the surface: collecting data is too time-consuming. The video clip shown in the above screen shot takes 10 seconds and has a frame rate of 30 frames per second. In each frame you have to click three times. So in total you must click 900 times. This is too time-consuming, extremely boring, and limits the number of experiments in an investigation enormously. What needs to be implemented in the computer learning environment is the possibility to track the motion of an object (e.g., a marker or an eye-catching point). Another feature of gait analysis software that we find interesting for the computer learning environment is the use of stick models. In video clips about body motion we are often only interested in the movements of body segments such as legs and arms, and more precisely the slope of the body segments. Measuring via stick models is easier than clicking on points in the video clip. Besides, these stick models are also valuable when it comes to modeling and comparing with the collected data.

2. Step Frequency versus Gait Speed

Using a treadmill with adjustable speed one can easily investigate the relationship between gait speed v and step frequency f . Humans choose a step frequency or stride length that minimizes metabolic energy consumption. The step frequency has been found empirically to obey the power law

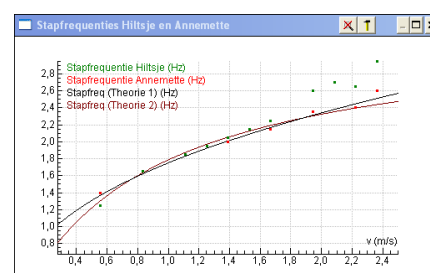
$$f = cv^b,$$

where c is a constant and $b \approx 0.52$ for normal gait speeds of adults (Bertram & Ruina, 2001). An alternative formula taken from (Bellemans, 1981) is

$$f = \frac{v}{av + b}.$$

It is based on the idea that a person who walks in a relaxed manner automatically adjusts his/her stride length to the gait speed in accordance with a linear relationship.

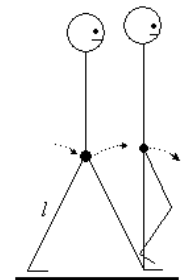
In the picture below we show the results for Hiltje and Annemette, who measured at the number of steps during 20 seconds of walking at various treadmill speeds. Which of the above formulas is better is less important; what is more interesting to note is that the graphs clearly reveal when the mathematical models are not so adequate anymore.



3. How Fast Can You Walk?

At high speed you cannot walk anymore, but you must run. An aerial phase of no support, i.e., in which both legs are off the ground, is introduced in the gait cycle. The main questions are when this change of gait pattern occurs and why it happens?

With the mathematical model of an inverted pendulum for the leg during walking one can already approximate the maximum walking speed. Assuming that the stance leg is kept straight while the foot is on the ground (a requirement in race walking), the hip moves forward in a series of arcs of circles. We neglect the weight of the legs and we assume that the center of mass of the body can be thought between, and just a little above, the hip joints. A body moving in a circle has acceleration towards the centre of the circle. If its speed is v and the radius of the circle is r , the acceleration is v^2/r . So, when the walker's body has velocity v at the stage of the stride when the leg is vertical, then at this stage the body has a downward acceleration v^2/l , where l is the length of the legs. The walker's feet are not glued to the floor, so the body cannot be pulled down; it can only fall under gravity. Consequently, the downward acceleration cannot be greater than the gravitational acceleration g . In other words: $v^2/l \leq g$.



Assuming a leg length of 0.9 m and a gravitational acceleration of 10 m/s^2 , the maximum speed of walking is about 3 m/s. Compute in this manner the maximum walking speed of this person when walking on the moon. Is this in agreement with television pictures of men walking on the moon under conditions of reduced gravity?

The (second) Froude number is defined by

$$\text{Froude number} = \frac{v^2}{gl}$$

So, at a Froude number less than or equal to 1 you can still walk. At a Froude number greater than 1 you must run, unless you are a race-walker. In that case you could reach a walking speed of 4 m/s. Do you have any idea how? See (Trowbridge, 1982) for a detailed mathematical model of race-walking. In practice people tend to change their gait from walking to running at a Froude number of 0.5, i.e., at a speed of about 2 m/s for adults. See also (Alexander, 2001) for a readable account on the use of mathematical models to explain the speed at which one changes from walking to running.

4. What is the Optimal Speed of Walking?

Not only the step frequency and stride length depend on the gait speed, but also the energy expenditure during walking changes as a function of speed. Like (Ralston, 1958) we use a model for the energy expenditure per unit time walked per unit weight, E , that has two components: a constant term representing the energy needed for quietly standing and a second term for the motion. Because of the well-known formula $\frac{1}{2}mv^2$ for the kinetic energy of a moving body, it is plausible that the second term is proportional to the square of the gait speed. So, $E = a + bv^2$, where a and b are experimentally determined constants and v is the gait speed. $a \approx 2$ and $b \approx 1.3$ (in SI units) for adults. It is a frequent practice to calculate energy expenditure in terms of distance walked. The mathematical formula relating energy expended per meter walked per kg, E_m , to speed v is deduced from the previous equation: $\frac{E}{v} = E_m = \frac{a}{v} + bv$. The diagram of E_m against v is a hyperbola with a minimum when a equals bv^2 . This condition can be deduced mathematically by computing the derivative of E_m with respect to v and setting it equal to 0. The above constants provide us with the following values at minimum energy expenditure:

$$v \approx 1.24 \text{ m/s}, \quad E \approx 4 \text{ W/kg}, \quad \text{and} \quad E_m \approx 3.5 \text{ Jm}^{-1}\text{kg}^{-1}.$$

So, when asked to walk in a relaxed manner, at a comfortable speed, most persons walk at a speed of about 4.5 km/hr. Similar results were reported in (Dumont & Waltham, 1997).

10. Conclusion

In this paper, we reported about our video-based laboratories for students in pre-university education carrying out an authentic investigation task. They were brought in contact with the field of gait analysis. After introductory activities to become familiar with the subject, the research methods, and the mathematical techniques, the students compared the cyclograms of two different gaits, which were recorded in the classroom with a web cam.

In summary, we are quite satisfied by the motivation and performance of the students. They were able to produce similar results as reported in the scientific literature about gait. A web cam and the computer learning environment Coach made this possible at student level. The most important input for the ongoing development of Coach is the need for a facility to track moving objects in a video clip so that the effort in collecting data in a video clip can be brought to an acceptable level.

The instructional setup contributed to the success of the students in the practical work. In our research we also found that the meaning of cyclograms in gait analysis stayed unclear for many a student. This can be remedied by paying more attention to the diagram and its purpose in the learning material and during the lessons. Luckily, quite a few students showed good understanding of the various possibilities of graphics to convey information about a particular phenomenon.

But maybe the most important conclusion of our classroom experiment is that low-cost ICT provides opportunities to bring the real world into the lessons in an attractive way. It makes high-quality authentic investigation tasks feasible at student level.

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