Homework 1

June 2, 2020

Pick one of the three questions below to answer.

**Question 1.** In the slides we wrote the probability that a majority out of the \( n \) total voters get it right as \( P(S_n \geq \frac{n+1}{2}) \). Assuming they all have the same competence \( p \), let’s call it \( M(n, p) \) here, to make it explicit that this is a function that depends on \( p \) as well. For instance, we have that:

\[
M(3, p) = \binom{3}{2} p^2 (1-p)^3 - 2 + \binom{3}{3} p^3 (1-p)^3 - 3 = 3p^2 (1-p) + p^3.
\]

Show that for a fixed odd \( n \) the function \( M(n, p) \) is increasing in the probability \( p \), i.e., that if \( p_1 < p_2 \) then \( M(n, p_1) < M(n, p_2) \).

**Attempt at an answer.** For \( n = 3 \):

\[
M(3, p) = 3p^2 (1-p) + p^3.
\]

If we see this is a function in \( p \), we want to show that it is increasing for \( p \in (0, 1) \). For that we can take the derivative with respect to \( p \):

\[
\frac{\partial M(3, p)}{\partial p} = . . . .
\]

We’re looking to see that the derivative is positive. The derivative turns out to be \(-6(p-1)p\), which is positive for the \( p \)'s we are interested in.
Question 2. In the presentation we always assumed \( n \) is odd, but what if \( n \) is even? The problem with that, of course, is that a strict majority winner might not exist. But suppose we modify the voting rule to say that if there is a tie between the two candidates, then we toss a (fair) coin to determine the winner.

The question, then, is: can you write down the formula for the probability that the true alternative is selected using this rule? Do you think the conclusions of the CJT still hold?

**Attempt at an answer.** Reduces to the case for \( n - 1 \).
Question 3. The CJT assumes that $p$, i.e., the probability that voters get it right, or, as we called it, their competence, is greater than $\frac{1}{2}$. This is like saying that they are better than random at getting it right. What, in general, would be a reason for $p < \frac{1}{2}$?

Attempt at an answer. Many reasons, but probably something to do with how we process information.