

51 Oracle Turing Machines

Turing also proposed a more general kind of machine, that is able to make use of an arbitrary amount of information from an external source.

Definition 1. Let S be a finite set with at least two elements. An *Oracle Turing machine over S* is a quadruple $M = (Q, q_0, q_1, \delta)$ of a finite set Q , distinct elements q_0 and q_1 of Q , and a partial map

$$\delta: (Q - \{q_0\}) \times S \times S \rightarrow Q \times S \times \{R, L\} \times \{R, N, L\}.$$

A *configuration* of M is an element of $S^* \times Q \times (\{-1\} \cup \omega) \times S^+$.

The above differs from Soare's definition in some respects. The most important is that we let the oracle tape move independently. The differences are irrelevant for the theory we shall develop.

We need only two symbols: 1 and B (blank). The oracle tape has a B in square n ($\in \omega$) if n does not belong to the oracle set. In a configuration (\bar{s}, q, k, \bar{t}) , the machine is in state q , watching the k -th square of the oracle tape and the first symbol of \bar{t} . Tapes can be extended arbitrarily — the oracle tape in only one direction. Q is the set of internal *states*; q_0 is the halting state, q_1 the initial state. The oracle tape may remain stationary (N) in relation to the eye watching it. I/O is as before; the Pythian eye begins on square -1 . With an oracle A , δ induces a transformation δ^A of configurations.

Definition 2. A *computation* of machine M with oracle A is a finite sequence (c_0, \dots, c_n) of configurations in which c_0 begins with q_1 and -1 , to the left of the beginning of the oracle tape, $c_{i+1} = \delta^A(c_i)$ for $0 \leq i < n$, and c_n contains q_0 . The *use* of a computation is the greatest *number* that appears in it.

52 Codes

As before, we can code instructions and programs (machines) into numbers, and we can number the program codes. We let \tilde{P}_e denote the program coded by the $(e + 1)$ th code number.

Exercise. Give a reasonably detailed description of the coding.