Homework 10 (due Monday 12 November)

Exercises 42.4 from the lecture notes.

1. (a) Show that the relation of divisibility, $m|n$, is representable. [5pts]
   (b) Show that $m|(k+n)$ and $m|n$ implies $m|k$. [5pts]

2. Numbers m and n are *relatively prime* if they have no prime factors in common. Show that m and n are relatively prime if and only if

$$\forall x \; (m|xn \Rightarrow m|x)$$

[10pts]

3. If m is relatively prime with n and k, then m is relatively prime with nk. [10pts]

4. The surjective pairing operation

$$\langle x, y \rangle := \frac{1}{2}(x^2 + 2xy + y^2 + 3x + y)$$

is strictly monotonic in both its arguments: if $m < n$, then $\langle m, k \rangle < \langle n, k \rangle$ and $\langle k, m \rangle < \langle k, n \rangle$. [10pts]