Appendix E

Errata (August 4, 2004)

Here is a list of all the more or less serious errata in the first edition of *Modal Logic* that we are aware of. Different from earlier versions of this list, we no longer list typos and errors that can be repaired in an obvious or trivial way. Please help us make this list as complete as possible by sending your comments to Yde Venema (yde@science.uva.nl).

The list refers to the version of the book published in June 2001 by *Cambridge University Press*. In this list, the notation '**p 2, +12**' refers to line 12 on page 2, counted from the head of the page; we use negative numbers if we count from the foot of the page.

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Chapter 1: Basic Concepts

Exercise 1.1.4 Replace '$r < t$ for all $t \in T$' with '$r < t$ for all $t \in T$ such that $r \neq t$'

p 15, +9 Replace 'variables' with 'formulas'

Exercise 1.5.1 Replace '$p \Vdash^g_{\mathsf{M(K)}} \Diamond p$' with $\Box p \Vdash^g_{\mathsf{M(K)}} p$

Exercise 1.5.3 The exercise should be about the class M of all models instead of about the class F of all frames.

Exercise 1.5.4 The first part of the exercise should be about the class M of all transitive models instead of about the class of all transitive frames. The second part of the exercise should refer to the class of all reflexive and transitive models.

Chapter 2: Models

p 57, -3 Replace '$n > 0$' with '$n \geq 0$'

p 62, +14 Replace '$f : \mathfrak{M} \to \mathfrak{M}'$' with '$f : \mathfrak{M} \to \mathfrak{M}'$ is a bounded morphism'

p 63, Exercise 2.1.5 In part (b), replace the sentence 'Give a ... to $(\mathfrak{N}, V_0)$' with 'Give a valuation V_1 on $\mathfrak{N}$ and a homomorphism from $(\mathfrak{B}, U_1)$ to $(\mathfrak{N}, V_1)$'. In part (c), delete '*surjective*'.

p 76, -5 Replace 'k' with '$k - n$.'

p 81, Definition 2.43 Replace the definition of clusters with the following:
A *cluster* on (W, R, V) is a maximal, nonempty equivalence class under R. That is, $C \subseteq W$ is a cluster if the restriction of R to C is an equivalence relation, and this is *not* the case for any other subset D extending C.

p 83, Exercise 2.3.9 Replace this exercise with the following:
Let $\mathfrak{M}^f$ be a finite, transitive filtration of a model based on the rationals with their usual ordering. Describe the possible shape of $\mathfrak{M}^f$ in terms of clusters and sets consisting of a single, irreflexive point. In particular, show that there is a natural way to impose a linear order on this collection of subsets of $\mathbb{Q}$. Can $\mathfrak{M}^f$ have two adjacent singleton clusters? Two adjacent singleton sets each consisting of an irreflexive point?

p 106, Lemma 2.73 Unfortunately, this proof is missing in Appendix A.

p 113, Example 2.80 Strictly spoken, the usage of the word 'safe' is not in accordance with the definition. One way to solve this problem would be to call a binary program constructor $\circ$ *safe for bisimulation* if the formula $\alpha(x, y)$ *defining* its accessibility relation is safe, given Definition 2.79. For instance, program composition is safe because the formula $\exists z\,(R_a xz \wedge R_b zy)$ is safe.

p 113, -7 The notion of complete additivity has been defined only for modal formulas. The obvious analog applies to first order formulas.

Chapter 3: Frames

p 137, Exercise 3.2.2 Replace the formula '$\Box((p \to \Box p) \to p) \to p$' with the proper version of Grzegorczyk's formula: $\Box(\Box(p \to \Box p) \to p) \to p$

p 137, Exercise 3.2.3 Replace 'Example 1.24' with 'Example 1.25'.

p 141, +14 Replace '$\neg\phi$ is false at u_w' with '$\neg\phi$ is true at u_w'.

p 142, Exercise 3.3.2(a) antisymmetry is $\forall xy(Rxy \wedge Ryx \to x = y)$.

p 143, -14 Replace 'generated submodel' with 'generated subframe'.

p 144, +9 In Lemma 3.20, replace 'for any frame $\mathfrak{G}$' with 'for any transitive frame $\mathfrak{G}$'.

p 144, Theorem 3.21 Replace 'Let K be a class of τ_0-frames' with 'Let K be a class of finite, transitive τ_0-frames'

Idem Replace 'τ-frames' with 'τ_0-frames'.

p 150, +2 Replace '$\forall yz\,(Rxy \wedge Ryz \to Rxz)$' with $\forall y\,(Rxy \to \exists z\,(Rxz \wedge Rzy))$.

p 156, -1 Replace ‘simple Sahlqvist formulas’ with ‘very simple Sahlqvist formulas’.

p 168, +8 Replace ‘contraposition’ with ‘reductio ad absurdum’.

p 168, -16/-15 The non-emptiness of C does follow from the fact that $\mathfrak{F} \models \forall y \exists z\,(Ryz \wedge y \neq z)$, but not immediately. Using ideas similar to those employed later on in the proof, one can construct a pair (Y, Z) in C.

p 181, +14, +16 Replace $\mathfrak{N}$ with $\mathfrak{N}'$.

p 182, +14 Replace ‘$f_t(i_j)$’ with ‘$f_t(j)$’.

Chapter 4: Completeness

p 196, +9 Replace ‘$\Lambda \subseteq \Lambda'$’ with Replace ‘$\Lambda' \subseteq \Lambda$’.

p 205, -10 Replace ‘$H(p \to p) \to (Hp \to Hp)$’ with ‘$H(p \to q) \to (Hp \to Hq)$’.

p 210, Exercise 4.3.3 In the second part, axioms for transitivity (in the case of **K4.3**) and for transitivity and reflexivity (in the case of **S4.3**) should be added.

p 210, Exercise 4.3.6 Replace ‘$\forall w Rww$’ with ‘$\forall wv Rwv$’

p 213, +24 Replace ‘ordinal’ with ‘cardinal’

p 214, +10 Replace ‘modal projections’ with ‘modal operations’

p 214, -5 Replace ‘$\mathbf{K}_t\mathbf{Tho}$’ with ‘$\mathbf{K}_t\mathbf{ThoM}$’

p 216, +24 Add ‘consistent’ to ‘normal modal logic’.

p 216, Exercise 4.4.4 The axiom E should read ‘$\Diamond(\Diamond p \wedge \Box q) \to \Box(\Diamond p \vee \Box q)$, and the axiom Q should read ‘$(\Diamond p \wedge \Box(p \to \Box p)) \to p$.
In (b), replace ‘both T and M’ with ‘both T and E’.

p 223, Exercise 4.5.6 In this context of reflexive orders, density refers to the property $\forall x \forall y\,((Rxy \wedge x \neq y) \to \exists z\,(Rxz \wedge x \neq z \wedge Rzy \wedge z \neq y))$.

p 226, -3 … p 227, +2 Replace all occurrences of ‘y’ with ‘x’

p 226, -5 Replace ‘$\nu(x)R^c\nu(y)$’ with ‘$\nu'(x)R^c\nu'(y)$’.

p 227, -4 Replace ‘$\nu(u)$’ with ‘$\nu(x)$’

p 233, +9 Replace the definition of $\theta(\mathcal{N}, t, s)$ with

$$\theta(\mathcal{N}, t, s) \quad := \quad \lambda(t) \wedge \textstyle\bigwedge_{s \neq v \in E(t)} \langle tv \rangle \theta(\mathcal{N}, v, t).$$

p 234, -1 Replace ‘u’ with ‘t’.

p 238, +18 Replace ‘If, in addition, $\mathfrak{M}$ is a versatile model for τ, then …’ with ‘More in particular, if $\mathfrak{M}$ is a bidirectional model for the basic temporal similarity type, then …’.

p 242, -11 Replace ‘Let A_1 be $\{\sigma_1\}$’ with ‘Let A_0 be $\{\phi\}$’.

p 243, Definition 4.85 Σ should be finite.

p 244, -5 Replace ‘$AS_{\pi^*}A$’ with ‘$A(S_\pi)^*A$’.

p 246, -4 Replace ‘$\vdash_{KL} \Box\Box\phi \to \Box\phi$’ with ‘$\vdash_{KL} \Box\phi \to \Box\Box\phi$’

Chapter 5: Algebras

p 265, +20 Replace the first 'Φ' with '$Form(\Phi)$'.

p 269, +1 Replace '$\psi \equiv_C \psi$' with '$\phi \equiv_C \psi$'.

p 269, +12 Replace '$x \cdot 1 = 1$' with '$x \cdot 1 = x$'.

p 271, +14 Replace '$[\phi] \cdot [\psi] := [\phi \wedge \psi]$' with '$[\phi] \cdot [\psi] = [\phi \wedge \psi]$'.

p 273, +19/+20 Replace 'any power set algebra is isomorphic to a subalgebra of a power of **2**' with 'any power set algebra is isomorphic to a power of **2**'

p 277, +21 Replace '$m_{R_\Delta}(\tilde{V}(\phi_1, \ldots, \phi_n))$' with '$m_{R_\Delta}(\tilde{V}(\phi_1), \ldots, \tilde{V}(\phi_n))$'

p 283, -13 Replace 'generated by a class of complex algebras' with 'generated by a class of full complex algebras'.

p 285, +7,+10 Replace 'd_1' with 'd_0'.

p 291, -1 Replace '$\mathsf{V}_\Lambda \subseteq \mathbf{Cm}\mathsf{K}$' with '$\mathsf{V}_\Lambda \subseteq \mathbf{SCm}\mathsf{K}$'.

p 296, -3 Replace 'from $\mathfrak{F}^+$ to $\mathfrak{F}'^+$' with 'from $\mathfrak{F}'^+$ to $\mathfrak{F}^+$'.

p 301, -9 Replace '$\mathfrak{A}$' with '$\mathfrak{Em}\mathfrak{A}$'.

p 305, -5/-6 Swap '$(\mathfrak{C}, V), u \Vdash \Diamond\Box p$' with '$(\mathfrak{C}, V), v \Vdash \Box p$'.

p 309, +18/19 Replace the last lines of Example 5.70 with '... consider for instance the set of co-finite subsets of S. This set has the finite intersection property, but there is no state in W that belongs to all co-finite subsets of T.'

p 312, +8 Replace '$.Q_f$' with 'm_{Q_f}'.

p 316, +15 Replace '$m_R(b) \subseteq b_1 \cap \cdots \cap b_n$' with '$m_R(b) \subseteq m_R(b_1) \cap \cdots \cap m_R(b_n)$'.

p 318, -5/-4 Replace '$s = t$' with '$s \approx t$' (3 times).

p 319, -2/-1 Replace the last sentences of Example 5.87 with 'Hence, there is a state t_n such that $t_n \Vdash p$. But then $s_n \Vdash \Diamond p$ and $r \Vdash \Diamond\Diamond p$.'

p 320, +14 Insert 'Assume that the formula holds on $\mathfrak{g}$; we will prove that it is also valid on $\mathfrak{F}$'.

p 324, -10 Replace '$u \in \bigcap_{U \lessdot V} V(\gamma')$' with '$u \in \bigcap_{U \lessdot V} V(\gamma)$'.

Chapter 6: Computability and complexity

p 332, -8 Replace 'revised' with 'reviewed'.

p 343, +22 Replace 'Λ is r.e. by Lemma 6.11.' with 'It is not difficult to see that Λ, being axiomatizable, is recursively enumerable.'

p 346, +12 Replace 'for all $0 \leq n < m \leq k$' with 'for all distinct m and n in W'.

p 349, +6 Replace 'Section A' with 'Appendix A'.

p 352, -5 Replace 'J' with '$\widehat{J}$'.

p 360 – 363 There are a number of inaccuracies in the subsection on mosaics.

p 360, -15 'let Cl_ϕ denote the smallest set containing ϕ, $H\bot$ and $F\top$ which is closed under taking subformulas and single negations'.

p 360, Definition 6.27 Reformulate the definition of bricks as follows,

(B-1) $F\top \in \Lambda$.
(B0) for all $F\psi \in \mathrm{Cl}_\phi$: if $F\psi \in \Lambda$ or $\psi \in \Lambda$ then $F\psi \in \Phi$.
(B1) for all $P\psi \in \mathrm{Cl}_\phi$: if $P\psi \in \Phi$ or $\psi \in \Phi$ then $P\psi \in \Lambda$.

p 360, -2 Add '$\in B$' after '(Φ, Λ)'.

p 362, +3 Replace '$\Lambda_{i+1} = \Phi_i$' with '$\Lambda_i = \Phi_{i+1}$'.

p 361, -9/-11 'For example, in order to prove S0, we consider the brick (Γ_0, Γ_k) if $k > 0$, and the brick (Γ_0, Γ_1) in case $k = 0$ (recall that k is the state where ϕ holds)'.

p 362, +3 Replace 'atoms' with 'Hintikka sets'.

p 362, -3 Replace '$\phi \in \lambda_0(L_0)$' with '$\phi \in \lambda_0(0) \cup \lambda(L_0)$'.

p 363, +15/+16 Given the present formulation there is a flaw in the backward induction argument due to the fact that the conditions (B0) and (B1) are formulated in terms of the universal modalities G and H while we take F and P as primitive. Given the above reformulation of the definition, this problem has been solved.

p 367, -15 Replace 'it is decidable' with 'KR has a decidable satisfiability problem'.

p 375, +25 Replace '$\Box\psi_j$' with '$\Diamond\psi_j$'.

p 383, -10 Actually the formula grows cubically in part (iv).

p 383, -11 Replace 'disjunct' with 'conjunct'.

p 384, +21 Replace '$\{\theta \mid \Box\theta \in H\}$' with '$\{\neg\theta \mid \neg\Diamond\theta \in H\}$'

p 389, +11 Replace '$\in S$' with '$\in X$'

p 390, +3 Replace '$\bigwedge_{\{i \mid \mathsf{Q}_i = \forall\}} \Box^i B_i$' with '$\bigwedge_{\{i \mid \mathsf{Q}_{i+1} = \forall\}} \Box^i B_i$'

p 404, +14/15 Replace the sentence 'As $\chi \in H'$ ... that $\langle\pi^*\rangle\chi \in H_{n-1}$' with 'As $\chi \in H'$ it follows by the definition of Hintikka sets that $\langle\pi^*\rangle\chi \in H' = H_n$, so inductively we find that $\langle\pi\rangle\langle\pi^*\rangle\chi \in H_{n-1}$; hence, by the definition of Hintikka sets, $\langle\pi^*\rangle\chi \in H_{n-1}$'.

Chapter 7: Extended Modal Logic

p 448, +12 Replace '$\exists\overline{y}\, R(x_{i_1}, \ldots, x_{i_n})$' with '$\exists x_{j_1} \ldots \exists x_{j_m}\, R(x_{i_1}, \ldots, x_{i_n})$'

p 454, +12 Replace 'a connected, directed and acyclic graph' with 'a connected and acyclic graph'.

p 454, -12 Replace '$A \subseteq \mathrm{range}(\alpha_t)$' with '$Q \subseteq \mathrm{range}(\alpha_t)$'.

p 455, +15 Add 'for ξ' to 'a perfect network'.

p 456, +14 Replace '$\{L(c,d) \mid c, d$ taken from $\overline{a}\overline{b}\}$' with '$\{L(c) \mid c$ taken from $\overline{a}\overline{b}\}$'. ∎

Appendices

p 488, Theorem A.2 Replace 'α' with 'γ' (three times)
p 490, +7 Replace '$\mathfrak{A}$ is an extension of $\mathfrak{A}$' with '$\mathfrak{A}$ is an extension of $\mathfrak{A}'$'.
p 493, +5 Replace the displayed equivalence with

$$R_U f_U^1 \dots f_U^n \text{ iff } \{i \in I \mid R_i f^1(i) \dots f^n(i)\} \in U.$$

p 493, +8 Replace the displayed equivalence with

$$F_U(f_U^1, \dots, f_U^m) = \{(i, F_i(f^1(i), \dots, f^m(i))) \mid i \in I\}_U.$$

p 494, +16 Replace '$a \mapsto f_a$' with '$a \mapsto (f_a)_U$'
p 494, +19 Replace '$a_1, \dots, a_n \in \prod_U \mathfrak{A}$' with '$a_1, \dots, a_n \in A$'
p 498, +7 Replace '$\mathfrak{B} = (A, \dots$' with '$\mathfrak{B} = (B, \dots$'
p 498, +15 Replace '$\mathfrak{A}'$' with '$\mathfrak{B}$'
p 510, -2 Replace '$x \in L_2$' with '$x \in L_1$'

Bibliography

p 542, [433] This item should be as follows:
S.K. Thomason. Categories of frames for modal logics. *Journal of Symbolic Logic*, 40: 439–442, 1975.

Index

basic hybrid language This item should appear just once and refer to page 435.
basic temporal language This item should appear just once and refer to the pages 11, 21 and 137.
atom Add '358'.

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