Data Semantics

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0. INTRODUCTION

The usual semantical explication of (logical) validity runs as follows: an argument is valid iff it cannot possibly occur that its premises are all true while its conclusion is not true. Compare this account of validity with the following one: an argument is valid iff it cannot possibly occur that its premises are all true *on the basis of the set of data available* while its conclusion is not true *on the basis of that set*. Do the principles of classical logic retain their validity when one changes over from the first explication of validity to the second one?

In what follows, my principal concern will be to answer this question for the case of propositional logic. Of course, the ultimate interest of the answer hangs on the claim that the second explication of validity conforms better than the first to what goes on in actual reasoning. I shall not try to support this claim in its full generality, but will restrict myself to showing that a number of problems which arise if one tries to analyse the logical behaviour of 'if ... then', 'must' and 'may' in terms of the first explication of validity simply vanish if one uses the second explication.

It will be clear that the question at issue cannot be answered until two other questions have been settled: (i) what is a set of data, and (ii) what does it mean for a sentence to be true on the basis of a set of data? Section 1 deals with the first of these questions, and Section 2 with the second. The final section is devoted to a discussion of some of the more salient features of the resulting logic.

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1. DATA SETS

The semantic system developed here differs in various respects from the kind of systems developed within the framework of possible worlds semantics. From an ontological point of view, the most important difference is that the models for a given language are built not on 'the set of possible worlds', but on 'the set of possible facts'.

I do not intend to say a great deal about the nature of facts. Yet I do want to maintain a few assumptions about them. To begin with, I trust that there is no harm in talking about *possible* facts. I shall take this notion in such a way that it is a truism to say 'All possible facts are such that it is possible for them to hold, though some of them will never actually do so'. Certain philosophers, following Quine[1], would claim that this way of speaking commits one to assuming that possible facts exist. I am not, in this respect, an unreserved follower of Quine. I doubt whether it makes much sense to speak of existence in connection with possible facts, even in the case of those possible facts which obtain here and now. However, this is not a crucial issue. I shall certainly quantify over possible facts, and if this commits me to assuming that they somehow 'exist', then I am ready to do so.

Second, I shall hold that the totality of all possible facts can be treated as a *set* (in the mathematical sense of the word). As far as I can see, the only conceivable objection to this might be that set theory does not admit sets with the properties we shall ascribe to the set of all possible facts. But in fact set theory does admit such sets.

Third, suppose we have two possible facts f and g. I shall assume that if f and g can obtain simultaneously, this simultaneous occurrence of f and g qualifies as another possible fact. This fact is called *the combination of* f *and* g. Since we would like to talk of the combination of f and g even if f and g cannot possibly hold together, we introduce as a technical convenience the so-called *improper fact*, and we stipulate that if f and g can not obtain simultaneously, the combination of f and g amounts to this improper fact.

These considerations taken jointly give the set of possible facts the structure of a semi-lattice:

DEFINITION 1. A *data lattice* is a triple $<\mathcal{F},\circ,0>$ with the following properties:

(i) $0 \in \mathcal{F}, \mathcal{F} \sim \{0\} \neq \emptyset$;

(ii) $\circ$ is a binary operation of $\mathcal{F}$ such that

(a) $f \circ f = f$

(b) $f \circ g = g \circ f$

(c) $(f \circ g) \circ h = f \circ (g \circ h)$

(d) $0 \circ f = 0$;

Explanation: The members of $\mathcal{F} \sim \{0\}$ are to be conceived of as the possible facts. '$f \circ g$' is to be read as 'the combination of f and g'. 0 is to be thought of as the improper fact. Given our informal remarks, it will be clear that the $\circ$-operation should have the properties laid down in (a) – (d). When $f \circ g = 0$, we shall often say that f *and* g *are incompatible*[2], and when $f \circ g = f$, we shall say that f *incorporates* g.

DEFINITION 2. Let $\langle \mathcal{F}, \circ, 0 \rangle$ be a data lattice. A possible world in $\langle \mathcal{F} \circ, 0 \rangle$ is a subset $\mathfrak{W}$ of $\mathcal{F}$ with the following properties:

(i) for every $f \in \mathcal{F}$, either $f \in \mathfrak{W}$ or $g \in \mathfrak{W}$ for some g such that $g \circ f = 0$

(ii) for no $f \in \mathcal{F}$, both $f \in \mathfrak{W}$ and $g \in \mathfrak{W}$ for some g such that $g \circ f = 0$

A possible world is a rather peculiar set of possible facts: it is complete in the sense that if a given fact f does not obtain in it, some fact g incompatible with f obtains in it; and it is consistent in the sense that no incompatible facts obtain in it. Actually, possible worlds are so peculiar that one might wonder wether they exist at all. In other words: given any data lattice $\langle \mathcal{F}, \circ, 0 \rangle$, are there subsets of $\mathcal{F}$ meeting both the requirements (i) and (ii)? A well known theorem in lattice theory tells us that we may rest assured that this is the case. Before we can state this result, we need one more definition.

DEFINITION 3. Let $\langle \mathcal{F}, \circ, 0 \rangle$ be a data lattice. A filter in $\langle \mathcal{F}, \circ, 0 \rangle$ is a subset $\mathfrak{D}$ of $\mathcal{F}$ such that $f, g \in \mathfrak{D}$ if and only if $f \circ g \in \mathfrak{D}$

A filter $\mathfrak{D}$ is proper iff $0 \notin \mathfrak{D}$

A proper filter $\mathfrak{D}$ is maximal iff there is no proper filter $\mathfrak{D}'$ such that $\mathfrak{D} \subseteq \mathfrak{D}'$ and $\mathfrak{D} \neq \mathfrak{D}'$

PROPOSITION 1. Let $\langle \mathcal{F}, \circ, 0 \rangle$ be a data lattice.

(i) If $\mathcal{E} \subseteq \mathcal{F}$ then $\mathcal{E}$ can be extended to a proper filter iff for every $f_1, \ldots, f_n \in \mathcal{E}$, $f_1 \circ \ldots \circ f_n \neq 0$.

(ii) Every proper filter can be extended to a maximal proper filter.

(iii) Every maximal proper filter is a possible world and vice versa.

PROOF. Omitted[3]

It remains to explain the notion of a possible set of data. Informally, every set of facts that might be obtained by investigating some possible world is a possible set of data – but of course, if both the facts f and g belong to the data, then so does the combination of f and g; and if the fact f belongs to the data, then so do the facts g incorporated by f. So it appears that, formally speaking, the proper filters in a given data lattice $\langle \mathcal{F}, \circ, 0 \rangle$ are the right candidates for the role of the possible data sets in $\langle \mathcal{F}, \circ, 0 \rangle$. Therefore, I shall from now on often refer to them in that way.

One last observation before we pass on to questions of semantics: notice that the theory of facts put forward here does not carry the metaphysical burden of many other theories. It is not assumed, for example, that there are facts of minimal complexity: any fact may incorporate other facts. Neither is it assumed that there are facts of maximal complexity: any fact may be incorporated by other facts. And finally, it is not assumed that there are *negative* facts: if a certain possible fact f does not obtain in a certain possible world, then some possible fact g incompatible with f obtains in it; but there does not have to be some particular fact g incompatible with f that obtains in every possible world in which f does not obtain.[4]

2. DATA SEMANTICS

What does it mean for a sentence to be true on the basis of a certain set of data? As indicated in the introduction, we shall answer this question only for a particular class of sentences. To be more specific, the sentences in question all belong to a formal language $\mathfrak{L}$ with

(i) a vocabulary consisting of countably many atomic sentences, two parentheses, three one place operators $\neg$, *must*, and *may*, and three two place operators $\wedge$, $\vee$ and $\rightarrow$; and

(ii) the formation rules that one would expect for a language with such a vocabulary.

The operators $\neg$, *must, may*, $\wedge$, and $\vee$ are meant as formal counterparts of "not", "must", "may", "and", and "or", respectively. The operator $\rightarrow$ should be read as "if ... then"; if ϕ and ψ are formal translations of the English sentence ϕ' and ψ', then $\phi \rightarrow \psi$ is meant to be a formal translation of the *indicative* conditional with antecedent ϕ' and consequent ψ'.

In presenting the semantics for this language $\mathfrak{L}$, I shall follow usual practice and first state how its non-logical symbols are to be understood.

DEFINITION 4. A *model* (*for* $\mathfrak{L}$) is a quadruple $<\mathcal{F},\circ,0,\mathfrak{I}>$ such that $<\mathcal{F},\circ,0>$ is a data lattice and $\mathfrak{I}$ is a function assigning some element of $\mathcal{F}$ to each atomic sentence of $\mathfrak{L}$. $\mathfrak{I}$ is called an *interpretation* (*of* $\mathfrak{L}$) *into* $<\mathcal{F},\circ,0>$.

Informally, Definition 4 can be put as follows: each atomic sentence describes a possible fact (or the improper fact). Hence, in a way the definition offers a final clue to the question of what possible facts are; apparently, possible facts are things that can be described by the most elementary kind of sentences.[5]

Let $\mathcal{M} = <\mathcal{F},\circ,0,\mathfrak{I}>$ be a model, $\mathfrak{D}$ a data set in $<\mathcal{F},\circ,0>$ and ϕ a sentence of $\mathfrak{L}$. In the sequel, "$\mathfrak{D} \Vdash_{\mathcal{M}} \phi$" abbreviates "$\phi$ is true (in $\mathcal{M}$) on the basis of $\mathfrak{D}$" and "$\mathfrak{D}_{\mathcal{M}} \dashv\!\Vert\, \phi$" abbreviates "$\phi$ is false (in $\mathcal{M}$) on the basis of $\mathfrak{D}$".

DEFINITION 5. Let $\mathcal{M} = <\mathcal{F}, \circ, 0, \mathfrak{J}>$ be a model and $\mathfrak{D}$ a data set in $<\mathcal{F}, \circ, 0>$.

- if ϕ is atomic,

  $\mathfrak{D} \Vdash_{\mathcal{M}} \phi$ iff $\mathfrak{J}(\phi) \in \mathfrak{D}$

  $\mathfrak{D}_{\mathcal{M}} \dashv\!\Vert\, \phi$ iff for some $f \in \mathfrak{D}$, $f \circ \mathfrak{J}(\phi) = 0$;
- $\mathfrak{D} \Vdash_{\mathcal{M}} \neg\phi$ iff $\mathfrak{D}_{\mathcal{M}} \dashv\!\Vert\, \phi$;

  $\mathfrak{D}_{\mathcal{M}} \dashv\!\Vert\, \neg\phi$ iff $\mathfrak{D} \Vdash_{\mathcal{M}} \phi$;
- $\mathfrak{D} \Vdash_{\mathcal{M}} (\phi \wedge \psi)$ iff $\mathfrak{D} \Vdash_{\mathcal{M}} \phi$ and $\mathfrak{D} \Vdash_{\mathcal{M}} \psi$;

  $\mathfrak{D}_{\mathcal{M}} \dashv\!\Vert\, (\phi \wedge \psi)$ iff $\mathfrak{D}_{\mathcal{M}} \dashv\!\Vert\, \phi$ or $\mathfrak{D}_{\mathcal{M}} \dashv\!\Vert\, \psi$;
- $\mathfrak{D} \Vdash_{\mathcal{M}} (\phi \vee \psi)$ iff $\mathfrak{D} \Vdash_{\mathcal{M}} \phi$ or $\mathfrak{D} \Vdash_{\mathcal{M}} \psi$;

  $\mathfrak{D}_{\mathcal{M}} \dashv\!\Vert\, (\phi \vee \psi)$ iff $\mathfrak{D}_{\mathcal{M}} \dashv\!\Vert\, \phi$ and $\mathfrak{D}_{\mathcal{M}} \dashv\!\Vert\, \psi$;
- $\mathfrak{D} \Vdash_{\mathcal{M}} (\phi \rightarrow \psi)$ iff for every data set $\mathfrak{D}' \supseteq \mathfrak{D}$, if $\mathfrak{D}' \Vdash_{\mathcal{M}} \phi$ then $\mathfrak{D}' \Vdash_{\mathcal{M}} \psi$;

  $\mathfrak{D}_{\mathcal{M}} \dashv\!\Vert\, (\phi \rightarrow \psi)$ iff for some data set $\mathfrak{D}' \supseteq \mathfrak{D}$, $\mathfrak{D}' \Vdash_{\mathcal{M}} \phi$ and $\mathfrak{D}'_{\mathcal{M}} \dashv\!\Vert\, \psi$;
- $\mathfrak{D} \Vdash_{\mathcal{M}}$ *may* ϕ iff for some data set $\mathfrak{D}' \supseteq \mathfrak{D}$, $\mathfrak{D}' \Vdash_{\mathcal{M}} \phi$;

  $\mathfrak{D}_{\mathcal{M}} \dashv\!\Vert$ *may* ϕ iff for no data set $\mathfrak{D}' \supseteq \mathfrak{D}$, $\mathfrak{D}' \Vdash_{\mathcal{M}} \phi$;
- $\mathfrak{D} \Vdash_{\mathcal{M}}$ *must* ϕ iff for no data set $\mathfrak{D}' \supseteq \mathfrak{D}$, $\mathfrak{D}'_{\mathcal{M}} \dashv\!\Vert\, \phi$;

  $\mathfrak{D}_{\mathcal{M}} \dashv\!\Vert$ *must* ϕ iff for some data set $\mathfrak{D}' \supseteq \mathfrak{D}$, $\mathfrak{D}'_{\mathcal{M}} \dashv\!\Vert\, \phi$.

The remainder of this section is devoted to a discussion of this definition. But first I need to introduce some concepts that will play a prominent part in that discussion.

DEFINITION 6. Let ϕ be a sentence.
ϕ is T-stable iff for every model $\mathcal{M}$ and data set $\mathfrak{D}$, if $\mathfrak{D} \Vdash_{\mathcal{M}} \phi$, then $\mathfrak{D}' \Vdash_{\mathcal{M}} \phi$ for every data set $\mathfrak{D}' \supseteq \mathfrak{D}$.
ϕ is F-stable iff for every model $\mathcal{M}$ and data set $\mathfrak{D}$, if $\mathfrak{D}_{\mathcal{M}} \dashv\!\Vert\, \phi$, then $\mathfrak{D}'_{\mathcal{M}} \dashv\!\Vert\, \phi$ for every data set $\mathfrak{D}' \supseteq \mathfrak{D}$.
ϕ is stable iff ϕ is both T-stable and F-stable.

So, informally, a sentence ϕ is T-stable iff it has the following property: once ϕ has turned out to be true on the basis of some set of data, ϕ will remain true, whatever additional data may come to light. Likewise an F-stable sentence has the property that once its falsity has been established, there is no possibility that further investigations will yield a set of data on the basis of which it is not false.

It is not the case that every English sentence is T-stable and F-stable in this sense. We shall meet examples of unstable sentences when we come to discuss the truth and falsity conditions of sentences of the form $(\phi \rightarrow \psi)$, *may* ϕ and *must* ϕ. But we shall discuss sentences of different forms first.

2.1. Atomic sentences

According to Definition 5, an atomic sentence ϕ is true on the basis of a certain set $\mathfrak{D}$ of data iff the fact described by ϕ belongs to $\mathfrak{D}$. And an atomic sentence ϕ is false on the basis of a certain set $\mathfrak{D}$ of data iff the fact described by ϕ is incompatible with some element of $\mathfrak{D}$.

Let $\mathcal{M}$ be a model and ϕ an atomic sentence. Notice:

- If $\mathfrak{D} \Vdash_{\mathcal{M}} \phi$, then $\mathfrak{D}' \Vdash_{\mathcal{M}} \phi$ for every data set $\mathfrak{D}' \supseteq \mathfrak{D}$.

  If $\mathfrak{D} \,{}_{\mathcal{M}}\!\dashv\!\!|\, \phi$, then $\mathfrak{D}' \,{}_{\mathcal{M}}\!\dashv\!\!|\, \phi$ for every data set $\mathfrak{D}' \supseteq \mathfrak{D}$.
- There are data sets $\mathfrak{D}$ such that neither $\mathfrak{D} \Vdash_{\mathcal{M}} \phi$ nor $\mathfrak{D} \,{}_{\mathcal{M}}\!\dashv\!\!|\, \phi$.
- There are no data sets $\mathfrak{D}$ such that both $\mathfrak{D} \Vdash_{\mathcal{M}} \phi$ and $\mathfrak{D} \,{}_{\mathcal{M}}\!\dashv\!\!|\, \phi$.
- If $\mathfrak{D}$ is a maximal data set, then either $\mathfrak{D} \Vdash_{\mathcal{M}} \phi$ or $\mathfrak{D} \,{}_{\mathcal{M}}\!\dashv\!\!|\, \phi$.

In other words, each atomic sentence ϕ is stable - once its truth or falsity has been established, it has been established for good. However, it is not always possible to decide on the basis of the data available whether ϕ is true or false. Of course, ϕ can never turn out to be both true and false. And ultimately ϕ must turn out to be either true or false.

The third and the fourth of the above observations apply to all sentences:

PROPOSITION 2. Let $\mathcal{M}$ be a model, $\mathfrak{D}$ a data set (pertaining to $\mathcal{M}$), and ϕ a sentence.

(i) It is not the case that both $\mathfrak{D} \Vdash_{\mathcal{M}} \phi$ and $\mathfrak{D} \,{}_{\mathcal{M}}\!\dashv\!\!|\, \phi$;

(ii) if $\mathfrak{D}$ is *maximal*, then either $\mathfrak{D} \Vdash_{\mathcal{M}} \phi$ or $\mathfrak{D} \,{}_{\mathcal{M}}\!\dashv\!\!|\, \phi$.

PROOF. Induction on the complexity of ϕ. □

It may very well be that a certain fact f does not occur in a certain set $\mathfrak{D}$ of data, but does hold in any possible world in which all facts in $\mathfrak{D}$ hold. According to Definition 5, a sentence ϕ describing f is *not* true on the basis of $\mathfrak{D}$ in such a case. Yet wouldn't it be plausible to call ϕ true on the basis of $\mathfrak{D}$ here?

I do not think so. Of course, if one keeps on adding more information to $\mathfrak{D}$, then $\mathfrak{D}$ will inevitably grow into a data set $\mathfrak{D}'$ on the basis of which ϕ is true. Consequently, I would not object if one were to call the sentence *must* ϕ true on the basis of the set $\mathfrak{D}$ of data. Nor would I object if one were to call the sentence ϕ just *true* - without an explicit reference to the evidence involved. I think, however, that it would blur an important distinction - that between *direct* and *indirect* evidence - if one were to maintain that it is simply and solely *on the basis of the set* $\mathfrak{D}$ *of data* that the sentence ϕ is true.[6]

2.2. Negation

I trust that the truth and falsity conditions for sentences of the form $\neg\phi$ do not need any further explanation. It may, however, be illuminating to compare these conditions with a few alternatives.

Presumably, it will not be difficult to convince the reader that the following stipulation would have been completely mistaken:

(*) $\mathfrak{D} \Vdash_{\mathfrak{M}} \neg\phi$ iff $\mathfrak{D} \nVdash_{\mathfrak{M}} \phi$.

If the few data presently at my disposal do not allow me to conclude that it is raining in Ipanema, this does not mean that they allow me to conclude that it is not raining there. Hence, (*) does not capture the meaning of English negation. Within the present framework, the equivalence expressed by (*) only holds in case $\mathfrak{D}$ is a maximal data set, but that is a rather exceptional case.

Readers familiar with Kripke's semantic analysis of intuitionistic logic or with model theoretic forcing[7] will be attracted to the following alternative to the account of negation given in Definition 5:

(**) $\mathfrak{D} \Vdash_{\mathfrak{M}} \neg\phi$ iff for every data set $\mathfrak{D}' \supseteq \mathfrak{D}, \mathfrak{D}' \nVdash_{\mathfrak{M}} \phi$.

I can hardly imagine that anyone would adhere to this (**)-definition and yet agree with the falsity conditions proposed in Definition 5; there seem to be no grounds for denying that the following two statements are equivalent:

(i) ϕ is false on the basis of the data;

(ii) the negation of ϕ is true on the basis of the data.

So I would expect the supporters of (**), if any, to completely reject our falsity conditions, rather than to reject the equivalence between (i) and (ii). The incorporation of (**) in Definition 5, therefore, would almost certainly bring a drastic revision of the entire system along with it.

At this moment, we are not yet in a position to explain in detail why Definition 5 offers a better analysis of the meaning of negation in English than (**) does. I shall here briefly sketch the relevant argument trusting that the remainder of this paper will enable the reader to fill in the details for himself.

To begin with, it is worth noting that the negation described by (**) is expressible within the framework presented here, albeit not by means of the operator $\neg$. Still,

$$\mathfrak{D}' \nVdash_{\mathcal{K}} \phi \text{ for every data set } \mathfrak{D}' \supseteq \mathfrak{D} \text{ iff } \mathfrak{D} \Vdash_{\mathcal{K}} \textit{must} \neg\phi.$$

Hence, the easiest way to compare the (**)-negation and the negation of Definition 5, is to study the different properties attributed by Definition 5 to sentences of the form *must* $\neg\phi$ on the one hand, and sentences of the form $\neg\phi$ on the other. By doing so for different kinds of sentences, one will undoubtedly sooner or later arrive at the conclusion that 'not' has more in common with the operator $\neg$ than with the operator *must* $\neg$. The reader is invited to test this for himself - the following cases are decisive: (i) ϕ is a sentence of the form $(\psi \rightarrow \chi)$; (ii) ϕ is a sentence of the form *must* ψ.

2.3. Disjunction and conjunction

English sentences of the form $\ulcorner\phi$ or $\psi\urcorner$ are often uttered in a context where the available data do not enable the speaker to decide which of the sentences ϕ and ψ are true, but only tell him that *at least one* of the sentences *has to be* true. Moreover, it would seem that sentences of the form $\ulcorner\phi$ or $\psi\urcorner$ are sometimes true, and indeed true on the basis of the data, when uttered in such a context. So it is quite possible, I think, that the police superintendent who says that either Mr. B. or Mr. C. killed Mrs. D. says something that is true on the basis of the available evidence, even though it may be weeks before the case of Mrs. D. is definitively solved.

If this observation is correct, it would seem that in most contexts the operator V cannot serve as the formal counterpart of 'or'. According to Definition 5, a sentence of the form $(\phi \vee \psi)$ is not true on the basis of the data unless it is possible to decide which of the sentences ϕ and ψ is true on that basis - and, on most occasions, this is a bit too much to ask.

Fortunately, the present system provides yet another possible analysis of disjunctive sentences: in place of a sentence of the form $(\phi \vee \psi)$, one can take a sentence *must*$(\phi \vee \psi)$ as their formal translation. *must*$(\phi \vee \psi)$ is true on the basis of the data set $\mathfrak{D}$ iff for no extension $\mathfrak{D}'$ of $\mathfrak{D}$, both ϕ and ψ are false on the basis of $\mathfrak{D}'$; in view of Proposition 3, this means that at least one of the sentences ϕ and ψ will eventually turn out to be true on the basis of the data if one continues to accumulate information.

At this point the reader may wonder why I did not assign to sentences of the form $(\phi \vee \psi)$ the truth and falsity conditions which are now associated with sentences of the form *must*$(\phi \vee \psi)$. Wouldn't that have been a more elegant procedure?

The reason that I did not proceed that way is this: sometimes disjunction *is* used in the manner formally captured by the truth and falsity conditions associated with the operator V. Here are a few examples:

- It is not the case that Mr. B. or Mr. C. killed Mrs. D.
- If Mr. E. or Mr. F. killed Mrs. D., then Mr. B. and Mr. C. are innocent.
- Maybe Mr. E. or Mr. F. killed Mrs. D.

Actually, from a syntactical point of view, there are only a few cases (the case where 'or' occurs as the main connective of the relevant sentence being the most obvious) in which the meaning of English disjunction does not seem to conform to the meaning of V. Yet I venture the hypothesis that even in these special cases the *literal* meaning of 'or' *can* be equated with the meaning of V, and that it is for *pragmatic* reasons that one is inclined to understand a statement of the form $\ulcorner\phi$ or $\psi\urcorner$ as $\ulcorner$ it must be the case that ϕ or $\psi\urcorner$: to put it briefly, if one were to take such a statement literally, one would be forced to assume that its utterer is violating the conversational *maxim of quantity*.[8] If, on the other hand, the relevant disjunction is embedded in a more complex sentence, then this predicament is less likely to arise and therefore one can in general take the disjunction at its face value in such cases.

The truth and falsity conditions pertaining to conjunction need no further comment - if indeed the reader is not inclined to barter the falsity conditions of $\ulcorner\phi$ and $\psi\urcorner$ for the truth conditions of $\ulcorner$ it cannot be that both ϕ and $\psi\urcorner$.

PROPOSITION 3. Suppose $\neg$, $\wedge$ and V are the only operators occurring in ϕ. Then ϕ is stable.

In the sequel, I shall sometimes discriminate between the sentences in which $\neg$, $\wedge$ and V are the only occurring operators and the other ones by calling the former *descriptive* and the latter *nondescriptive.* The difference between these two kinds of sentences amounts to this: by uttering a descriptive sentence a speaker only informs his audience of the data he has gathered *so far.* By uttering a non-descriptive sentence he also gives words to his expectations about the outcome of *further* investigations.

2.4. Implication

According to Definition 5, a sentence of the form $\ulcorner$If ϕ then $\psi\urcorner$ is true on the basis of a set $\mathfrak{D}$ of data iff there is no possibility of extending $\mathfrak{D}$ into a data set $\mathfrak{D}'$ on the basis of which ϕ is true and ψ is not true: if, by any chance, further investigations should reveal that ϕ is true, they will reveal that ψ is true too. Furthermore, it is stated that $\ulcorner$If ϕ then $\psi\urcorner$ is false on the basis of a set $\mathfrak{D}$ of data iff, given $\mathfrak{D}$, it is still possible that further investigations will yield an extension $\mathfrak{D}'$ of $\mathfrak{D}$ on the basis of which ϕ is true and ψ is false.

It will be clear that on this account a sentence of the form $\ulcorner$If ϕ then $\psi\urcorner$ is not necessarily F-stable. This, I hope, conforms to the reader's intuitions. Consider for instance the sentence 'If Mary went to the party, then John went there, too', and suppose that John's best friend is Peter. Peter happens not to know that John has fallen in love with Mary, and, accordingly, his data allow for the possibility that Mary attended the party and John did not do so. So, on the basis of the limited set of data available to Peter, the sentence 'If Mary went to the party, then John went there, too' is false. On the other hand, it is very likely that Peter will be able to exclude this possibility – knowing John for what he is – as soon as he learns that John has fallen in love again. So, on the basis of this extension of Peter's data, the sentence 'If Mary went to the party, then John went there, too' will probably not be false anymore. Hence, it is not F-stable.

Let ϕ be F-stable and suppose that ϕ is false on the basis of the data set $\mathfrak{D}$. Then according to Definition 5, $\ulcorner$If ϕ then $\psi\urcorner$ is true on the basis of $\mathfrak{D}$ for any sentence ψ. Likewise: let ψ be T-stable and suppose that ψ is true on the basis of $\mathfrak{D}$. Then $\ulcorner$If ϕ then $\psi\urcorner$ is true on the basis of $\mathfrak{D}$ for any sentence ϕ.

In other words, the present treatment of conditionals does not meet the requirement that a sentence of the form $\ulcorner$If ϕ then $\psi\urcorner$ should *never* be true unless the antecedent ϕ is somehow 'relevant' to the consequent ψ.[9]

Should we regret this?

There is, I think, no need to do so: pragmatic constraints ensure that a conditional will normally be uttered only in circumstances where the antecedent is somehow 'relevant' to the consequent. Hence, there is no need to incorporate relevance into the semantics.

Let me indicate why I think that relevance can be delegated to the pragmatics.

(i) The most natural context of utterance for an indicative conditional $\ulcorner$If ϕ then $\psi\urcorner$ – and here I restrict myself to the case where both ϕ and ψ are *descriptive* – is one in which the following conditions are satisfied: (a) it is not the case that ψ is true on the basis of the data available, though (b) it is possible that ψ will on further investigation turn out to be true; (c) it is not the case that ϕ is true on the basis of the data available, though (d) it is possible that ϕ will on further investigation turn out to be true.[10] (If condition (a) is not satisfied, then by the maxims of quantity and manner ψ should be uttered rather than $\ulcorner$If ϕ then $\psi\urcorner$, for ψ is both stronger and less wordy than ⌜If ϕ then ψ⌝. Likewise, if condition (d) is not fulfilled, ⌜It must be the case that not ϕ⌝ should be uttered rather than ⌜If ϕ then ψ⌝. Furthermore, if both (a) and (d) are satisfied and either (b) or (c) are not satisfied, then ⌜If ϕ then ψ⌝ is false on the basis of the data available. Thus, in view of the maxim of quality, it is forbidden to utter ⌜If ϕ then ψ⌝ in either of these cases.)

(ii) Now, if a sentence of the form ⌜If ϕ then ψ⌝ is uttered in the circumstances appropriate to its use, then the present truth condition by itself guarantees that this sentence cannot be true unless the antecedent ϕ is highly relevant to the consequent ψ: whenever the available data are extended in a way that results in ϕ being true on the basis of the new data set, ψ must be true on the basis of that extended data set too. It will be clear that there must be some positive connection between ϕ and ψ if this is to be so in circumstances where in particular the conditions (a) and (d) are satisfied.

2.5. may *and* must

The clearest examples of T-unstable sentences are found among sentences of the form ⌜it may be the case that ϕ⌝. A sentence of this form - take 'it may be snowing' - will often at first (as you awake one winter morning) be true on the basis of the data available, and then (open the curtains and what do you see?) turn out false as soon as new data become available. In view of Definition 5, this should be a very common occurrence, for the definition states (i) that a sentence of the form ⌜it may be the case that ϕ⌝ is true on the basis of the data $\mathfrak{D}$ as long as it is possible for $\mathfrak{D}$ on further investigation to grow into a set of data on the basis of which ϕ is true; and (ii) that such a sentence is false on the basis of the data as soon as this possibility can be excluded.

In the previous pages I have hinted several times at the truth and falsity conditions associated with the operator *must*. According to Definition 5, a sentence of the form ⌜It must be the case that ϕ⌝ is true on the basis of the available data iff there is no possibility that this data set will on further investigation grow into a set of data on the basis of which ϕ is false. (Hence, as the investigation proceeds, the data will inevitably grow into a set on the basis of which ϕ is true.) However, as long as this possibility is not excluded, ⌜it must be the case that ϕ⌝ is false on the basis of the data.[11]

It is worth noting that this analysis predicts that in many cases, notably if ϕ is descriptive, a sentence of the form ⌜It must be the case that ϕ⌝ is weaker than the corresponding sentence ϕ. If a descriptive sentence ϕ is true on the basis of the data, then ⌜It must be the case that ϕ⌝ is true on that basis as well, but ⌜It must be the case that ϕ⌝ can be true on the basis of the data without ϕ being true on that basis.

That 'it must be the case that ϕ' is on most occasions weaker than ϕ itself, has been noticed by a number of authors. Lauri Karttunen[12] illustrates this phenomenon with the following examples:

(a) *John must have left*

(b) *John has left*

His informal explanation fits in neatly with our formal analysis: 'Intuitively, (a) makes a weaker claim than (b). In general, one would use (a) the epistemic *must* only in circumstances where it is not yet an established fact that John has left. In (a), the speaker indicates that he has no first-hand evidence about John's departure, and neither has it been reported to him by trustworthy sources. Instead (a) seems to say that the truth of *John has left* in some way logically follows from other facts the speaker knows and some reasonable assumptions that he is willing to entertain. A man who has actually seen John leave or has read about it in the newspaper would not ordinarily assert (a), since he is in the position to make the stronger claim in (b)'.

Similar remarks can be found in Groenendijk & Stokhof (1975) and Lyons (1977). Yet despite this unanimity, so far no formal theory has been proposed which actually predicts that for descriptive sentences ϕ, ⌜It must be the case that ϕ⌝ is a logical consequence of ϕ. Most theories treat *may* and *must* as epistemic modalities and depending on whether the underlying epistemic notion is either knowledge or belief, *must* ϕ turns out to be either stronger than ϕ or independent of it.

(Notice in passing that the present theory does not predict that ϕ is stronger than *must* ϕ for all sentences ϕ. Example: let $\phi = \neg(\psi \rightarrow \chi)$ and ψ, χ be descriptive. $\mathfrak{D} \Vdash_{\mathfrak{M}}$ *must* $\neg(\psi \rightarrow \chi)$ iff $\mathfrak{D} \Vdash_{\mathfrak{M}}$ *must* $(\psi \wedge \neg\chi)$, whereas $\mathfrak{D} \Vdash_{\mathfrak{M}} \neg(\psi \rightarrow \chi)$ iff $\mathfrak{D} \Vdash_{\mathfrak{M}}$ *may* $(\psi \wedge \neg\chi)$. Hence, *must* ϕ turns out stronger than ϕ here.)

The next proposition is an immediate consequence of Proposition 2.

PROPOSITION 4. Let $\mathfrak{M}$ be a model and let $\mathfrak{D}$ be a *maximal* data set (pertaining to $\mathfrak{M}$).

$\mathfrak{D} \Vdash_{\mathfrak{M}} \neg\phi$ iff $\mathfrak{D} \nVdash_{\mathfrak{M}} \phi$

$\mathfrak{D} \Vdash_{\mathfrak{M}} \phi \wedge \psi$ iff $\mathfrak{D} \Vdash_{\mathfrak{M}} \phi$ and $\mathfrak{D} \Vdash_{\mathfrak{M}} \psi$

$\mathfrak{D} \Vdash_{\mathfrak{M}} \phi \vee \psi$ iff $\mathfrak{D} \Vdash_{\mathfrak{M}} \phi$ or $\mathfrak{D} \Vdash_{\mathfrak{M}} \psi$

$\mathfrak{D} \Vdash_{\mathfrak{M}} \phi \rightarrow \psi$ iff $\mathfrak{D} \nVdash_{\mathfrak{M}} \phi$ or $\mathfrak{D} \Vdash_{\mathfrak{M}} \psi$

$\mathfrak{D} \Vdash_{\overline{\mathcal{M}}}$ *may* ϕ iff $\mathfrak{D} \Vdash_{\overline{\mathcal{M}}} \phi$
$\mathfrak{D} \Vdash_{\overline{\mathcal{M}}}$ *must* ϕ iff $\mathfrak{D} \Vdash_{\overline{\mathcal{M}}} \phi$.

In other words, it does not make much sense to use the phrases 'if ... then', 'must', and 'may' in a context where the data set is maximal: in such a context, 'if ... then' gets the meaning of the material conditional while both ⌜it must be the case that ϕ⌝ and ⌜it may be the case that ϕ⌝ turn out equivalent to ϕ. However, in such an ideal case there is no need to use nondescriptive sentences - the data set is *complete*; so, what could possibly be the good of speculations on the outcome of *further* investigations?

3. DATA LOGIC

DEFINITION 7. Let ϕ be a sentence and let Δ be a set of sentences. $\Delta \Vdash \phi$ iff there is no model $\mathcal{M} = \langle \mathcal{F}, \circ, 0, \mathfrak{I} \rangle$ such that for some data set $\mathfrak{D}$ in $\langle \mathcal{F}, \circ, 0, \mathfrak{I} \rangle$, $\mathfrak{D} \Vdash \psi$ for every $\psi \in \Delta$ while $\mathfrak{D} \not\Vdash \phi$.

'$\Delta \Vdash \phi$' abbreviates 'the argument Δ / ϕ (i.e. the argument with the set Δ of premises and conclusion ϕ) is valid'. We shall feel free to write '$\Vdash \phi$' instead of '$\emptyset \Vdash \phi$' and '$\Delta, \psi_1, \ldots, \psi_n \Vdash \phi$' instead of '$\Delta \cup \{\psi_1, \ldots, \psi_n\} \Vdash \phi$'. Read '$\Vdash \phi$' as '$\phi$ is valid'.

The following remarks should give the reader an idea of how the logic generated by the above definition works. A more systematic account will be given in a subsequent paper.[13]

3.1. Data logic and classical logic

Our first observations concern the initial question of this paper: In what respects does data logic differ from classical logic? The following list shows that many classical principles are valid in the sense of Definition 7 as well.

(i) $\Delta, \phi \wedge \psi \Vdash \phi$; $\Delta, \phi \wedge \psi \Vdash \psi$

(ii) $\Delta, \varphi, \psi \Vdash \varphi \wedge \psi$

(iii) $\Delta, \phi \Vdash \phi \vee \psi$; $\Delta, \psi \Vdash \phi \vee \psi$

(iv) If $\Delta, \phi \Vdash \chi$ and $\Delta, \psi \Vdash \chi$, then $\Delta, \phi \vee \psi \Vdash \chi$

(v) $\Gamma, \phi, \neg\phi \Vdash \psi$

(vi) $\Delta, \phi, \phi \rightarrow \psi \Vdash \psi$.

The reader will notice that this list is made up entirely of principles which underly the classical system of natural deduction. Actually, only two of the principles underlying that system are missing. These principles fail within the present context: It is not generally so that

If $\Delta,\phi \Vdash \psi$, then $\Delta \Vdash \phi \rightarrow \psi$.

Nor does it hold that

If $\Delta,\neg\phi \Vdash \phi$, then $\Delta \Vdash \phi$.

The next two principles partially make up for this:

(vii) If $\Delta,\phi \Vdash \psi$ and each $\chi \in \Delta$ is T-stable, then $\Delta \Vdash \phi \rightarrow \psi$;

(viii) If $\Delta,\neg\phi \Vdash \phi$ and each $\chi \in \Delta$ is T-stable, then $\Delta \Vdash$ *must* ϕ.

Illustrations:

- It is easy to check that $\neg(\phi \vee \neg\phi) \Vdash \phi \vee \neg\phi$.

Yet if ϕ is a descriptive sentence, then $\nVdash \phi \vee \neg\phi$. (Actually, there are no valid descriptive sentences at all.)

must$(\phi \vee \neg\phi)$, on the other hand, is valid whether ϕ is descriptive or not. In this connection, it is worth noting that also *must* ϕ V $\neg$(*must* ϕ) and *must* ϕ V *must* $\neg\phi$ V (*may* ϕ $\wedge$ *may* $\neg\phi$) are valid for any ϕ.

- Suppose ϕ is an atomic sentence.

Then we have that *may* ϕ, $\neg\phi \Vdash \phi$, whereas neither *may* $\phi \Vdash \phi$ nor *may* $\phi \Vdash$ *must* ϕ.

What is notable here, is not so much the invalidity of *may* ϕ / ϕ and *may* ϕ / *must* ϕ as the validity of *may* ϕ, $\neg\phi$ / ϕ. Actually, according to the present theory, any conclusion can be drawn from the premises *may* ϕ and $\neg\phi$. To put it differently, by the standards here applied, the sentence

(a) *It may be raining in Ipanema now and it isn't*

is just a contradictory as

(b) *It is raining in Ipanema now and it isn't.*

These examples show that the present theory of 'may' differs widely from the theories of 'may' developed within the framework of possible worlds semantics and pragmatics. According to the latter[14], a sentence like (a) can be perfectly true although no one can assert it without violating the *maxim of quality*; consequently, the argument *may* ϕ, $\neg\phi$ / ψ is considered not as *logically* valid, but at best as *pragmatically* valid.

(Is there any evidence in favour of the claim that arguments of the form *may* ϕ, $\neg\phi$ / ψ – with ϕ atomic, or at least F-stable – are pragmatically valid rather than logically valid? Clearly, this evidence should consist in an informal example which shows that the putative 'seeming' inconsistency of the premises of an argument of this form can, in principle, be cancelled. I am pretty sure, however, that no such example can be found[15].)

- Let ϕ and ψ be two distinct atomic sentences.

It goes without saying that $\phi,\psi \Vdash \psi$ and $may\ \phi,\psi \Vdash may\ \phi$.
Furthermore, $\phi \Vdash \psi \rightarrow \phi$ but $may\ \phi \not\Vdash \psi \rightarrow may\ \phi$.
Hence, the present theory labels the first of the following arguments as valid and the second as invalid.

(a) *John's bicycle is red. Therefore, if John's bicycle is green, then it is red*

(b) *Maybe John's bicycle is red. Therefore, if John's bicycle is green, then it may be red.*

Perhaps the reader finds it difficult to accept the validity of (a). If so, he is invited to read the conclusion once more without losing sight of the premise - The conclusion does not say that John's bicycle would be red if it *had been* green.

If this does not help, then presumably the problem is that (i) the conclusion suggests that there is some positive connection between the supposed greenness of John's bicycle and its actual redness, wheras (ii) no such connection can possibly exist. However, that there is no positive connection between the antecedent and the consequent of the conclusion does not imply that the conclusion does not hold. That would only follow if the relevant conditional had been uttered in circumstances appropriate to its use (see Section 2.4). But this particular conditional is uttered in rather exceptional circumstances: given the premise of the argument, the antecedent of the conclusion is false on the basis of the data (and it will remain false if the data are extended), and its consequent is true on the basis of the data (and it will remain true if the data are extended). So we see that it is *pragmatically incorrect* to utter the conclusion in the circumstances described by the premise. However, we also see that if one does utter it anyway, then one can count the resulting statement as trivially true.

Notice in passing that the conclusion of (a) is trivially false if it is uttered in circumstances where neither the truth nor the falsity of either the antecedent or the consequent have been established. Assuming, then, that an addressee expects a speaker to observe all conversational rules, it is quite understandable that ones first reaction to (a) might be one of protest.

Let us now turn to argument (b). It is illuminating to compare this argument with the following.

(b′) *Maybe John's bicycle is red. Therefore, if John's bicycle turns out green, then it is still true on the basis of the data presently at my disposal that John's bicycle may be red.*

Unlike (b), the argument (b′) is valid. Roughly, the difference is this: in the consequent of the conclusion of (b′) explicit reference is made to the data available at the time of utterance. The consequent of the conclusion of (b), on the other hand, implicitly refers to the potential sets of data available after John's bicycle has turned out to be green.

3.2. Substitution and Replacement

Let ϕ and ψ be two distinct atomic sentences. It is only one step from the validity of ϕ / $\psi \rightarrow \phi$ to the validity of $\phi \rightarrow (\psi \rightarrow \phi)$ and from the invalidity of *may* ϕ / $\psi \rightarrow$ *may* ϕ to the invalidity of *may* $\phi \rightarrow (\psi \rightarrow$ *may* $\phi)$. Still, it is worthwhile to take these steps, for the resulting examples show that the *Principle of Substitution* cannot be carried over from classical logic to data logic without modification. In general, only uniform substitution of a *stable* sentence for an atomic sentence will transform a valid sentence into a valid one (Uniform substitution of an *instable* sentence may yield an invalid sentence.)

Also the *Principle of Replacement* needs to be treated with some care. Let us call the sentence ϕ and ψ *weakly equivalent* if both $\phi \Vdash \psi$ and $\psi \Vdash \phi$, and *strongly equivalent* iff $\phi \Vdash \psi$, $\psi \Vdash \phi$, $\neg\phi \Vdash \neg\psi$ and $\neg\psi \Vdash \neg\phi$. This distinction is important. Consider, for example, the sentences $\neg(\phi \vee \neg\phi)$ and $\neg(\psi \vee \neg\psi)$ where ϕ and ψ are two distinct atomic sentences. $\neg(\phi \vee \neg\phi)$ and $\neg(\psi \vee \neg\psi)$ are weakly equivalent but not strongly equivalent. If the occurrence of $\neg(\phi \vee \neg\phi)$ in $\neg\neg(\phi \vee \neg\phi)$ is replaced by an occurrence of $\neg(\psi \vee \neg\psi)$, then the resulting sentence $\neg\neg(\psi \vee \neg\psi)$ is *not* weakly equivalent to the original $\neg\neg(\phi \vee \neg\phi)$. Hence, the Principle of Replacement fails for weak equivalents. Yet it does hold for strong equivalents: if two sentences ϕ and ψ are strongly equivalent, then replacement of an occurrence of ϕ in a sentence χ by an occurrence of ψ will always yield a sentence χ' which is strongly equivalent to the original χ.

EXAMPLES.

- $\neg\neg\phi$ is strongly equivalent to ϕ
- $\phi \vee \psi$ is strongly equivalent to $\neg(\neg\phi \wedge \neg\psi)$
- $\phi \wedge \psi$ is strongly equivalent to $\neg(\neg\phi \vee \neg\psi)$
- *may* ϕ is strongly equivalent to $\neg(\phi \rightarrow \neg\phi)$
- *must* ϕ is strongly equivalent to $\neg\phi \rightarrow \phi$[26].

So we see that $\phi V \psi$, $\phi \Lambda \psi$, *may* ϕ, and *must* ϕ can be considered as mere abbreviations of $\neg(\neg\phi\Lambda\neg\psi)$, $\neg(\neg\phi V \neg\psi)$, $\neg(\phi\rightarrow\neg\phi)$ and $\neg\phi\rightarrow\phi$, respectively. In other words, in principle it is possible to give a more economical presentation of the present system by taking Λ, $\neg$, $\rightarrow$ (or alternatively V, $\neg$, $\rightarrow$) as primitive operators and defining the other operators in terms of them. (It is not possible to find three other operators among the ones given with which one can do the same.)

3.3. may *and* must

From the above observations it is clear that the logical properties of *may* and *must* are completely determined by the properties of $\neg$ and $\rightarrow$. Still, it seems worthwile to examine to what extent *must* and *may* behave like standard model operators.

- *may* ϕ is strongly equivalent to $\neg$*must* $\neg\phi$
- If $\Vdash \phi$, then $\Vdash$ *must* ϕ
- If ψ is F-stable, then $\Vdash$ *must*$(\phi\rightarrow\psi) \rightarrow$ (*must* $\phi \rightarrow$ *must* ψ)
- $\nVdash$ *must* $\phi \rightarrow \phi$
- $\Vdash$ *must*(*must* $\phi \rightarrow \phi$)
- $\Vdash$ *must* $\phi \rightarrow$ *may* ϕ
- $\Vdash$ *must* $\phi \rightarrow$ *must must* ϕ.

Thus, at first sight, it would seem that *must* and *may* behave like the obligation and permission operators of some system of *deontic* logic. But we also find:

- If ϕ is F-stable, then $\Vdash$ *must may* $\phi \rightarrow$ *must* ϕ, which would be a rather strong result for a system which is marked as deontic.
- $\Vdash \phi \rightarrow$ *may* ϕ, which gives the logic of *may* an *alethic* flavour.
- If ϕ is T-stable then $\Vdash \phi \rightarrow$ *must* ϕ. Cf. Section 2.5.

3.4. Implication

Let us now take a closer look at the logical properties of the operator $\rightarrow$. In many respects $\rightarrow$ behaves like *intuitionistic* implication.

PROPOSITION 5. Suppose that Λ, V and $\rightarrow$ are the only operators occuring in the sentence of the argument Δ / ϕ. Then, $\Delta \Vdash \phi$ iff Δ / ϕ is intuitionistically valid.

I shall not prove this proposition here.[17]

The above result does not hold if we permit other connectives to occur in the sentence of an argument. We encountered some counterexamples earlier: every sentence of the form $\neg\neg\phi \rightarrow \phi$ is valid in the sense of Definition 7, but a sentence of that form is in general not intuitionistically valid. On the other hand, every argument of the form $(\phi \wedge \psi) \rightarrow \chi \ / \ \phi \rightarrow (\psi \rightarrow \chi)$ is intuitionistically valid, whereas according to the present theory the validity of an argument of that form depends on the T-stability of ϕ: If ϕ is T-stable, then $(\phi \wedge \psi) \rightarrow \chi \Vdash \phi \rightarrow (\psi \rightarrow \chi)$, but if ϕ is not T-stable, then it is very well possible that $(\phi \wedge \psi) \rightarrow \chi \nVdash \phi \rightarrow (\psi \rightarrow \chi)$.

In one important respect the behaviour of $\rightarrow$ matches with the behaviour of the *strict implications* occurring in the Lewis Systems[18]: $\neg(\phi \rightarrow \psi) \Vdash$ *may*$(\phi \wedge \neg \psi)$ and *may*$(\phi \wedge \neg \psi) \Vdash \neg(\phi \rightarrow \psi)$. This is exactly what one would find if $\rightarrow$ were the implication and *may* the possibility operator of another extension of S 0.5.

However, $\neg(\phi \rightarrow \psi)$ and *may*$(\phi \wedge \neg \psi)$ are only weakly equivalent and not strongly equivalent. Although we do find that $\neg\neg(\phi \rightarrow \psi) \Vdash \neg$*may*$(\phi \wedge \neg \psi)$ or, equivalently, that $(\phi \rightarrow \psi) \Vdash$ *must*$(\neg \phi \vee \psi)$, it is not the case that *must*$(\neg \phi \vee \psi) \Vdash (\phi \rightarrow \psi)$; at best we have that *must*$(\neg \phi \vee \psi) \Vdash \phi \rightarrow$ *must* ψ, and even this only for T-stable sentences ϕ.

Our final observations with respect to $\rightarrow$ concern the Principle of *Modus Tollens*. This principle, which holds both in intuitionistic logic and in the systems of strict implication and also in such a weak system as the system R of *Relevance logic*, fails here[19]. Only if ψ is atomic, the argument $\varphi \rightarrow \psi, \ \neg \psi \ / \ \neg \phi$ is valid. For more complex ψ the closest approximation available is this: if ψ is F-stable, then $\phi \rightarrow \psi, \ \neg \ \psi \Vdash$ *must* $\neg \phi$.

If ψ is not F-stable, even this weakened version of *Modus Tollens* does not hold. Consider, for example the premises $\phi \rightarrow (\psi \rightarrow \chi)$ and $\neg(\psi \rightarrow \chi)$, where ϕ, ψ and χ are three distinct atomic sentences. Neither $\neg \phi$ nor *must* $\neg \phi$ follow from these premises; we only have that $\phi \rightarrow (\psi \rightarrow \chi)$, $\neg(\psi \rightarrow \chi) \Vdash$ *may* $\neg \phi$.

An example showing that the Principle of Modus Tollens fails in natural language is due here.

Three persons are involved, Allen, Brown and Carr. Perhaps the reader met the three of them before in connection with Lewis Carroll's barbershop paradox[20]. Well, they still run a barbershop, but nowadays they do so according to the following rules: (i) At all times at least one of them must be in the shop. (ii) None of them may ever leave the shop without one of the others accompanying him.

Which of them, do you think, will be in the shop right now? It is clear, of course, that *if Carr is in, then Allen is in if Brown is in.* Furthermore, it may very well be that Allen is out in the company of Carr, while Brown minds the shop, *So, it is not the case that if Brown is in, Allen is in.* Now, by an application of the Principle of *Modus Tollens,* it would follow from the italicized sentences that Carrr is out; and then, by a similar argument, one might prove that also Brown and Allen are out...

3.5. Descriptive arguments

Even if one likes the way in which the theory presented here deals with nondescriptive arguments, one may still regret the divergences from classical logic in reasonings with descriptive sentences. However, the departure from classical logic is not as drastic as one might fear at first sight:

PROPOSITION 6. Suppose $\wedge$, V and $\neg$ are the only operators occurring in the sentences of Δ / ϕ. If Δ / ϕ is classically valid, then $\Delta \Vdash$ *must* ϕ.

The proof, which is based on Proposition 3 and Proposition 4 is left to the reader.

In other words, if by the standards of classical logic the descriptive sentence ϕ must follow from the descriptive premises Δ, then *at least* 'it must be the case that ϕ' follows from Δ be the standards set here.

NOTES

1. See especially his by now classic 'On What There is', reprinted as Chapter 1 in Quine (1961).
2. I want to stress that the improper fact is introduced merely as a technical convenience. In principle, one can dispense with it by taking a *partial* combination operation and calling two facts f and g incompatible iff the combination of f and g is not defined.
3. The proof is identical to the proof of the analogous theorem for Boolean Algebras. For details, see Bell & Slomson 1969, pp. 13-15.
4. The position on 'negative facts' taken here is not so different from Mr. Demos' position, which is discussed by Bertrand Russell in 'The Philosophy of Logical Atomism'. See the relevant chapter in Russell 1956.
5. Admittedly, in the absence of a clear cut grammatical criterion to determine which English sentences count as most elementary, this remark is not very illuminating.
6. On the present account, the falsity of an atomic sentence is always established *indirectly*. In view of Proposition 1(i) and Definition 5 we have for atomic φ: $\mathfrak{D} \,{}_{\overline{\mathcal{M}}}\!\!\dashv\!\vert\, \varphi$ iff for every $\mathfrak{D}' \supseteq \mathfrak{D}, \mathfrak{D}' \nVdash_{\mathcal{M}} \varphi$ iff $\mathfrak{D} \Vdash_{\overline{\mathcal{M}}}$ *must* $\neg\varphi$. See Veltman (forthcoming) for further discussion.
7. See Kripke 1965 and Keisler 1977. It will be obvious to anyone familiar with the subject that the present paper found some of its inspiration in the notion of forcing.
8. Throughout this paper, I shall assume that the reader is familiar with Grice 1975.
9. I am referring here to the requirements set by the authors and co-authors of the sections on Relevance Logic in Anderson & Belnap 1975.
10. See also Gazdar 1979, pp. 59-61.
11. An obvious alternative to the truth and falsity conditions of sentences of the form *must* ϕ and *may* ϕ is the following:

$\mathfrak{D} \Vdash_{\overline{\mathcal{M}}}$ *must* ϕ iff for every *maximal* data set $\mathfrak{D}' \supseteq \mathfrak{D}$, $\mathfrak{D}' \Vdash_{\overline{\mathcal{M}}} \phi$

$\mathfrak{D} \,{}_{\overline{\mathcal{M}}}\!\!\dashv\!\vert$ *must* ϕ iff for some *maximal* data set $\mathfrak{D}' \supseteq \mathfrak{D}$, $\mathfrak{D}' \,{}_{\overline{\mathcal{M}}}\!\!\dashv\!\vert\, \phi$

$\mathfrak{D} \Vdash_{\mathcal{M}}$ *may* ϕ iff for some *maximal* data set $\mathfrak{D}' \supseteq \mathfrak{D}$, $\mathfrak{D} \Vdash_{\mathcal{M}} \phi$

$\mathfrak{D} \,_{\mathcal{M}}\!\dashv\!\!\!\Vdash$ *may* ϕ iff for every *maximal* data set $\mathfrak{D}' \supseteq \mathfrak{D}$, $\mathfrak{D} \,_{\mathcal{M}}\!\dashv\!\!\!\Vdash \phi$.

Notice that for stable sentences ϕ, the above conditions are equivalent to the ones included in Definition 5.

The main reason that I prefer the clauses of Definition 5 to the ones given above is methodological in nature. In the above clauses reference is made to the *maximal* proper extensions of data sets. However, it can only be proved by using powerful set theoretic methods that data sets have any maximal extensions. To be more specific, Proposition 1(ii) though somewhat weaker than the Axiom of Choice, is independent of the axioms of Zermelo Fraenkel Set Theory. Its status as a mathematical truth is not as solidly based as it is for these axioms. Now, if we want the above clauses for *must* and *may* to really work, we must rely on this proposition. The clauses for *must* and *may* given in Definition 5 on the other hand, do not presuppose Proposition 1(ii) or any other equally questionable set theoretic proposition. Therefore, from a methodological point of view, the clauses of Definition 5 are to be preferred.

12. See Karttunen 1972, p. 12.
13. See Veltman (forthcoming).
14. See Groenendijk & Stokhof 1975, pp. 83-84.
15. So far, no elaborate pragmatic theory has succeeded in drawing the dividing line between logical and pragmatical-but-not-logical validity precisely as the criterion of cancellability prescribes. It appears that in particular the conclusions of arguments which owe their pragmatical validity exclusively to the maxim of quality defy any attempt to cancellation. (See Gazdar 1979, p. 46.) It is, therefore, perhaps a little premature to suppose that because the inconsistency of the premises cannot be cancelled, it follows that arguments of the form *may* ϕ, $\neg\phi$ / ψ (with F-stable ϕ) are logically rather than just pragmatically valid. Consider, however, the following version of the maxim of quality: *Do not utter a sentence* ϕ *unless* ϕ *is true on the basis of the data at your disposal.* Every argument owing its pragmatical validity exclusively to this version of the maxim of quality is logically valid in the sense of 'logically valid' discussed here, too. So, presumably, data semantics allows for a pragmatics in which 'cancellability' *can* serve as a condition that an argument *must* satisfy in order to be classified as pragmatically but not logically valid.
16. Hence, principle (viii) of Section 3.1 is in fact a special case of principle (vii).
17. See Veltman (forthcoming).
18. These systems are extensively discussed in Hughes & Cresswell 1972.
19. Modus Tollens does fail in the theory of conditionals put forward in Cooper 1978. However, the evidence and explanation offered by Cooper are quite different from the evidence and explanation offered here.
20. The present example is a slight variant of this paradox, which first appeared in Carrol 1894. I can hardly imagine that nobody has ever thought of this variant before. In my view, it is much more powerful than the rather innocent barbershop paradox itself. Yet even Cooper, who discusses Carroll's paradox at some length, does not refer to it. Cf. Cooper 1978, pp. 204-205.

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