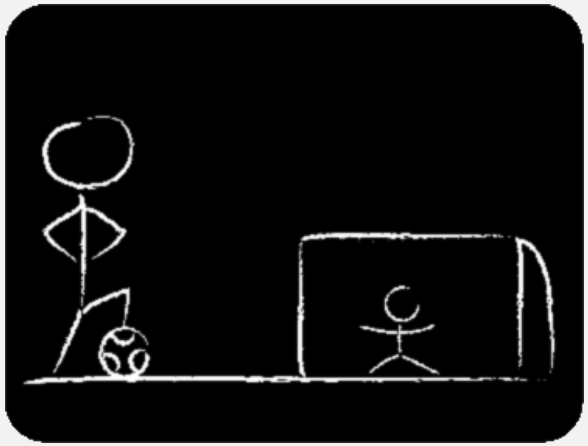
HEINRICH HEINE
UNIVERSITÄT DÜSSELDORF

Interference in Judgment Aggregation

Dorothea Baumeister, Gábor Erdélyi, Olivia Erdélyi, and Jörg Rothe

ILLC Workshop on Collective Decision Making, April 2013

Judgment Aggregation



	Penalty Area	Foul	Penalty
	Yes	Yes	Yes
	Yes	No	No
	No	Yes	No
Majority	Yes	Yes	No

Doctrinal Paradox / Discursive Dilemma

Outline

- Formal Framework
- Manipulation
  - Types of preferences
  - Strategyproofness
- Bribery
- Control by …
  - Adding Judges
  - Deleting Judges
  - Replacing Judges

Formal Framework

Judges

Agenda

Premises — Conclusions

	Penalty Area	Foul		Penalty
Referee 1	Yes	Yes		Yes
Referee 2	Yes	No		No
Referee 3	No	Yes		No
Quota ½	Yes	Yes	⇨	Yes

Individual Judgment Sets
Yes / No

Quota
fraction for each premise

Collective Judgment Set
Yes if quota is reached

Requirements:
- Agenda is closed under propositional variables
- Premises consists of all literals

= Complete and consistent outcome

Variants:
- Uniform quota
- Constant quota

We focus on:
- PBP: Uniform premise-based quota rules for quota ½
- Uniform constant premise-based quota rules

Forms of Interference

Manipulation:
Provide untruthful information to obtain a better result.

Bribery:
Briber judges to obtain a better result.

Control:
Change the structure to obtain a better result.

Widely studied in voting from a computational point of view!

Manipulation

Incentive:
Provide untruthful information to obtain a better result.

- Information = individual judgment set
- Result = collective outcome
- Better = ?

Different assumptions on the preferences:

- Unrestricted
- Top-respecting
- Closeness-respecting
- Hamming-distance induced

Preferences over collective JS

Preferences with respect to JS **1 0 0 1 1**

- Unrestriced (U): every preference is possible
- Top-respecting (TR): **1 0 0 1 1** > ? ? ? ? ?
- Closeness-respecting (CR): **1** ? ? ? **1** > **1** 1 1 0 **1**
- Hamming-distance induced (HD):

0 **0** **0** 0 **1** > **1** 1 1 0 **1**

The only complete relation is HD (by allowing equalities)

A judgment aggregation procedure is **strategyproof** if a judge prefers the acutual outcome to all outcomes resulting from untruthful individual judgment sets of him.

Strategyproofness

Fix some induced preference $>$:

A judge **necessarily** prefers X to Y if $X \succ Y$ in *every* complete extension of $>$.

A judge **possibly** prefers X to Y if $X \succ Y$ in *some* complete extension of $>$.

A judgment aggregation procedure is **necessarily/ possibly strategyproof** if a judge necessarily/possible prefers the acutual outcome to all outcomes resulting from untruthful individual judgment sets of him.

Manipulation

A	F	A ∧ F
Yes	Yes	Yes
Yes	No	No
No	Yes	No
Yes	Yes	Yes

Manipulative judge

Question: Is it possible to obtain a „better outcome“ by reporting an inscincere judgment set?

A	F	A ∧ F
Yes	Yes	Yes
Yes	No	No
No	No	No
Yes	No	No

HD, TR, CR-preferences regarding A ∧ F, Exact

Results for Manipulation

Preferences	Necessary Manipulation	Possible Manipulation
Unrestricted	?	in P
Top-respecting	NP-complete	in P
Closeness-respecting	strategyproof	?
Hamming Distance	NP-complete	
Exact	strategyproof	

Complete desired judgment set

Also holds for general quotas

Bribery (HD + Exact)

A	F	A ∧ F
Yes	Yes	Yes
Yes	No	No
No	Yes	No
Yes	Yes	Yes

No

Bribe 1 judge

A	F	A ∧ F
Yes	Yes	Yes
Yes	No	No
No	No	No
Yes	No	No

- Desired judgment set
- Budget k

Microbribery:
= Change up to k premise entries

Question: Is it possible to obtain a „better outcome“ by bribing at most k judges?

Exact Variant: Is it possible to reach the desired judgment set by bribing at most k judges?

Results for Bribery

	Bribery	Exact Bribery	MicroBribery	Exact MicroBribery
# judges	NP-comp.		NP-comp.	NP-comp.
# of bribes	NP-comp.	W[2]-hard	X	X
# of microbribes	X	X	NP-comp.	NP-comp.
General problem	NP-comp.	NP-comp.	NP-comp.	in P

Reduction from Dominating Set

Generalization of Optimal Lobbying

Desired Judgment set:
- complete
- contains all premises
- contains only premises

Control by Adding Judges

A	F	A ∧ F
Yes	Yes	Yes
Yes	No	No
No	Yes	No
Yes	Yes	Yes

No

Add 2 judges

No No No

A	F	A ∧ F
Yes	Yes	Yes
Yes	No	No
No	Yes	No
No	No	No
No	No	No
No	No	No

- Desired judgment set
- Set of potential new judges
- Positive integer k

Question: Is it possible to obtain a
adding at most k judges?

Non-constant number of judges:
= Difference between uniform and uniform constant premise-based quota rule

Exact Variant: Is it possible to reach the desired judgment set by adding at most k judges?

Control by Deleting Judges

A	F	A ∧ F
Yes	Yes	Yes
Yes	No	No
No	Yes	No
Yes	Yes	Yes

No

Delete 2 judges

A	F	A ∧ F
No	Yes	No
No	Yes	No

- Desired judgment set
- Positive integer k

Non-constant number of judges:
= Difference between uniform and uniform constant premise based quota rule

Question: Is it possible to obtain a „better outcome“ by deleting at most k judges?

Exact Variant: Is it possible to reach the desired judgment set by deleting at most k judges?

Control by Replacing Judges

A	F	A ∧ F
Yes	Yes	Yes
Yes	No	No
No	Yes	No
Yes	Yes	Yes
		No

Replace 1 judge

No No No

A	F	A ∧ F
Yes	Yes	Yes
Yes	No	No
No	No	No
Yes	No	No

- Desired judgment set
- Set of potential new judges
- Positive integer k

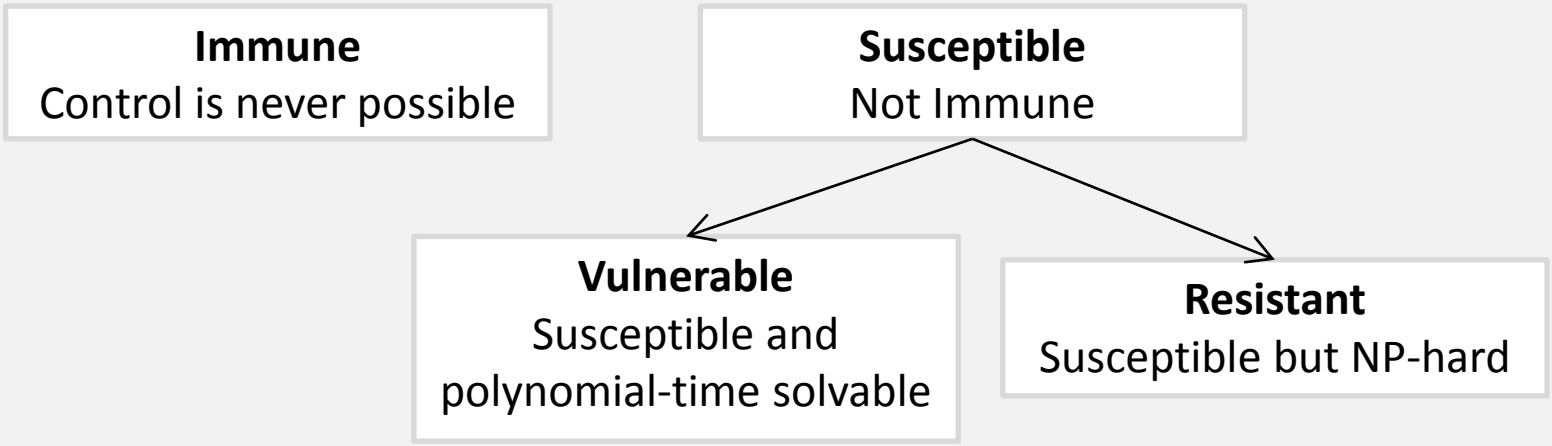
Constant number of judges:
= No difference between uniform and uniform constant premise-based quota rule

Question: Is it possible to obtain a „better outcome“ by replacing at most k judges?

Exact Variant: Is it possible to reach the desired judgment set by replacing at most k judges?

Approach

Control is usually an undesired behavior



= Computational hardness can be seen as a barrier against control

Results for Control

	Uniform Constant Quota	Uniform Quota = ½	Uniform Quota
Adding Judges (HD)	Resistant	Resistant	
Adding Judges (Exact)	Resistant	Resistant	
Deleting Judges (HD)	Resistant	Resistant	
Deleting Judges (Exact)	Resistant	Resistant	
Replacing Judges (HD)	Resistant	Resistant	Resistant
Replacing Judges (Exact)	Resistant	Resistant	Resistant

Reduction from Dominating Set

Reduction from Exact Cover by 3-Sets

Agenda contains only premises

Concluding Remarks

- Different Aggregation Procedures
- New Control Problems
- Typical-case analysis
- Different types of induced preferences for Bribery and Control

Thank you for your attention!