Multi-Issue Elections: A New Hope?

Framework and Initial experiments

Stéphane Airiau

LAMSADE

Universiteit van Amsterdam

ILLC Workshop on Collective Decision Making

Voting in Combinatorial domains

- **toy example:** choose a unique menu
  - first course: soup, salad, paté
  - main course: vegetarian, beef, chicken, fish
  - dessert: cheese, cake, ice cream
  - wine: light red, strong red, white, sparkling
  - ⇒ number of possible menus quickly becomes large!
- during an election in the US, many times voters also vote for many referenda (questions, elect judges, etc)

⇒ the number of candidates is exponential and it may be difficult to elect a winner

Voting in Combinatorial domains

starter	main dish	wine
salad s	veal v	red r
oyster o	truit t	white w

voter 1: $\texttt{svr} \succ \texttt{svw} \succ \texttt{ovw} \sim \texttt{stw} \succ \texttt{str} \sim \texttt{ovr} \succ \texttt{otw} \succ \texttt{otr}$

voter 2: $\texttt{ovw} \succ \texttt{svr} \sim \texttt{otw} \succ \texttt{stw} \succ \texttt{otr} \sim \texttt{ovr} \sim \texttt{str} \sim \texttt{svw}$

voter 3: $\texttt{stw} \succ \texttt{svr} \sim \texttt{otw} \succ \texttt{ovw} \succ \texttt{otr} \sim \texttt{ovr} \sim \texttt{str} \sim \texttt{svw}$

- **plurality**: due to the large number of candidates, each candidate may receive few votes, the tie-breaking rule will play an important role.
- **Borda**: need to rank all candidates, which is costly for large number of issues.
- **voting issue-by-issue**: may have paradoxical outcomes, e.g., may elect a winner that is bad for every voters. Also, may not be clear how to vote.

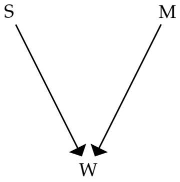
Preferential Dependencies

We say that issue X *depends* on issue Y if there exists a situation where you need to know the value of Y for telling which value for X should be weakly preferred.

Definition (Preferential dependencies)

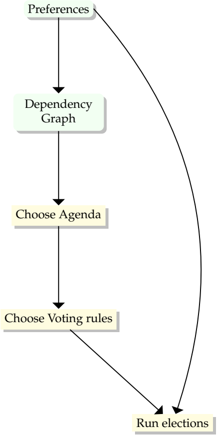
Issue $i \in \mathcal{I}$ is **preferentially dependent** on issue $j \in \mathcal{I}$ given preference relation $\succeq$, if there exist values $x, x' \in D_i$, $y, y' \in D_j$, and a vector of values $\vec{z} \in \mathcal{D}[\mathcal{I} \setminus \{i,j\}]$ for the remaining domains such that $x.y.\vec{z} \succeq x'.y.\vec{z}$ but $x.y'.\vec{z} \not\succeq x'.y'.\vec{z}$.

The Dependency Graphs of voter 1:



$\texttt{svr} \succ \texttt{svw} \succ \texttt{ovw} \sim \texttt{stw} \succ \texttt{str} \sim \texttt{ovr} \succ \texttt{otw} \succ \texttt{otr}$

Approach: Sequential Voting with Complex Agendas



An approach to designing voting procedures for multi-issue elections:

1 Elicit some basic information from the voters (here: everyone's *dependency graph* over the issues at stake).

2 Choose an *agenda* (which issues to vote on together in local elections + order of local elections), based on dependencies.

3 Choose a *local voting procedure* for each local election.

Basic Meta-Agenda Choice Functions (MACFs)

All procedures given below map a profile of dependency graphs into a single collective dependency graph: $F : \mathrm{DG}(\mathcal{I})^{\mathcal{N}} \to \mathrm{DG}(\mathcal{I})$.
We can then *condense* the collective graph to get a meta-agenda.

- *Majority aggregation:* include edge if a majority of voters do
- *Quota-based aggregation:* include edge if $\geqslant q\%$ of voters do
- *Canonical aggregation:* take the union of the input graphs
- *Distance-based aggregation:* choose a graph that is closest to the input profile, for a given metric (e.g., sum of Hamming distances)
- *Constraint-based aggregation:* choose a graph with clusters $\leqslant \ell$ that generates $\leqslant k$ dependency violations (there a several ways of counting violations: sum of all violations; no. of voter/election pairs where the voter experiences at least one uncertainty; ...)

Axiomatic Analysis

We can apply the axiomatic method to the study of MACFs.
For example, *quota-based procedures* satisfy all of these axioms:

- *Anonymity:* symmetry wrt. input graphs
- *Dependency-neutrality:* for dependencies (a,b) and (a',b'), if each voter accepts both or neither, then so does the meta-agenda
- *Reinforcement:* if the intersection S of sets of meta-agendas for two subelectorates is $\neq \emptyset$, then S is the outcome for their union

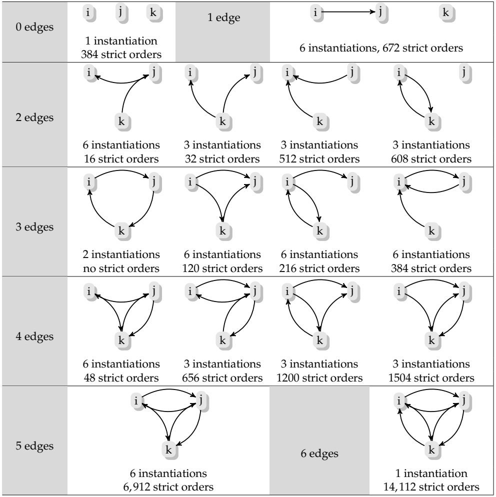
For *distance-based procedures*, some axiomatic properties are inherited from properties of the distances chosen:

- Any MACF defined in terms of a *neutral* distance (= invariant under renaming of vertices) on graphs is *dependency-neutral*.
- Any MACF defined in terms of a *symmetric* operator for extending distances between pairs of graphs to a distance between a graph and a set of graphs is *anonymous*.

... but one weird voter seems enough to force a single election with all issues!

if an oracle could tell us that the voter is not pivotal, we could use the voting protocol.

Lesson from linear orders with 3 issues



- a small proportion of strict linear orders have an acyclic dependency graph (6,864 preferences, i.e. 17.02% of all strict linear orders)
- 3080 different strict linear orders that are compatible with issue-by-issue voting, 7.64% of all possible strict linear orders.

With more issues

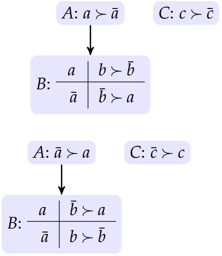
Likelihood that the dependency graph of a given strict preference order is the full graph

# of issues	2	3	4	5
proportion of s.o. with full graph	$\frac{1}{3}$	$\frac{7}{20}$	0.578	0.9345

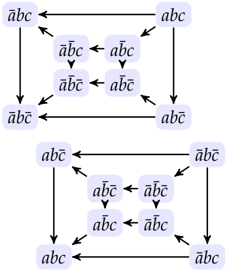
The impartial culture assumption is quite restrictive

If this assumption is realistic, sequential voting will not be a good solution and the voters need to pay a high cost to elicit the preferences.

Working with pre-orders



CP-net representation



Naive representation

- **for Borda**: the score of a candidate as the number of candidates she dominates.
- two agendas compatible with the dependencies of all the voters can elect different winners!
  $\{A\} \rhd \{B\} \rhd \{C\}$: winner is decided by tie-breaking rule, e.g., $\bar{a}\bar{b}\bar{c}$ if the tie-breaking rule chooses $\bar{a}$ over a, $\bar{b}$ over b and $\bar{c}$ over c.
  $\{A, B, C\}$ tie between abc and $\bar{a}b\bar{c}$

➩ are there tie-breaking rules that avoid this problem?

Bounding the size of the largest election

If the preferential dependency is violated, a voter is **uncertain** about his preference. We consider these three basic behaviours:

- **abstain** a voter can decide not to vote for that election
- **optimistic** a voter vote as if the best outcome is selected (wishful thinking).
- **pessimistic** a voter vote as if the worse outcome is selected.

optimistic and pessimistic are easy to compute if the CP-net is acyclic. If it is cyclic, it becomes hard.

Initial experiments

data generation:
Assumption 1: there exists a "true" dependency graph G_o and some voters make mistake.

- add an edge to G_o with probability r_1
- remove an edge from G_o with probability r_2

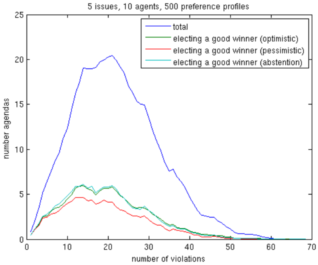
Then, generate random CP-tables that respect the dependencies.
Assumption 2: voters can rank up to 8 candidates
(i.e. voters can vote on combinaison of 3 issues at most).

Results with acyclic dependency graphs

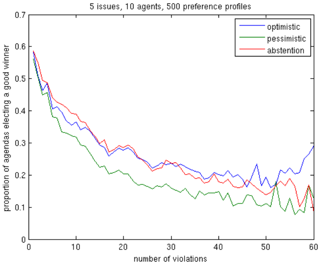
experiments with $|\mathcal{I}| = 5$ binary issues, $|\mathcal{N}| = 10$ voters, average over 500 preference profiles.
In 28% of the preference profiles generated, the largest election of the canonical agenda is less than 3, hence it produces a legitimate winner.
For the remaining profiles, we generate all possible agendas with election size no larger than **3** issues.

- about half the candidates can be elected
- a "legitimate winner" is elected is about 29% of the agendas (22% with pessimistic, 29% with optimistic and abstain)
- ⇨ 49% a "legitimate winner" is elected
- if we select an agenda minimizing the number of violations, a "legitimate winner" is elected 65% of the time.

Results with acyclic dependency graphs



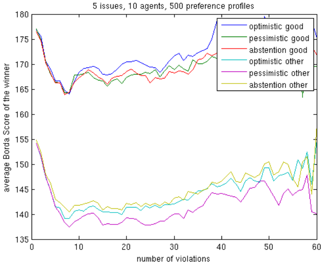
(a) number of agendas



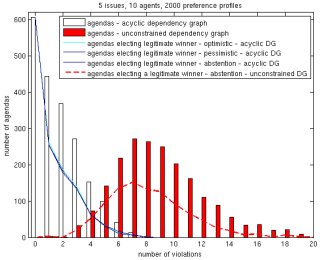
(b) proportion of agendas electing a legitimate winner

Results with acyclic dependency graphs

Quality of the winners



(a) Winners' avg Borda score over $\mathcal{G}_3(\mathcal{I})$



(b) Agendas minimizing the number of violations.

Results with unconstrained dependency graphs

- **none** of the canonical agendas is in $\mathcal{G}_3(\mathcal{I})$
- a legitimate winner was elected in 28.3% over all agendas in $\mathcal{G}_3(\mathcal{I})$
- if we concentrate on agenda that minimize the number of violations, a "legitimate winner" is elected in about 49% of the time

Conclusion and future works

- we need some real data, at least check with other types of data
- test with larger number of issues
- compute a likelihood of being pivotal given the dependency graph of the voters

current work:

- check if we can solve more profiles if we check the results a posteriori (a voter could cast a ballot indicating his preferential dependencies for the issues at stake).
- estimate/compute likelihood of electing a legitimate winner