Theodore Groves: Incentives in Teams

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General organization team model: $T = [I, (S, \mathscr{S}, P), \{B_i, i \in I\}, \omega_0]$

$(n+1)$-person game: $G = [I, (S, \mathscr{S}, P), \{B_i, i \in I\}, \{\omega_i, i \in I\}]$

1. Set of decision makers: $I = \{0, 1, \ldots, n\}$
   - The organisation head: 0
   - His employees: $1, \ldots, n$
2. Probability space of alternative states: $(S, \mathscr{S}, P)$
   - State space: S
   - σ-algebra over S: $\mathscr{S}$ (family of subsets of S that includes S and is closed under complementation and countable unions)
   - Probabilty measure: P (countable additive function $\mathscr{S} \to [0; 1]$ s.t. $P(\emptyset) = 0$ and $P(S) = 1$)
3. Set of alternative strategies for decision maker i: B_i $(i \in I)$

   $\rightarrow$ Set of joint strategies: $B = \bigtimes_{i=0}^{n} B_i$
4. Payoff (compensation) function for decision maker i:
   $\omega_i : B \times S \to \mathbb{R}$ (assumed to be P-integrable for every $\beta \in B$)

Expected value of the payoff function for decision maker i:
$\bar{\omega}_i : B \to \mathbb{R}$ defined by

$$\bar{\omega}_i(\beta) = \int_S \omega_i(\beta, s)\,\mathrm{d}P(s)$$

A joint strategy $\beta^* \in B$ is optimal if

$$\bar{\omega}_0(\beta^*) = \max_{\beta \in B} \bar{\omega}_0(\beta)$$

Assumption A: There exists a $\beta^* \in B$ such that
$\overline{\omega}_0(\beta^*) \geq \overline{\omega}_0(\beta)$ for all $\beta \in B$
$\overline{\omega}_0(\beta^*) > \overline{\omega}_0(\beta^*/\beta_i)$ for all $\beta_i \in B_i, \beta_i \neq \beta_i^*$ $(i = 1, \ldots, n)$

For a joint strategy $\beta = (\beta_0, \ldots, \beta_n)$ and a strategy β_i' for decision maker i, β/β_i' is $(\beta_0, \ldots, \beta_{i-1}, \beta_i', \beta_{i+1}, \ldots, \beta_n)$

Incentive structure: A set $W = \{\omega_i, i = 1, \ldots, n\}$ of employee payoff functions.

An incentive structure $W^* = \{\omega_i^*, i = 1, \ldots, n\}$ is optimal if

$$\bar{\omega}_i^*(\beta^*) = \max_{\beta_i \in B_i} \bar{\omega}_i^*(\beta^*/\beta_i) \qquad \text{uniquely for all } i = 1, \ldots, n$$

(the optimal joint strategy is in a strong sense a Nash equilibrium)

The incentive problem: To find an optimal incentive structure.

The paid worker incentive structure $W^0 = (\omega_1^0, \ldots, \omega_n^0)$ is defined by

$$\omega_i^0(\beta, s) = \begin{cases} 1 & \text{if } \beta_i = \beta_i^* \\ 0 & \text{otherwise} \end{cases} \qquad (i = 1, \ldots, n)$$

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The profit-sharing incentive structure $W^I = (\omega_1^I, \ldots, \omega_n^I)$ is defined by

$$\omega_i^I(\beta, s) = \alpha_i \omega_0(\beta, s) + A_i \qquad (i = 1, \ldots, n)$$

where α_i is a positive constant and A_i is any constant

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	W^0	W^I	W^{II}
Compensation by individual performance	✓	✗	✓
Only requires limited knowledge of the head	✗	✓	✓

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Condition S.1
(the decision makers)

$I = \{0, 1, \ldots, n\}$, where $i = 0$ is the head and $i = 1, \ldots, n$ the subunit managers.

Condition S.2
(independence of subunits)

$(S, \mathscr{S}, P) = \left(\bigtimes_{i=0}^{n} S_i, \sigma\left[\bigtimes_{i=0}^{n} \mathscr{S}_i\right], \prod_{i=0}^{n} P_i\right)$, where $(S_i, \mathscr{S}_i, P_i)$ is the probability space of the ith component's environmental state variable and $\sigma\left[\bigtimes_{i=0}^{n} \mathscr{S}_i\right]$ is the σ-algebra of subsets of S generated by the σ-algebras $\mathscr{S}_i$, $i = 0, \ldots, n$

Condition S.3

(a strategy contains an observation strategy, a message strategy, and a decision strategy, and the subunit managers only communicate with the head)

If $\beta_i \in B_i$ then $\beta_i = (\zeta_i, \gamma_i, \delta_i)$ for some

- observation strategy $\zeta_i : S_i \to \overline{Y}_i$
- message strategy $\gamma_i : Y_i \to \overline{Y}_i$
  - except $\gamma_0 : Y_0 \to \overline{Y}_0^n$ of the form $\lambda x \gamma_0(x) = \lambda x(\gamma_0^1(x), \dots, \gamma_0^n(x))$ ($\gamma_0^i : Y_0 \to \overline{Y}_0$ and $\gamma_0^i(x)$ is interpreted as the message from the head to the *i*th subunit)
- and decision strategy $\delta_i : Y_i \to D_i$

where $Y_0 = \overline{Y}_0 \times \cdots \times \overline{Y}_n$ and $Y_i = \overline{Y}_i \times \overline{Y}_0$ are information sets and $D_0, \dots, D_n$ are decision sets.

For given observation and message strategies, information functions $y_i : S \to Y_i$ satisfy

$$y_i(s) = \left[\zeta_i(s_i), \gamma_0^i(y_0(s))\right] \qquad (i = 1, \dots, n)$$

$$y_0(s) = [\zeta_0(s_0), \gamma_1(y_1(s)), \dots, \gamma_n(y_n(s))]$$

Condition S.4

(payoff for the head is the sum of the profits of the subunits and the central administration)

The payoff function for the head is of the form

$$\omega_0(\beta, s) = \sum_{i=1}^{n} \nu_i \left[\delta_i(y_i(s)), \delta_0(y_0(s)), s_i\right] + \nu_0 \left[\delta_0(y_0(s)), s_0\right]$$

where $\nu_i : D_i \times D_0 \times S_i \to \mathbb{R}$, $i = 1, \ldots, n$ and $\nu_0 : D_0 \times S_0 \to \mathbb{R}$ (profit functions)

Condition S.5
(The profit of a subunit accrues directly to that subunit)

$$\omega_i(\beta, s) = \nu_i \left[\delta_i(y_i(s)), \delta_0(y_0(s)), s_i\right] + \ldots ???$$

The class $\mathscr{I}$ of all incentive structures requiring the head to know no more than $y_0(s)$: The class of all tuples $(\omega_1, \ldots, \omega_n)$ where

$$\omega_i(\beta, s) = \nu_i \left[\delta_i(y_i(s)), \delta_0(y_0(s)), s_i\right] + C_i(y_0(s))$$

where again $C_i : Y_0 \to \mathbb{R}$

Conditional expected value:

For (measurable) subsets $U \subseteq S$:

$$\begin{aligned} E\left[f(s) | s \in U\right] &= \int_{s \in U} f(s)\, \mathrm{d}\frac{P(s)}{P(U)} \\ &= \int_{s \in U} f(s)\, \mathrm{d}\hat{P}_U(s) \\ &= \int_{s \in U} f(s)\, \mathrm{d}\hat{P}(s) \end{aligned}$$

The own profit incentive structure $W^{II} = (\omega_1^{II}, \dots, \omega_n^{II})$ is defined by

$$\omega_i^{II}(\beta, s) = \nu_i\left[\delta_i(y_i(s)), \delta_0(y_0(s)), s_i\right] + C_i^{II}(y_0(s)) \qquad (i = 1, \dots, n)$$

where for all $y_0 \in Y_0$

$$C_i^{II}(y_0) = \sum_{j \neq i} \int_{\{s \in S | y_0^*(s) = y_0\}} \nu_j\left[\delta_j^*(y_j^*(s)), \delta_0^*(y_0^*(s)), s_j\right] \mathrm{d}\hat{P}(s) - A_i \qquad (i = 1, \dots, n)$$

where again

$$y_j^*(s) = \left[\zeta_j^*(s_j), \gamma_0^{j*}(y_0^*(s))\right] \qquad (j = 1, \dots, n)$$

$$y_0^*(s) = \left[\zeta_0^*(s_0), \gamma_1^*(y_1^*(s)), \dots, \gamma_n^*(y_n^*(s))\right],$$

and

A_i is any constant $\qquad (i = 1, \dots, n)$

Theorem

Given the organization model $T = [I, (S, \mathscr{S}, P), \{B_i, i \in I\}, \omega_0]$ *with the conglomerate specifications S.1-S.5, if* T *satisfies Assumption* A*, then* W^{II} *is an optimal incentive structure in the class* $\mathscr{I}$.

Theorem

Given the organization model $T = [I, (S, \mathscr{S}, P), \{B_i, i \in I\}, \omega_0]$ *with the conglomerate specifications S.2-S.4, if T satisfies Assumption A and* $\gamma_i^*[Y_i] = \overline{Y}_i$ *and* $\forall \overline{y}_i \in \overline{Y}_i : P\{s \in S | \gamma_i^*(y_i^*(s)) = \overline{y}_i\} > 0$ $(I = 1, \ldots, n)$, *then* W^{II} *is an optimal incentive structure in the class* $\mathscr{I}$.

Theorem

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$$\omega_i(\beta, s) = \nu_i [\delta_i(y_i(s)), \delta_0(y_0(s)), s_i] + C_i(y_0(s))$$

$$\omega_i^{II}(\beta, s) = \nu_i [\delta_i(y_i(s)), \delta_0(y_0(s)), s_i] + C_i^{II}(y_0(s))$$

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To be shown: $\bar{\omega}_i^{II}(\beta^*) = \max_{\beta_i \in B_i} \bar{\omega}_i^{II}(\beta^*/\beta_i)$ uniquely for all $i = 1, \ldots, n$

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Sufficient to show:
$\bar{\omega}_i^{II}(\beta^*/\beta_i) + A_i = \bar{\omega}_0(\beta^*/\beta_i)$ for all $\beta_i \in B_i, i = 1, \ldots, n$

$$
\begin{aligned}
y_j^*(s) &= \left[\zeta_j^*(s_j), \gamma_0^{j*}(y_0^*(s))\right] && (j = 1, \ldots, n) \\
y_0^*(s) &= \left[\zeta_0^*(s_0), \gamma_1^*(y_1^*(s)), \ldots, \gamma_n^*(y_n^*(s))\right] \\
\hat{y}_j(s) &= \left[\zeta_j^*(s_j), \gamma_0^{j*}(\hat{y}_0(s))\right] && (j = 1, \ldots, n; j \neq i) \\
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$$
\bar{\omega}_i^{II}(\beta^*/\beta_i) + A_i = \int_{s \in S} \nu_i \left[\delta_i(\hat{y}_i(s)), \delta_0^*(\hat{y}_0(s)), s_i\right] \mathrm{d}P(s) +
$$

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\sum_{j \neq i} \int_{s \in S} \int_{\{s' \in S | y_0^*(s') = \hat{y}_0(s)\}} \nu_j \left[\delta_j^*(y_j^*(s')), \delta_0^*(y_0^*(s')), s_j'\right] \mathrm{d}\hat{P}(s') \mathrm{d}P(s)
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\bar{\omega}_0(\beta^*/\beta_i) = \int_{s\in S} \nu_i \left[\delta_i(\hat{y}_i(s)), \delta_0^*(\hat{y}_0(s)), s_i\right] \mathrm{d}P(s) +
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$$= \int_{s\in S}\int_{\{s'\in S|y_0^*(s')=\hat{y}_0(s)\}} \nu_j\left[\delta_j^*\left(\zeta_j^*(s_j'), \gamma_0^{j*}(y_0^*(s'))\right), \delta_0^*(y_0^*(s')), s_j'\right] \mathrm{d}\hat{P}(s')\mathrm{d}P(s)$$

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$$=\int_{s\in S}\int_{\{s_j'\in S_j|\exists s_0',\dots,s_{j-1}',s_{j+1}',\dots,s_n':y_0^*(s_0',\dots,s_j',\dots,s_n')=\hat{y}_0(s_0,\dots,s_j,\dots,s_n)\}}$$

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