

Modern Classics in Social Choice Theory

MSc Logic - Project June 2009

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Classics

- ▶ Kenneth J. Arrow. A difficulty in the Concept of Social Welfare. *The Journal of Political Economy*, 1950.
- ▶ Duncan Black. On the Rationale of Group Decision-making. *The Journal of Political Economy*, 1948.
- ▶ Kenneth O. May. A Set of Independent Necessary and Sufficient Conditions for Simple Majority Decision. *Econometrica*, 1952.
- ▶ Amartya K. Sen. A Possibility Theorem on Majority Decisions. *Econometrica*, 1966.
- ▶ Amartya K. Sen. The Impossibility of a Paretian Liberal. *The Journal of Political Economy*, 1970.
- ▶ Allan Gibbard. Manipulation of Voting Schemes: A General Result. *Econometrica*, 1973.
- ▶ Mark A. Satterthwaite. Strategy-proofness and Arrow's conditions: Existence and correspondence theorems for voting procedures and social welfare functions. *Journal of Economic Theory*, 1975.
- ▶ Edward H. Clarke. Multipart Pricing of Public Goods. *Public Choice*, 1971.
- ▶ Theodore Groves. Incentives in Teams. *Econometrica*, 1973.
- ▶ John Duggan, Thomas Schwartz. Strategic Manipulability without resoluteness or shared beliefs: Gibbard-Satterthwaite generalized. *Social Choice and Welfare*, 2000

Arrow: A difficulty in the Concept of Social Welfare, 1950

- ▶ Introduction of a notation for preferences,
- ▶ Axiomatization of preferences relations and discussion of the properties,
- ▶ Definition of social welfare function,
- ▶ Conditions on social welfare function, definition of the independence of irrelevant alternatives (IIA).
- ▶ Arrow's theorem:

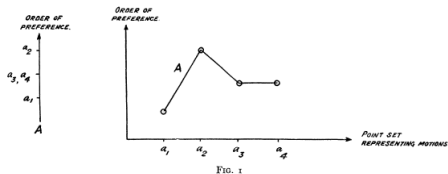
Theorem

"If there are at least three alternatives among which the members of the society are free to order in any way, then every social welfare function satisfying Conditions 2 [positive monotonicity, as May calls it] and 3 [(IIA)] and yielding a social ordering satisfying Axioms I [connectedness] and II [transitivity] must be either imposed or dictatorial."

- ▶ There's a mistake in the proof, see Blau 1957.

Black: On the Rationale of Group Decision-making, 1948

- ▶ Definition of single-peaked preference orderings and profiles:



- ▶ Proof of the result concerning the efficacy of majority rule on single-peaked profiles:
- ▶ For single-peaked profiles majority rules produces non-cyclic outcomes, it elects alternatives which get simple majority over every other.

May: A Set of Independent Necessary and Sufficient Conditions for Simple Majority Decision, 1952

- ▶ Four conditions characterizing simple majority rule:
 - ▶ Decisive (functionality and totality),
 - ▶ Anonymity (equality among voters),
 - ▶ Neutrality (the method does not favor any alternative),
 - ▶ Positive responsiveness (stronger than Arrow's positive monotonicity)

Sen: A Possibility Theorem on Majority Decisions, 1966

- ▶ Generalization of the conditions avoiding cycles (single peakedness, single cavedness,...),
- ▶ Condition of Value-restriction:
“A set of individual preferences over a triple of alternatives such that there exist one alternative and one value with the characteristic that the alternative never has that value in any individual’s preference ordering, is called a Value Restricted Preference pattern over that triple for those individuals”.
- ▶ Possibility theorem for valued restricted preferences.
- ▶ Comparison with the previous conditions avoiding cycles (e.g. single-peakendens.)

Sen: The Impossibility of a Paretian Liberal, 1970

- ▶ Sen's Liberalism: "The intention is to permit each individual at least one social choice".
- ▶ (L): For each individual i , there is at least one pair of alternatives, say (x, y) , such that if this individual prefers x to y , then the society should prefer x to y , and if this individual prefers y to x , then the society should prefer y to x .

▶ Theorem

There is no social decision function that can simultaneously satisfy condition U (unrestricted domain), P (weak Pareto principle) and L

- ▶ Minimal liberalism (L^*): There are at least two individuals such that for each of them there is at least one pair of alternatives over which he is decisive.

▶ Theorem

There is no social decision function that can simultaneously satisfy condition U (unrestricted domain), P (weak Pareto principle) and L^ .*

Gibbard: Manipulation of Voting Schemes: A General Result, 1973

- ▶ The general result: “any non-dictatorial voting scheme with at least three possible outcomes is subject to individual manipulation”. (The same result was proved also by Satterthwaite in his PhD thesis, with a different construction.)
- ▶ Formal approach, definition of *Game form* as a function from individuals to sets of strategies, where strategies are here the possible orderings of the alternatives at issue.
- ▶ The result on manipulability are stated in terms of game forms:

Theorem

Every straightforward game form with at least three possible outcomes is dictatorial

- ▶ From this result, one obtains as a corollary that every voting scheme with at least three outcomes is either dictatorial or manipulable.

Satterthwaite: Strategy-proofness and Arrow's conditions: Existence and correspondence theorems for voting procedures and social welfare functions, 1975

- ▶ Definition of sincere strategy, strategy-proof voting procedure.
- ▶ Three results:
- ▶ Every strategy-proof voting procedure on three or more alternatives is dictatorial (Gibbard - Satterthwaite);
- ▶ Correspondence between strategy-proof voting procedures and social welfare functions satisfying Arrow's conditions of rationality, non-negative response, citizens' sovereignty and independence of irrelevant alternatives.
- ▶ New proof of Arrow's theorem using the first two results.

Clark: Multipart Pricing of Public Goods, 1971

Groves: Incentives in Teams, 1973

- ▶ Results in mechanism design: is it possible to design a mechanism that makes truth-telling a dominant strategy?
- ▶ “The proposed system requires an assignment by society of cost responsibilities, and relies on a multipart pricing procedure to elicit reliable demand information” (Clarke '71).
- ▶ “The main result of the paper is given in a theorem that exhibits a system of compensation rules for employees of a conglomerate organization that will induce them to behave as a team” (Groves '73).
- ▶ Using cost responsibilities, under some hypotheses on utility functions (quasi-linearity), it is possible to avoid Gibbard-Satterthwaite result.

Duggan, Schwartz: Strategic Manipulability without resoluteness or shared beliefs: Gibbard-Satterthwaite generalized, 2000

- ▶ Manipulability is little affected by ties or beliefs about them,
- ▶ Generalization of Gibbard-Satterthwaite theorem on manipulability to the case including ties and multi-member choice sets (voting correspondences).

Suggestions for topics

- 1 Concept of strategy-proofness: used in both voting and mechanism design, leading to both impossibility and possibility results; comparison; investigate origin of concept; etc.
- 2 Same as above for a different "concept" of your choice featuring in different segments of the social choice literature.
- 3 What do social choice theorists mean when they speak of "the axiomatic method"? How did this approach develop? How does it compare to work in (mathematical/symbolic) logic?
- 4 Philosophical/ethical justification for axioms: What are the most important axioms used in the field? How do people justify them?

Suggestions for topics

- 5 History: How did the different "classics" (those discussed and maybe others) influence each other and the field as a whole?
- 6 Social choice and the real world: To what extent do results in social choice (in particular those discussed in the course) impact on the real world? Are some of the mechanisms proposed actually used? Do some of the negative results have serious consequences for how people organise society or make decisions?
- 7 Embedding of social choice theory into the "social and economic sciences" as a whole: How does social choice relate to other areas, including game theory, decision theory, welfare economics, microeconomics, political science, etc.