Formalization: methodology and concrete examples

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The famous muddy children (in prose).

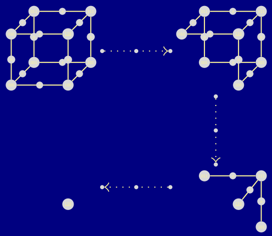
Three brilliant children go to the park to play. When their father comes to find them, he sees that two of them have mud on their foreheads. He then says, "At least one of you has mud on your forehead", and then asks, "Do you know if you have mud on your forehead?" The children simultaneously respond, "No".

The father repeats his question, "Do you know if you have mud on your forehead?" and this time the two children with muddy foreheads simultaneously answer, "Yes, I have mud on my forehead!" while the remaining child answers, "No, I don't know".

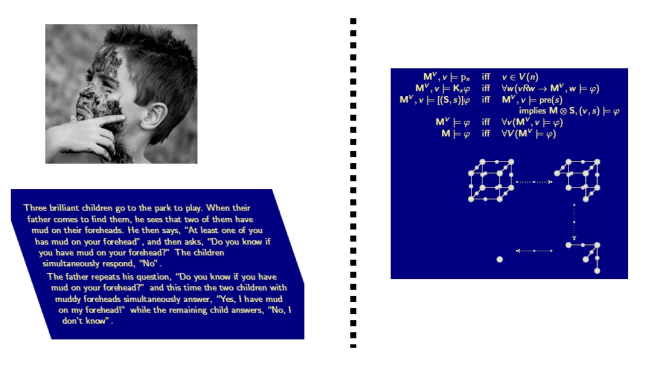
The famous muddy children (in symbols).

Semantics of dynamic epistemic logic.

$$\begin{array}{rcl}
\mathbf{M}^V, v \models \mathrm{p}_n & \text{iff} & v \in V(n) \\
\mathbf{M}^V, v \models \mathbf{K}_x\varphi & \text{iff} & \forall w(vRw \rightarrow \mathbf{M}^V, w \models \varphi) \\
\mathbf{M}^V, v \models [(\mathbf{S}, s)]\varphi & \text{iff} & \mathbf{M}^V, v \models \mathrm{pre}(s) \\
 & & \quad \text{implies } \mathbf{M} \otimes \mathbf{S}, (v, s) \models \varphi \\
\mathbf{M}^V \models \varphi & \text{iff} & \forall v(\mathbf{M}^V, v \models \varphi) \\
\mathbf{M} \models \varphi & \text{iff} & \forall V(\mathbf{M}^V \models \varphi)
\end{array}$$



What happened here?



We have informal "real world" data on the left side and replaced it with a formal representation on the right side to explain the story.

The task of the logician.

Transforming "real world" data into formal representations: *modelling* gives us a representation of the phenomena we want to understand in a formal language that we can manipulate and apply the deductive method to.

But how does this translation from the real world data to the formal language work?

First lesson (1).

Modelling is a representation: the model is not the same object as the modelled reality.

Pierre Duhem, La théorie physique: son objet, et sa structure (1906)

First lesson (2).

"And then came the grandest idea of all! We actually made a map of the country, on the scale of a mile to the mile!"

"Have you used it much?" I enquired.

"It has never been spread out, yet," said Mein Herr: "the farmers objected: they said it would cover the whole country, and shut out the sunlight! So we now use the country itself, as its own map, and I assure you it does nearly as well."

Lewis Carroll (Charles Dodgson)
Sylvie and Bruno Concluded (1893)

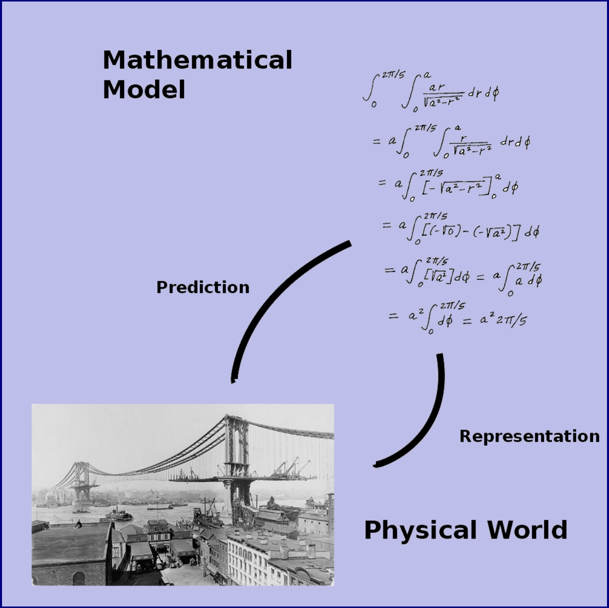
First lesson (3).

And yet:

- "Knowledge is an equivalence relation".
- "Natural numbers are von Neumann ordinals".

First lesson. So, we keep in mind that modelling is a process that replaces one thing with another and that we need to make sure that the model stays close to the modelled object. How do we measure whether the representation is close to the modelled object?

Mathematical modelling.



The hermeneutic circle.

Johann Conrad Dannhauer
1603–1666

Christian Weise
1642–1708

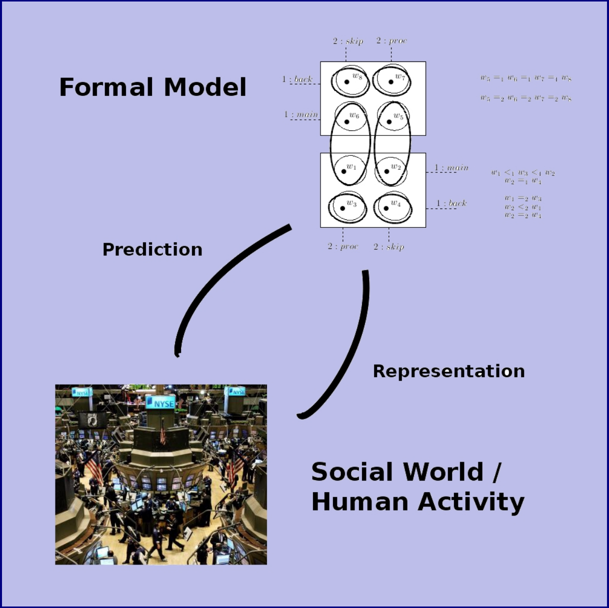
Modelling is a hermeneutic process, starting from hermeneutical presumptions (*in bonam partem interpretari*) and aiming to reach *reflective equilibrium* between the model and the modelled.

Step 1. Modelling. Design of a mathematical model based on pre-theoretic world knowledge.

Step 2. Phenomenology. On the basis of predictions of the mathematical model, identify phenomena of the physical world.

Step 3. Critical assessment. Critically discuss the model in light of the results from **Step 2** and come up with changes for a new **Step 1**.

Formal modelling of human activity



Second lesson.

Since modelling is a hermeneutic process, we need explicitly name something that plays the role of the external physical world in our modelling process and define what would constitute a *modelling success*.

Methodologically, we need to make sure that we separate the steps of *Modelling* and *Phenomenology*.

In the case of philosophical concepts, what is the external benchmark we are testing our formal representations against?

The traditional answer.

The traditional answer is *Expert Intuition*.

In philosophy, we're dealing with concepts that have a history of hundreds if not thousands of years. Who better, if not the experts in the history of these concepts, i.e., the philosophers, to judge whether a formalisation is in line with the concepts.

Problems with the traditional answer.

- *Clash between two different forms of expert intuition:* native speaker vs trained linguist.
- *Clash between two experts.*

Mates, B.: 1958, On the verification of statements about ordinary language. *Inquiry* 1, 161171:

> *[T]he intuitive findings of different people, even of different experts, are often inconsistent. Thus, for example, while Prof. Ryle tells us that "voluntary" and "involuntary" in their ordinary use are applied only to actions which ought not to be done, his colleague Prof. Austin states in another connection: ". . . for example, take 'voluntary' and 'involuntary': we may join the army or make a gift voluntarily, we may hiccough or make a small gesture involuntarily . . ." If agreement about usage cannot be reached within so restricted a sample as the class of Oxford Professors of Philosophy, what are the prospects when the sample is enlarged? (p. 165)*

Experimental philosophy vs. traditional philosophy (1).

Expert intuition has been criticized by a new species of philosophers, the so-called *experimental philosophers*:

Knobe, J., and S. Nichols, *An Experimental Philosophy Manifesto*, in: Knobe, J., and S. Nichols, editors (2008), *Experimental Philosophy*, Oxford University Press, New York, pp. 3–16.

Experimental philosophy vs. traditional philosophy (2).

Weinberg, J. M., S. Nichols, and S. Stich (2001), "Normativity and Epistemic Intuitions", *Philosophical Topics* **29**(1/2):429–460.

Williamson, T. (2004), "Philosophical 'Intuitions' and Scepticism about Judgment", *Dialectica* **58**(1):109–153.

Sosa, E. (2007), "Experimental Philosophy and Philosophical Intuition", *Philosophical Studies* **132**:99–107.

Symons, J. (2008), "Intuition and Philosophical Methodology", *Axiomathes* **18**:67–89.

There is no such thing as unbiased empirical data. Each set of data has its own bias that derives from the time, method, location of its collection. If you are doing research on the correlation between brain activity and some thought processes and try to argue that certain things are hard-wired and independent of culture, keep in mind that 99% of all brain scans that exist were made of relatively affluent people from Western industrial nations who have health insurance.

Examples from empirical studies in language.

If you are trying to get information about the semantics and pragmatics of words in English, you need to restrict your test group.

The third lesson.

There is no such thing as a universal empirical basis: we need to make a principled decision what our empirical basis should be and why.

Concrete example 1 (1).

Paul Benacerraf, What numbers could not be?, *Philosophical Review* 74 (1965):47–73.

- Two boys, Ernie and Johnny, whose respective logicist parents decide that they should learn their mathematics starting from its very foundations and so are first taught logic and set theory.
- Only once this is complete, the two boys are told about numbers: they are simply informed which parts of their set-theoretical knowledge are called 'numbers' by other, ordinary people.
- Ernie: 0 is $\varnothing$ and the successor of n is $n \cup \{n\}$.
- Johnny: 0 is $\varnothing$ and the successor of n is $\{n\}$.
- Ernie: n has n elements; Johnny: n has one element.
- Ernie & Johnny: $16 \in 17$; Ernie: $3 \in 17$; Johnny: $3 \notin 17$.

Concrete example 1 (2).

Benacerraf uses this thought experiment as an argument to show that numbers are not sets (*the problem of arbitrary identification*).

In fact what is going on is that each of the systems of Johnny and Ernie are different *formalization* of a pre-formal concept of number reflecting correctly all of the number-theoretic properties ascribed to the pre-formal concept of number.

But both formalizations (Zermelo and von Neumann) add new expressive power that wasn't there before: to the *ordinary people*, numbers are not collections of other numbers, and thus the question whether $3 \in 17$ doesn't even make sense.

How do we check whether our formalization is adequate? ("ordinary people" vs mathematicians) If you had organized a poll of mathematicians in 1900, no one would have accepted $3 \in 17$. If you did it today, a sizable number would accept this.

Participant observation / participatory action research.

Concrete example 2 (1).

Formalisations of narratives. Human beings are able to consider narratives abstractly and compare whether two narratives are "essentially the same":

- Romeo & Juliet vs West Side Story.
- A popular book (say, Lord of the Rings or Harry Potter) and its movie version.

This process of comparison seems to be an abstraction process and thus it is natural to assume that it could be formalized.

Concrete example 2 (2).

Which formal framework do you pick? What are the criteria for a good language?

Benedikt Löwe, Methodological remarks about comparing formal frameworks for narratives, in: Patrick Allo, Giuseppe Primiero (eds.), Third Workshop in the Philosophy of Information, Contactforum van de Koninklijke Vlaamse Academie van België voor Wetenschappen en Kunsten, Brussel 2011, pp. 10-28

Rens Bod, Bernhard Fisseni, Aadil Kurji, Benedikt Löwe, Objectivity and reproducibility of Proppian annotations, in: Mark A. Finlayson (ed.), The Third Workshop on Computational Models of Narrative, Cambridge MA 2012, pp. 17-21

Bernhard Fisseni, Benedikt Löwe, Which dimensions of narrative are relevant for human judgments of story equivalence?, in: Mark A. Finlayson (ed.), The Third Workshop on Computational Models of Narrative, Cambridge MA 2012, pp. 114-118

Our three lessons.

- **First lesson.** Modelling is a process that replaces the object of study by formal objects. We need to keep the object of study and its formal representation apart.
- **Second lesson.** The quality of modelling is determined by the degree of agreement between the object of study and the model. We need to keep the methodologies of modelling and of testing the model against the external data separate.
- **Third lesson.** There is no neutral choice for the empirical basis of our testing: the choice will have an effect on our results and therefore has to be part of our principled set-up and properly documented.