WinKE Questions

Question 1. Work on the (three) problems in file `cs3aur-1.ke`, which you can download from the course web site.

Question 2. An article on computational complexity in the *New York Times* from 13 July 1999 starts like this:

> *"Anyone trying to cast a play or plan a social event has come face-to-face with what scientists call a satisfiability problem. Suppose that a theatrical director feels obligated to cast either his ingénue, Actress Alvarez, or his nephew, Actor Cohen, in a production. But Miss Alvarez won't be in a play with Cohen (her former lover), and she demands that the cast include her new flame, Actor Davenport. The producer, with her own favors to repay, insists that Actor Branislavsky have a part. But Branislavsky won't be in any play with Miss Alvarez or Davenport. [...]"*

Is there a possible casting (and if there is, who will play)?

Formalise the problem and enter it into WinKE. Then saturate the corresponding KE proof tree. If there is an open branch left it will correspond to a model (a possible casting). You can use the *Countermodel* option from the *Analysis* menu to generate it. (WinKE uses the term 'countermodel', because a model constitutes a counterexample for an attempted KE proof.)

Question 3.

(a) Try to prove $(\forall x)(P(x) \vee Q(x)) \models (\forall x)P(x) \vee (\forall x)Q(x)$.

(b) Proving the statement in (a) is not possible (because it is false). Try to get WinKE to generate a countermodel for you. To do this you need to expand one of the branches (in case there are several) until it is 'obvious' that further rule applications would never succeed in closing it. (In the WinKE help system, under *useful instantiation*, you can read how WinKE can decide in *some* cases whether further rule applications would be futile; but note that there can be no general strategy of this kind as FOL is undecidable.)

(c) WinKE's description of models is somewhat informal. Write down the model suggested by WinKE in the formal way introduced during classes.

Question 4. Use WinKE to generate as many models as you can for the following set of sentences:

$$\{\ \mathit{loves}(\mathit{Arabella}, \mathit{Carlo}),\ \mathit{loves}(\mathit{Carlo}, \mathit{Bianca}) \rightarrow \mathit{loves}(\mathit{Dino}, \mathit{Bianca}),\ \mathit{loves}(\mathit{Eduardo}, \mathit{Arabella}) \vee \mathit{loves}(\mathit{Eduardo}, \mathit{Bianca})\ \}$$

Note that for this particular type of example, WinKE can provide a visualisation of the generated model. Try it.

Question 5. Use WinKE to prove the following statements. Part of the exercise is about understanding what exactly these statements mean (and how to enter them into WinKE).

(a) $(\exists x)P(x) \rightarrow (\forall x)P(x),\ P(a) \models P(b)$

(b) The set $\{A \vee B, A \vee \neg B, \neg A \vee B, \neg A \vee \neg B\}$ is not satisfiable.

(c) $(\forall x)(A(x) \vee B(x)),\ \neg((\exists x)A(x)),\ (\exists x)\neg B(x) \models \bot$

(d) $(\exists x)(P(x) \vee Q(x)) \leftrightarrow (\exists x)P(x) \vee (\exists x)Q(x)$ is a theorem.

(e) $\models (\exists x)(\forall y)(\forall z)[(P(y) \rightarrow Q(z)) \rightarrow (P(x) \rightarrow Q(x))]$

Question 6. The objective of this question is to prove that any binary relation that is symmetric, transitive, and serial (a relation is called serial if for any element we have an element to which the former is related) must also be reflexive.

Proceed as follows. Given the following abbreviations:

$$
\begin{aligned}
\textit{reflexive} &= (\forall x)R(x,x) \\
\textit{symmetric} &= (\forall x)(\forall y)(R(x,y) \rightarrow R(y,x)) \\
\textit{transitive} &= (\forall x)(\forall y)(\forall z)(R(x,y) \wedge R(y,z) \rightarrow R(x,z)) \\
\textit{serial} &= (\forall x)(\exists y)R(x,y)
\end{aligned}
$$

Use WinKE to show: $\textit{symmetric} \wedge \textit{transitive} \wedge \textit{serial} \models \textit{reflexive}$.