Homework #1

Deadline: Tuesday, 13 September 2022, 19:00

Exercise 1 (10 points)

This exercise concerns *uniform quota rules* in judgment aggregation.

(a) Consider the uniform quota rule F_λ with quota $\lambda = \frac{2}{3}n$. Which of the following properties are satisfied by F_λ: anonymity, neutrality, independence, completeness, complement-freeness? Justify your answers, writing one sentence per property.

(b) Consider the agenda $\Phi = \{p, \neg p, q, \neg q, r, \neg r, p \vee q \vee r, \neg(p \vee q \vee r)\}$. Characterise the class of all uniform quota rules (in terms of their quota λ) that are guaranteed to return a consistent judgment set for any admissible (i.e., complete and consistent) profile over Φ. Briefly justify your answer.

Exercise 2 (10 points)

Intuitively speaking, when there is unanimous agreement on something by all agents involved in a decision making scenario, then we would like to see that agreement be reflected by the collective decision taken. Here are two possible ways in which we could formalise this intuitive idea as an axiom for judgment aggregation rules F:

- *Propositionwise unanimity:* F is unanimous if, for every formula $\varphi \in \Phi$ and every profile $\boldsymbol{J} \in \mathcal{J}(\Phi)^n$, it is the case that $\varphi \in J_i$ for all agents $i \in N$ implies $\varphi \in F(\boldsymbol{J})$. That is, if every agent accepts φ, then so should the aggregation rule.
- *Simple unanimity:* F is unanimous if, for every judgment set $J \in \mathcal{J}(\Phi)$, it is the case that $F(J, \ldots, J) = J$. That is, if all agents report exactly the same judgment set, then that same set should also be the output of the aggregation rule.

Show that these two definitions define two different concepts. Then show that, when restricted to judgment aggregation rules that are independent and that always return complement-free outcomes, the two definitions coincide.

Exercise 3 (10 points)

Consider the doctrinal paradox agenda $\Phi = \{p, \neg p, q, \neg q, p \wedge q, \neg(p \wedge q)\}$. Suppose there are objectively correct truth values for each of these agenda items (so-called *ground truth*). Furthermore, suppose that, when an agent is asked for her judgment regarding one of the two premises, she will give the correct answer with a probability of 70%, and she will do so independently of any of the judgments made by anyone. This exercise is about comparing the epistemic performance of the premise-based and the conclusion-based rules in this scenario. We want to understand how likely each of these rules is to return the correct judgment for the conclusion, and what parameters this probability depends on (if any).

Observe that we can distinguish four situations regarding the ground truth: (1) pq (both p and q are objectively true); (2) $p\bar{q}$ (only p is objectively true); (3) $\bar{p}q$ (only q is objectively true); and (4) $\bar{p}\bar{q}$ (both p and q are objectively false). So in the first situation $p \wedge q$ is true and in the other three situations it is false. Note that the second and the third situation are essentially symmetric, so it will suffice to analyse only one of them. When answering the following questions, consider the possibility that the performance of a rule might depend on the ground truth (i.e., the situation we are in). For example, a given rule might be less likely to return the correct truth value for $p \wedge q$ in situation pq than in situation $\bar{p}\bar{q}$.

(a) For each of the four situations identified (i.e., for each possible ground truth), what is the probability of a given agent to report the correct truth value for the conclusion?

(b) Let n be an odd number. When there are n agents participating, what is the probability of the majority judgment regarding a given premise being correct? Express this probability as a formula and calculate it for the three values $n = 3, 9, 99$. Does this probability depend on the ground truth?

(c) As n goes to infinity, what is the probability of the premise-based rule to return the correct truth value for the conclusion, and what is the same probability for the conclusion-based rule? Discuss to what extent these probabilities depend on the ground truth.