Homework #1

Deadline: Wednesday, 7 April 2021, 18:00

Question 1 (10 marks)

In analogy to the definition of a Condorcet winner, a *Condorcet loser* is an alternative that would lose against every other alternative in a pairwise majority contest.

(a) Give an example that shows that the plurality rule *can* elect a Condorcet loser.

(b) Prove that the Borda rule *never* elects a Condorcet loser.

Remark: It is in fact possible to show that the Borda rule is the *only* positional scoring rule (with a strictly descending scoring vector) that satisfies this property. This is not part of the exercise, but you may still want to think about it.

Question 2 (10 marks)

Recall that, when you design a voting rule, it is not always possible to achieve resoluteness if we also require anonymity and neutrality. For example, it is easy to see that this combinations of desiderata is impossible to satisfy when there are two alternatives and two voters.

For this exercise, we focus on elections with *three alternatives* and n *voters*. Suppose we accept that resoluteness is hard to achieve, but that we at least want to have a voting rule that *never returns a three-way tie* between all three alternatives, besides being *anonymous* and *neutral*. For some values of n this is possible, while for others it is impossible. Provide a full characterisation of when it is possible and when it is impossible. For the cases for which it is possible, define a voting rule that has all the desired properties. For the cases for which it is impossible, prove that this really is so.