Homework #2

Deadline: Monday, 29 April 2019, 18:00

Question 1 (10 marks)

In analogy to the definition of a Condorcet winner, a *Condorcet loser* is an alternative that would lose against every other alternative in a pairwise majority contest.

(a) Give an example that shows that the plurality rule *can* elect a Condorcet loser.

(b) Prove that STV *never* elects a Condorcet loser.

(c) Recall that the Slater rule is a Condorcet extension. Prove that Slater *never* elects a Condorcet loser. Is this true for every Condorcet extension? Justify your answer.

(d) Prove that the Borda rule *never* elects a Condorcet loser.

(e) Prove that for $m \geqslant 3$ alternatives and every $k \in \{1, \ldots, m-1\}$, the k-approval rule will elect a Condorcet loser in at least one profile for some number n of voters.

Question 2 (10 marks)

Recall that, when you design a voting rule, it is not always possible to achieve resoluteness if we also require anonymity and neutrality. For example, it is easy to see that this combinations of desiderata is impossible to satisfy when there are two alternatives and two voters.

For this exercise, we focus on elections with *three alternatives* and *n voters*. Suppose we accept that resoluteness is hard to achieve, but that we at least want to have a voting rule that *never returns a three-way tie* between all three alternatives, besides being *anonymous* and *neutral*. For some values of n this is possible, while for others it is impossible. Provide a full characterisation of when it is possible and when it is impossible. For the cases for which it is possible, define a voting rule that has all the desired properties. For the cases for which it is impossible, prove that this really is so.

Question 3 (10 marks)

The purpose of this exercise is to investigate what happens to Arrow's Theorem, in its formulation for resolute social choice functions discussed in class, if we replace the Pareto Principle by the seemingly more basic surjectivity condition. Recall that we had defined surjectivity in the context of our discussion of the Muller-Satterthwaite Theorem.

(a) Show that the Pareto Principle is strictly stronger than surjectivity. That is, show that every Paretian resolute social choice function is surjective and that there exists a surjective resolute social choice function that is not Paretian.

(b) Show that Arrow's Theorem ceases to hold when we replace the Pareto Principle by surjectivity. That is, show that there exists a resolute social choice function that is surjective, independent, and nondictatorial.

Question 4 (10 marks)

The purpose of this exercise is to explore the boundaries of some of the impossibility theorems we had discussed in class. Answer the following questions:

(a) Does the Muller-Satterthwaite Theorem continue to hold when we replace strong monotonicity by weak monotonicity?

(b) Does the Gibbard-Satterthwaite Theorem continue to hold when we drop surjectivity?

(c) Does the Duggan-Schwartz Theorem continue to hold when we replace the condition of immunity against manipulation by both optimistic and pessimistic voters by immunity against manipulation by optimistic voters only?

(d) Does the Duggan-Schwartz Theorem continue to hold when we replace the condition of immunity against manipulation by both optimistic and pessimistic voters by immunity against manipulation by pessimistic voters only?

(e) Let us call a voter *cautious* if she prefers a set of alternatives A to another set B only if she ranks her least preferred alternative in A above her most preferred alternative in B. That is, such a voter would only consider manipulating if the worst way of breaking ties would yield a better result for her than the best way of breaking ties when she votes truthfully. Does the Duggan-Schwartz Theorem continue to hold when we replace the condition of immunity against manipulation by both optimistic and pessimistic voters by immunity against manipulation by cautious voters?

Justify your answers. If you show that a given theorem ceases to hold under the changed conditions by proving a specific voting rule meets all the requirements stated, also indicate why that same voting rule does not constitute a counterexample to the original theorem.