Homework #3

Deadline: Tuesday, 2 May 2017, 19:00

Submit your solutions for (up to) three of the five exercises below. If you solve more than that, we will consult a random number generator to decide which three to look at and grade.

Question 1 (10 marks)

Attend the special session in honour of Kenneth Arrow of the Dutch Social Choice Colloquium on 21 April 2017 and write a summary of 500–750 words of one of the talks given. Your summary should be accessible to and of some interest for people taking this course who have missed the talk in question. Where appropriate, adapt the terminology and notation used by the speaker to what we are using in this course. Include a word count.

Question 2 (10 marks)

The purpose of this exercise is to explore the boundaries of some of the impossibility theorems we have discussed in class. Answer the following questions:

(a) Does the Muller-Satterthwaite Theorem continue to hold when we replace strong monotonicity by weak monotonicity?

(b) Does the Gibbard-Satterthwaite Theorem continue to hold when we drop surjectivity?

(c) Does the Duggan-Schwartz Theorem continue to hold when we replace the condition of immunity against manipulation by both optimistic and pessimistic voters by immunity against manipulation by optimistic voters only?

(d) Does the Duggan-Schwartz Theorem continue to hold when we replace the condition of immunity against manipulation by both optimistic and pessimistic voters by immunity against manipulation by pessimistic voters only?

(e) Let us call a voter *cautious* if she prefers a set of alternatives A to another set B only if she ranks her least preferred alternative in A above her most preferred alternative in B. That is, such a voter would only consider manipulating if the worst way of breaking ties would yield a better result for her than the best way of breaking ties when she votes truthfully. Does the Duggan-Schwartz Theorem continue to hold when we replace the condition of immunity against manipulation by both optimistic and pessimistic voters by immunity against manipulation by cautious voters?

Justify your answers. If you show that a given theorem ceases to hold under the changed conditions by proving a specific voting rule meets all the requirements stated, also indicate why that same voting rule does not constitute a counterexample to the original theorem.

Question 3 (10 marks)

For voting scenarios with n voters and m alternatives there are $(m!)^n$ possible preference profiles. How many of them are single-peaked? Justify your answer.

Question 4 (10 marks)

When investigating the computational complexity of problems, it is often more convenient to consider *decision problems* rather than *search problems*, even if the latter is what we are ultimately interested in. This exercise is about formulating a suitable decision problem for a specific search problem at hand. Let F be a *social welfare function* (SWF) that returns a single linear order for every given profile of linear orders, i.e., it is a function of the form $F : \mathcal{L}(X)^n \to \mathcal{L}(X)$. We are interested in the search problem of finding $F(\boldsymbol{R})$ when given a profile $\boldsymbol{R} \in \mathcal{L}(X)^n$ as input. Answer the following questions:

(a) Formulate a decision problem Q such that you can construct $F(\boldsymbol{R})$ in polynomial time by making multiple queries to an oracle for Q. Assume that every such query to the oracle requires only a single unit of time. Be very explicit about the formulation of Q: specify what the *input* is, and specify what the *question* is that is to be decided. Then explain *how* your can solve the original search problem in polynomial time if you have access to an oracle for Q (this is called a *polynomial-time Turing reduction*).

(b) Suppose Q is decidable in polynomial time. What can you conclude about the complexity of the search problem of finding $F(\boldsymbol{R})$ given $\boldsymbol{R}$ as input? Why? Explain.

(c) Now, instead, suppose that Q is not decidable in polynomial time. Can you show that the problem of finding $F(\boldsymbol{R})$ is also not solvable in polynomial time? If so, explain how. If not, explain why not.

Question 5 (10 marks)

We have seen that any nondictatorial voting rule can be manipulated when we want that rule to operate on all possible preference profiles. We have also seen that this problem can be avoided when we restrict the domain of possible profiles appropriately, e.g., to single-peaked profiles. What we have not discussed is the *frequency of manipulability:* how often will we encounter a profile in which a voter has an incentive to manipulate? One way of studying this problem is by means of simulations: generate a large number of profiles and check for which proportion of them the problem under consideration (here, manipulability) occurs. The standard method for generating profiles is to make the *impartial culture assumption*, under which every logically possible preference order has the same probability of occurring. Write a program to analyse and compare the frequency of manipulability of the *plurality*, *Borda*, and *Copeland* rule under the impartial culture assumption. You may collaborate in groups of up to three people. Submit your commented code, instructions for how to run your program, and a report detailing your approach and findings.

Remark: This is not required for this exercise, but note that you could take this much further and look for more convincing assumptions than the impartial culture assumption. You could also carry out a similar study using real data.