Coursework #1

Deadline: Tuesday, 6 March 2007, 11:15am

Question 1 (10 marks)

A social welfare function is said to satisfy the axiom of *non-imposition* (NI) if any social preference ordering is achievable by *some* profile of individual preference orderings:

$$(\forall P \in \mathcal{P})(\exists \boldsymbol{P'} \in \mathcal{P}^n)(\forall x, y \in A)[xPy \leftrightarrow xP'y]$$

In other words, a social welfare function satisfying (NI) does not impose any restrictions that would *a priori* exclude a particular social preference ordering.

(a) Show that the Pareto condition (P) implies (NI).

(b) Show that Arrow's Theorem breaks down if we replace (P) by (NI).

(Adapted from A.D. Taylor, *Social Choice and the Mathem. of Manipulation*, CUP, 2005.)

Question 2 (10 marks)

Two candidates, A and B, compete in an election. Of the n citizens, k support candidate A and $m = n - k$ support candidate B. Each citizen decides either to vote, at a cost c (with $0 < c < 1$), for the candidate they support, or to abstain. A citizen who abstains receives the payoffs of 2 if their candidate wins, 1 if there is a draw, and 0 if their candidate loses. A citizen who does vote receives the payoffs $2 - c$, $1 - c$, and $-c$, respectively.

(a) For $k = m = 1$, is this the same as any of the games introduced in class?

(b) For $k = m$, identify the set of all Nash equilibria.

You may find considering some of the following questions helpful: Is the situation where everyone votes in equilibrium? Is there a Nash equilibrium in which the candidates tie and not everyone votes? Is there a Nash equilibrium in which one of the candidates wins by just one vote? Is there a Nash equilibrium in which one of the candidates wins by two or more votes?

(c) What is the set of Nash equilibria for $k < m$?

(Adapted from M.J. Osborne, *An Introduction to Game Theory*, OUP, 2004.)

(Please turn over)

Question 3 (10 marks)

Compute all mixed Nash equilibria for each of the following two games. Show your working.

(a)

	L	R
T	2/3	5/3
B	5/4	3/3

(b)

	L	R
T	3/0	2/1
B	2/4	8/2

Question 4 (10 marks)

The numbers in a game matrix mean slightly different things depending on whether we do or do not admit mixed strategies. If only pure strategies are considered, then the numbers represent ordinal preference relations for the players involved, and the cardinal intensities of payoffs do not matter. This question is about games with pure strategies only. Let us call two such games equivalent iff the game matrices involved represent the same ordinal preference relations. Answer the following questions (and justify your answers):

(a) How many different two-player games with two actions per player are there?

(b) How many of these do not have a (pure) Nash equilibrium?

(c) A finer notion of equivalence would also regard two games as being the same when we can obtain one from the other by swapping the columns, the rows, or both (but not the players). Answer parts (a) and (b) also with respect to this refinement.