SAT-Based Automated Mechanism Design for False-Name-Proof Facility Location

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Idea

"In this paper we apply a very recent Automated Mechanism Design approach based on Boolean Satisfiability (SAT) to the mechanism design of false-name-proof facility location."

Outline

Facility Location

Agents are (ideally) located on the vertices of a graph and a public good (or bad) is to be located on one of them.

- An undirected, connected graph $\Gamma = (V, E)$
- The distance function $d : V^2 \to \mathbb{N}$ defined as $d(v, w) = |\{e \in E \mid e \in s(v, w)\}|$ for all $v, w \in V$, where $s(v, w)$ is a shortest path from v to w.
- $\mathcal{N}$ the set of potential agents
- $N \subseteq \mathcal{N}$ a set of participating agents
- Each agent $i \in N$ has type $\theta_i \in V$ (i is located on vertex θ_i)
- $\theta = (\theta_i)_{\imath \in N}$, the profile of types of the agents
- $I(\theta) \subseteq V$ is the set of vertices on which at least one agent is located.

$$I(\theta) = \bigcup_{i \in N} \theta_i$$

- $\succeq_v$ is the preference of the agent located at vertex v.
  $\succeq_v$ is single peaked if $w \succ_v x$ iff $d(v, w) < d(v, x)$ and $w \sim_v x$ iff $d(v, w) = d(v, x)$
  Similarly, single dipped.

Facility Location

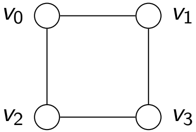
A mechanism is a mapping from the set of possible profiles to the set of vertices.

Agents might pretend to be multiple agents, so mechanisms are defined for different-sized profiles.

$$f = (f_N)_{N \subseteq \mathcal{N}}, \text{ where } f_N : V^{|N|} \to V$$

Assumption. Mechanisms are anonymous.

Example (2-by-2 grid):



$$\theta = (v_0, v_1, v_1)$$

$$\succeq_{v_0}: v_0 > v_1, v_2 > v_3$$
$$\succeq_{v_1}: v_1 > v_0, v_3 > v_2$$

False-Name-Proofness (FNP). A mechanism is false-name-proof if for any $N, \theta, i \in N, \theta_i \in V, \theta'_i \in V, \Phi_i \subseteq \mathcal{N} \setminus N$, and $\Phi_{\Phi_i} \in V^{|\Phi_i|}$

$$f(\theta) \succeq_{\theta_i} f(\theta'_i, \theta_\Phi, \theta_{-i}).$$

Pareto Efficiency (PE). An outcome $v \in V$ Pareto dominates $w \in V$ under θ if $v \succeq_{\theta_i} w$ for all $i \in N$ and $v \succ_{\theta_j} w$ for some $j \in N$. A mechanism f is Pareto efficient if for any N anf θ, no outcome $v \in V$ Pareto dominates $f(\theta)$.

Ignoring Duplicate Ballots (IDB). A mechanism is ignoring duplicate ballots if for any pair θ, θ',

$$I(\theta) = I(\theta') \text{ implies } f(\theta) = f(\theta')$$

If we only consider PE and FNP mechanisms with single peaked (or single dipped) preferences, then we can focus on IDB mechanisms without loss of generality.

Now we can consider only mechanisms that do not count the number of agents located on each vertex

$$f : 2^V \to V$$

Encoding

There is one variable $c_{S,v}$ for every set $S \in 2^V \setminus \{\emptyset\}$ and every vertex $v \in PE(S)$.

The variables for a 2-by-2 grid are:

$$c_{0,0}, c_{1,1}, c_{2,2}, c_{3,3},$$
$$c_{01,0}, c_{01,1}, c_{02,0}, c_{02,2}, c_{03,0}, c_{03,1}, c_{03,2}, c_{03,3},$$
$$c_{12,0}, c_{12,1}, c_{12,2}, c_{12,3}, c_{13,1}, c_{13,3}, c_{23,2}, c_{23,3},$$
$$c_{012,0}, c_{012,1}, c_{012,2}, c_{013,0}, c_{013,1}, c_{013,3}$$
$$c_{023,0}, c_{023,2}, c_{023,3}, c_{123,1}, c_{123,2}, c_{123,3}$$
$$c_{0123,0}, c_{0123,1}, c_{0123,2}, c_{0123,3}$$

Then, feasibility and false-name-proofness constraints are encoded in CNF.

Results

1. Replication of known results.
   There is no mechanism that satisfies false-name-proofness and Pareto efficiency on a cycle graph C_k for any integer $k \geq 6$. However, for every $k \leq 5$ there is such mechanism.
2. Finding new mechanisms.
   They found new FNP and PE mechanisms for the location of a public good (or bad) in a 2-by-2 grid.
3. An Impossibility.
   There are mechanisms for locating a public bad that satisfy both FNP and PE for 2-by-3 grid. But there are no mechanisms that satisfy FNP, PE and **surjectivity**.

Results

4 Extension to a new variant of facility location.
There is a FNP and PE mechanism for the location of a pubic good in a cycle graph with six vertices if we restrict the set of manipulators by considering one trusted vertex.

Summary

- They used the SAT solver approach to describe mechanisms with some desired properties.
- Their focus was more on a practical problem.
- Nevertheless, they were able to derive *theoretical* results.