Voting cycles in a computational electoral competition model with endogenous interest groups

Vjollca Sadiraj*, Jan Tuinstra† and Frans van Winden‡

Abstract

We develop a computational electoral model by extending the benchmark model of spatial competition in two directions. First, political parties do not have complete information about voter preferences but behave adaptively and use polls to find policy platforms that maximize the probability of winning an election. Second, we allow voters to organize in different interest groups endogenously and depending upon the incumbent's policy platform. These interest groups transmit information about voter preferences to political parties and coordinate voting behavior. We use computational methods to investigate the convergence properties of this model. We find that the introduction of endogenous interest groups increases the separation between parties platforms, inhibits convergence to the center of the distribution of voter preferences, and increases the size of the winning set. Moreover, the presence of interest groups in an environment with adaptively searching political parties increases the likelihood of voting cycles, even when a dominant point exists. We also investigate the dynamics of this agent-based spatial model of electoral competition by looking at the mean-dynamics, i.e. by replacing stochastic variables by their expected values. The resulting Markov process shows that voting cycles exist. The mechanism driving these voting cycles may explain some empirical regularities found in the political science literature.

Keywords: Computational political economy, interest groups, spatial competition, polling, campaign contributions.

JEL classification code: D72; D83.

*Department of Economics, Andrew Young School of Policy Studies, Georgia State University, vsadiraj@gsu.edu.

†Department of Quantitative Economics and CeNDEF, University of Amsterdam, the Netherlands, J.Tuinstra@uva.nl. This research has been supported by the Netherlands Organisation for Scientific Research (NWO) under a MaGW-Pionier grant.

‡Department of Economics and CREED, University of Amsterdam, the Netherlands, F.A.A.M.vanWinden@uva.nl.

1 Introduction

Existing models of electoral competition typically make strong assumptions about the information political parties and voters use. In Downs-Hotelling spatial competition models, for example, the preferred policy of a voter is modeled as a point in an issue space and voters vote for the party whose policy platform is 'closest' to this ideal point. Each voter is assumed to be able to evaluate the consequences of all policy positions and each party is assumed to have complete information about the distribution of the voters' ideal points. These assumptions are not very realistic and in this paper we will take the informational constraints in politics explicitly into account, using a spatial competition model with two office motivated parties. Starting point is the observation that parties have to find out about voter preferences through some kind of polling. However, this search activity is costly. Voters may be willing to contribute in the form of effort or money because this allows them to affect the election outcome as well as policy platforms. Conditioning takes place by making contributions only available for polling in that part of the political issue space that the voter is mostly concerned about. For simplicity, we will have voters contribute to an 'interest group' which conditionally transfers the contributions to the parties. In line with recent literature contributions are assumed to be primarily driven by dissatisfaction with existing policies on issues of particular concern to the voter. Note that by getting politically involved in this way voters are likely to identify themselves with the policy stances they go for. In our model it is assumed, therefore, that some coordination of voting will occur. This coordination of voting may affect policies.

Our study is related to Kollman *et al.* (1992), which investigates the relevance of the theoretical "chaos" results for multi-dimensional issue spaces, which predict that, in general, the challenging party can always defeat the incumbent. They found convergence of the parties' platforms to the center of the distribution of voters' ideal positions. Sadiraj *et al.* (2006) presents extensive simulation studies of a spatial competition model with endogenous emergence of interest groups and shows that their presence increases separation between policy platforms and increases the probability of winning for the challenger. In this paper we provide a theoretical underpinning of these results by considering the *mean dynamics*, where we replace the stochastic elements of the model by their expected values and study the asymptotic properties of the resulting Markov model. It turns out that the steady state distribution of policy outcomes depends critically upon the way interest groups transmit information about the electoral landscape to the political parties. The model with interest groups may help explain some "stylized facts" concerning empirical data on policy outcomes.

The rest of the paper is organized as follows. Section 2 introduces the computational electoral competition model and the mean dynamics are introduced and studied in Section 3. Section 4 presents a general result on voting cycles and Section 5 concerns a replication of some stylized facts. Section 6 concludes.

2 The spatial competition model[1]

2.1 Incompletely informed political parties

Policy platforms are represented as points in a discrete two-dimensional issue spaces $\mathcal{X} = \{1, \ldots, K\} \times \{1, \ldots, K\}$. There is a population of N voters, where utility voter j attaches to policy outcome $y = (y_1, y_2)$ is given by

$$u_j(y) = -\sum_{i=1}^{2} s_{ji} (x_{ji} - y_i)^2, \tag{1}$$

with $x_j \in \mathcal{X}$ the voter's *ideal point* and s_{ji} the *strength* voter j attaches to issue i, where $s_j = (s_{j1}, s_{j2}) \in \mathcal{S} \times \mathcal{S}$ with $\mathcal{S} = \{s_0, s_1, \ldots, s_c\}$ and $0 \leq s_0 < \ldots < s_c \leq 1$. A configuration of voters is generated by drawing, for each j, an ideal position x_j from the discrete uniform distribution on $\mathcal{X}$ and strengths s_{ji} from a discrete distribution on $\mathcal{S}$. There are two political parties entering the election, the incumbent and the challenger. The incumbent does not change its policy position y from the previous period. Each voter votes for the political candidate yielding him the highest utility as given by (1). Then for each position z the height of the *electoral landscape*, $h(z \mid y)$, is given by the fraction of voters voting for the challenger, if it would select that position. For every z with $h(z \mid y) > (<) \frac{1}{2}$, the challenger wins (loses) the election. (If $h(z \mid y) = \frac{1}{2}$, the challenger wins with probability $\frac{1}{2}$). The objective for the challenger is to find maxima of the electoral landscape. Instead of assuming that political parties or candidates have complete information about the electoral landscape, we follow Kollman, Miller and Page (1992) in assuming that political parties have incomplete information about voter preferences and select policy platforms *adaptively* as follows. The challenger randomly draws a number of positions from the issue space and runs a poll there. Such a poll consists of, for example, a randomly drawn 10% of the voters. The challenger observes the fraction of this poll which favors his policy over the incumbents policy and uses this as an estimate of the true height of the electoral landscape at that position. If the best polling result indicates a height of at least $\frac{1}{2}$ then the challenger chooses that position. Otherwise it chooses the incumbent position, where it has probability $\frac{1}{2}$ of winning the election. If the true height of the landscape at the position selected by the challenger is above (below) $\frac{1}{2}$, the challenger (incumbent) wins the election. If the height is exactly $\frac{1}{2}$, each political party has a probability $\frac{1}{2}$ of winning the election. This procedure is then repeated for each election. Figure 1 shows some simulation results (taken from Sadiraj *et al.* 2006) for $K = 5$, $\mathcal{S} = \{0, 0.5, 1\}$, $N = 301$, 20 elections, 10 polls of 30 voters (10%) per election and 100 trials.

The solid lines in Figure 1 show the value of four different measures describing the outcomes of the model averaged over 100 trials. For each trial a new configuration of voters is drawn. The measure 'convergence' (upper left panel)

[1] The model, results and discussion in this section are taken from Sadiraj *et al.* (2006).

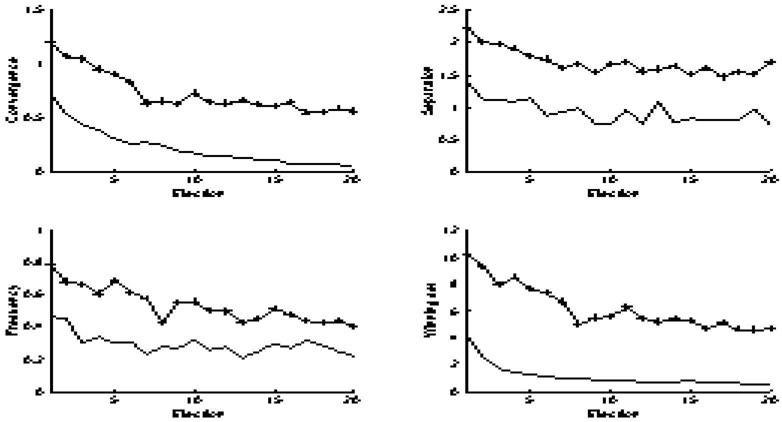

Figure 1: Time series of different measures, averaged over 100 trials, for the basic moden ($-$) and for the interest group model ($+$).

gives the Euclidean distance between the election outcome and the *center* of the distribution and decreases in the number of elections. The measure 'separation' (upper right panel) gives the Euclidean distance between the incumbent and challenger, which is also decreasing over time. The lower left panel shows the empirical frequency of election victories for the challenger and the lower right panel shows the size of the winning set (i.e. the number of positions that defeats the incumbent). These simulation results replicate the finding of Kollman *et al.* (1992) that policy platforms tend to converge to the center of the distribution of voter preferences.

2.2 Interest groups

We model interest groups as endogenously emerging institutions, arising from social interaction and dissatisfaction. Our approach differs from most of the literature which focuses on lobbying and campaign contributions and uses game theoretic models to describe the interaction between political parties and interest groups.

Interest groups emerge as follows. Voters with the same ideal position on one of the issues may decide to organize in interest groups in order to play a role in determining the election outcome. Now let n_k^i be the total number of voters having position $k \in \{1, \ldots, K\}$ on issue $i \in \{1, 2\}$. Clearly, $\sum_{k=1}^K n_k^1 = \sum_{k=1}^K n_k^2 = N$. Along each of the $2K$ "lines" in the issue space an interest group emerges. Prior to each election interest group formation takes place, where each voter determines whether he\she joins zero, one or two interest groups. After this process of interest group formation is over, total funds collected by the interest groups determines which interest groups become "active".

Joining an interest group provides a means to exert some political influence. Since this influence in interest group size, so is the willingness to join, which might even be reinforced by identification with the group. Furthermore, we assume voters are more inclined to join if the current policy position is farther away from their own position on that issue. On the other hand, there may be costs c of joining, which we assume to be exogenous and fixed. The process is now modeled as follows. Potential members are drawn in a random order and sequentially determine whether to join or not. This procedure is repeated once, so each voter decides whether to join or not one or two times. Let $m^i_{k,s-1}$ (with $m^i_{k,0} = 0$) be the size of the interest group at position k of issue i after $s-1$ voters have decided. The s'th voter then decides on the basis of a decision rule of the form $v^i_{js}(k) = V\left(\frac{m^i_{k,s-1}}{n^i_k}, k - y_i, c\right)$ which is increasing in s_{ji}, $k - y_i$ and $m^i_{k,s-1}/n^i_k$ and decreasing in c. This process leads to $2K$ different interest groups with typically different sizes. The total size of the interest groups decides which of them become active.

Interest groups try to influence the election process by coordinating voting behavior of their members and, conditionally, providing information about the electoral landscape to the candidates. Each active interest group finances some polls of the challenger conditional on: *i)* that the challenger runs these polls in policy positions coinciding with the interest group's position on the relevant issue; *ii)* that the challenger commits to select the platform with the highest poll result, provided this platform has a height of at least $\frac{1}{2}$. The interest group's members vote according to the interest group's advice, which is determined as follows. If one candidate is closer to the interest group's position than the other candidate, the former is supported. If the distance of the candidates from the interest groups positions on the relevant issue is the same,interest group members votes according to (1).

During an electoral campaign the challenger also runs some polls at randomly selected policy positons, next to those financed by the interest groups. It then chooses the position with the best polling result. All voters organized in interest groups vote according to the interest group's advice (if they belong to two interest groups they follow the interest group with the highest value of $v^i_{js}(\cdot)$, all other voters vote according to (1). The party with the majority of votes wins the election.

The simplifying assumptions about the symmetry and the uniform distribution of preferences, as well as the small number of issues and positions typically imply that the *generalized median voter* exists. The position of this median voter, once located, can not be defeated by any other platform, and will be reached with probability 1 because of the finiteness of the issue space. The model therefore predicts that, in the absence of interest groups, the incumbent converges to the median in the long run. One effect of interest groups is that typically they increase the winning set (see Sadiraj *et al.* 2005), since interest groups are more likely to form far away from the incumbent and hence tilt the electoral landscape at the expense of the incumbent. This leads to a higher probability for the challenger to win an election. Furthermore, if the location of the incumbent favors the organization of the median voters, conditional polling makes sure that the median is located much faster than in the benchmark model. On the other hand, if the distribution of voters allows for formation of interest groups asymmetric to the median and cycles in winning platforms may appear. Consider for example the case where, once the incumbent is at the median, two groups located on different issues and different from the median organize in interest groups. The policy position corresponding to their intersection may then in fact defeat the center, only due to the fact that interest groups coordinate voting behavior. Figure 1 confirms this intuition conveyed above. The winning set for the interest group model is larger, and consequently, so is separation between platforms and the probability that the challenger wins.

3 Mean dynamics

The dynamics of the computational model depends critically upon the random initial configuration of the population of voters. We now derive analytical results by replacing stochastic variables by their expected values and study the resulting stationary Markov process, which can be seen as the deterministic skeleton of the original process. These so-called *mean dynamics* can give us useful information about the stochastic electoral competition model.

3.1 Electoral competition as a Markov process

Consider again a population of N voters, with ideal positions x_j drawn from the uniform discrete distribution on $\mathcal{X} = \{1, 2, \ldots, K\} \times \{1, 2, \ldots, K\}$ (with K odd). The distribution of strengths $s_j = (s_{j1}, s_{j2}) \in \mathcal{S} \times \mathcal{S}$ satisfies $p_s = \Pr(S_{j1} = s_{j1}, S_{j2} = s_{j2}) = \Pr(S_{j1} = s_{j1}) \Pr(S_{j2} = s_{j2}) = p_{s_1} p_{s_2}$. Let $y^{t-1} \in \mathcal{X}$ be the winning platform of the election at time $t - 1$.

Definition 1 *Let* $\mathcal{R} = \{R : \exists i_1, i_2 \in \{1, \ldots, K\} \text{ s.t. } R^2 = i_1^2 + i_2^2\}$ *and* $\mathcal{U} = \{U(R), R \in \mathcal{R}\}$ *the family of subsets of* $\mathcal{X}$ *with* $U(R) = \{x \in \mathcal{X} : \|x - C\| = R\}$.

We use $\mathcal{U}$ as the new state space since, due to symmetry, all platforms that belong to the same element $U(R)$ can be treated equivalently. Notice that $\mathcal{U}$ has only $n = \sum_{k=1}^{\frac{1}{2}(K+1)} k << K^2 = |\mathcal{X}|$ elements and that each element of $\mathcal{U}$ contains 1, 4 or 8 elements of $\mathcal{X}$.

Proposition 2 *The family* $\mathcal{U}$ *satisfies the following properties.*

i) It forms a partition for the space $\mathcal{X}$.

ii) For all R *and* R' *and for all* $y^t, (y^t)' \in U_R$,

$$\Pr\left(y^{t+1} \in U_{R'} \mid y^t\right) = \Pr\left(y^{t+1} \in U_{R'} \mid \left(y^t\right)'\right).$$

The second property states that the probability of moving from any platform z in U_R to platforms in $U_{R'}$ is independent of the particular platform z. The electoral competition model corresponds to a Markov process with stationary transition probabilities on $\mathcal{U}$. Denote the n elements of $\mathcal{U}$ as $\{U_1, \ldots, U_n\} \equiv \{U(0), U(1), U(\sqrt{2}), \ldots, U(\sqrt{2}K)\}$. We can then, for given political institutions, voter preferences and interest group formation process, compute the $n \times n$ transition matrix P_r, where r indicates the number of polls. Element (i, j) of P_r gives the probability that, if the incumbent is in U_i, the election outcome will be in U_j.

Let the initial policy platform, y^0, follow some discrete distribution π_0 on $\mathcal{U}$. Then, at election t, the distribution of policy platforms over the different states U_i is given by $\pi_t = \pi_0 (P_r)^t$. We are, for every time period t, interested in two variables: the distance of the incumbent from the center of the issue space, which is given by $E(\|y^t - C\|) = \sum_{R \in \mathcal{R}} R\pi_t$ and the probability that the challenger wins the election, which is $\Pr(\text{the challenger wins at time } t) = \pi_t w$, where $w = (w_R)_{R \in \mathcal{R}}$ with $w_R = \Pr(\text{challenger wins} \mid \text{incumbent } \in U_R)$. An algorithm outlining how to compute the transition matrix P_r and the vector w, for the different models can be found in Sadiraj (2002).

In the next two subsections we will investigate the mean dynamics for the benchmark model and the interest group model for $K = 5$, and $\mathcal{S} = \{0, \frac{1}{2}, 1\}$. with $\Pr(s_{ji} = 0) = \Pr(s_{ji} = 1) = \frac{1}{4}$ and $\Pr(s_{ji} = \frac{1}{2}) = \frac{1}{2}$, for $j \in \{1, 2, \ldots, N\}$. The state space becomes $\mathcal{U} = \{U_R \mid R \in \{0, 1, \sqrt{2}, 2, \sqrt{5}, 2\sqrt{2}\}\}$ and we assume that the initial policy position is drawn from the uniform distribution on $\mathcal{X}$ which implies $\pi_0 = \left[\frac{1}{25}, \frac{4}{25}, \frac{4}{25}, \frac{4}{25}, \frac{8}{25}, \frac{4}{25}\right]$.

3.2 Dynamics for the benchmark model

For the benchmark model without interest groups and $r = 10$ random polls we obtain

$$P_{10} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0.400 & 0.600 & 0 & 0 & 0 & 0 \\ 0.400 & 0.543 & 0.057 & 0 & 0 & 0 \\ 0.250 & 0.495 & 0.253 & 0.002 & 0 & 0 \\ 0.152 & 0.533 & 0.308 & 0.006 & 0.001 & 0 \\ 0.090 & 0.407 & 0.422 & 0.001 & 0.080 & 0 \end{pmatrix}, w_{(10)} = \begin{pmatrix} 0.50000 \\ 0.70000 \\ 0.97174 \\ 0.99878 \\ 1.00000 \\ 1.00000 \end{pmatrix}$$

Let $P_r(i,j)$ be the element in the i-th row and j-th column of P_r. Then $P_r^n(i,i) = [P_r(i,i)]^n$, since P_r is a lower diagonal matrix. Hence, for all $i = 2, \ldots, 6$, $\sum_n P_r^n(i,i) < \infty$ as a geometric series with term $|P_r(i,i)| < 1$. Thus all states U_R, $R > 0$ are transient since transience (persistence) of a state j is equivalent to $\sum_n P^n(j,j) < \infty$ $(= \infty)$ Furthermore, $P_{11} = 1$ implies that $\{U_0\}$ is a closed set[2] and U_0 a persistent state. Thus, the stationary distribution

[2] A set B in S is *closed* if $\sum_{j \in B} P(i,j) = 1$ for $i \in B$: once the system enters B it cannot leave.

is $\pi^* = [1, 0, 0, 0, 0, 0]$, and in the long run: *(i)* the policy platform ends up in C and stays there forever; and *(ii)* the probability the challenger wins at an election converges to $0.5 (= \lim_{t\to\infty} \pi_t w = \pi^* w = w_0)$.[3]

3.3 Dynamics for the model with interest groups

Interest groups influence the election process by: *(i)* coordinating voting behavior; *(ii)* providing information about the electoral landscape to the candidates; and *(iii)* putting conditions on polling. In order to be able to disentangle the impact of the latter from the first two we present results of the model with interest groups for 'unconditional' and 'conditional' polling separately. For analytical tractability we assume that all voters organize in interest groups and all interest groups become active.

3.3.1 Unconditional polling

Our first research question is to investigate the effects of the new (if any) properties of the electoral landscape in the dynamics of the electoral outcomes. For this we assume that the challenger runs r random polls. It should be clear by now that this case is exactly the same as the benchmark model, corrected for the fact that strength profiles of interest group members change from s to $(1, 0)$ or $(0, 1)$. The transition matrix, P^I_{10} and the vector, $w^I_{10^u}$ of winning probabilities for the model with interest groups, turns out to be

$$P^I_{10^u} = \begin{pmatrix} 1.000 & 0 & 0 & 0 & 0 & 0 \\ 0.152 & 0.848 & 0 & 0 & 0 & 0 \\ 0.400 & 0.425 & 0.176 & 0 & 0 & 0 \\ 0.007 & 0.443 & 0.407 & 0.142 & 0 & 0 \\ 0.152 & 0.444 & 0.307 & 0.007 & 0.090 & 0 \\ 0.028 & 0.407 & 0.542 & 0.000 & 0.023 & 0 \end{pmatrix}, w^I_{(10^u)} = \begin{pmatrix} 0.50000 \\ 0.99878 \\ 0.99985 \\ 1.00000 \\ 1.00000 \\ 1.00000 \end{pmatrix}$$

As for the basic model, we find that there is one and only one closed set, the elements of which are all persistent states, which is $\{U_0\}$. All states $U \in \mathcal{U} \backslash U_0$ are transient. However, there is a difference in the speed with which the system convergence to the center as the following shows. Figure 2 gives, for the 3 different cases, diagrams with $E(\|y^t - C\|)$ and Pr (the challenger wins at time t), respectively. First consider the left panel of Figure 2. From the highest to the lowest curve we have: benchmark model with 2 random polls, interest group model with 10 random polls, benchmark model with 10 random polls. From this we find that an increase in the number of (unconditional) polls decreases the expected separation between the winning platform and the center of the distribution. Secondly, for the interest group model expected separation is larger than for the basic model with the same number of polls. For the right panel of Figure 2 the highest to the lowest curve (as measured at election 6)

[3] Recall that, if the challenger does not find a platform with $h(z \mid y) > 0.5$, it chooses the incumbent's platform y.

are respectively: the interest group model with 10 random polls, the basic model with 10 polls and the basic model with 2 polls. From this it follows that the presence of interest groups increases the probability of winning an election. One of the findings in Sadiraj *et al.* (2006) was that the presence of interest groups appears to increase the winning set. That result is confirmed here as well. Given the state of the incumbent, we find that the size of the winning set equals: *(a)* $\begin{pmatrix} 0 & 1 & 5 & 9 & 14 & 21 \end{pmatrix}'$ for the basic model, and *(b)* $\begin{pmatrix} 0 & 9 & 11 & 17 & 19 & 22 \end{pmatrix}'$ for the model with interest groups (recall that $|\mathcal{X}| = 25$). Figure 2 suggests that typically the size of the winning set increases in the presence of interest groups.[4]

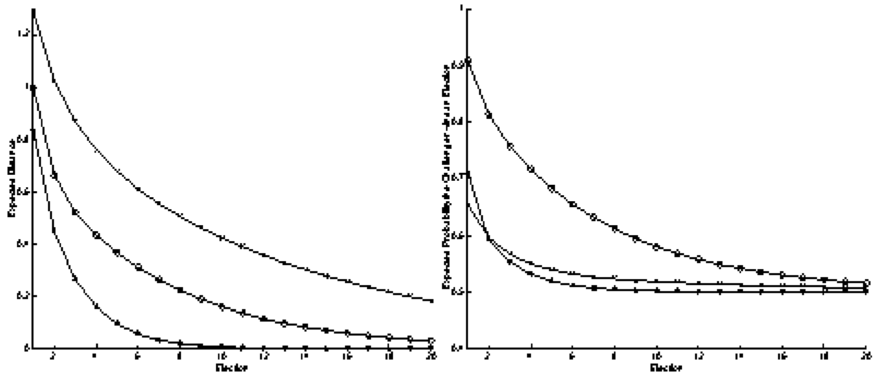

Figure 2: **Left panel**: Time series of the expected distance between the incumbent and the center of the space. The curve $-\times$ denotes the benchmark model with 2 polls, the curve $-\circ$ denotes the interest group model with 10 polls and $-*$ denotes the benchmark model with 10 polls. **Right panel**: Time series of the expected probabilities with which the challenger defeats the incumbent. The curve $-\times$ denotes the benchmark model with 2 polls, the curve $-\circ$ denotes the interest group model with 10 polls and $-*$ denotes the benchmark model with 10 polls.

3.3.2 Conditional polling

As mentioned above, the interest groups influence the election process by providing information about the electoral landscape to the political parties. The interest groups finance polls ran by the challenger conditional on: *i)* running a number of polls[5] in policy positions coinciding with the interest group's position

[4] The result is robust to changes in all parameter settings we have investigated. We have derived similar results for different distributions p on $\mathcal{S}$, and different number of positions per issue ($K \in \{3, \ldots, 11\}$).

[5] Remember that the number of polls that an interest group can finance is determined by the cost of running a poll and the size of the fund that the group possesses.

	5	×	×	.538	×	×
	4	×	×	.590	.538	.508
Issue 2	*3*	×	.500	.575	.500	×
	2	×	×	.590	.538	.508
	1	×	×	.538	×	×
		1	*2*	*3*	*4*	*5*
				Issue 1		

Table 1: Fractions of voters who prefer a position $z = (i, j)$ to $(2, 3)$ ($\times$ refers to fractions less than 0.5).

on the relevant issue; *ii)* commitment of the challenger to select the platform with the highest poll result, if this platform has a height of at least $\frac{1}{2}$. Furthermore, it is assumed that each interest group knows the median of the distribution of its group's members on the other issue and finances a poll there. Let r_1 be the number of random polls and r_2 the number of conditional polls. Let the challenger first run r_2 conditioned polls and then r_1 random polls. Removing from the policy space the positions where the conditioned polls are run one can compute the transition probabilities for the conditional polling procedure.

For the specified model and $r_2 = 8, r_1 = 2$, we find

$$P^I_{10^c} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0.882 & 0.118 & 0 & 0 & 0 \end{pmatrix}, w^I_{10^c} = \begin{pmatrix} 0.5 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

A new persistent state appears. In addition to state U_0 which remains a persistent state with the property '$\{U_0\}$ is a closed set', state U_1 becomes a persistent state as well with the property '$\{U_1\}$ is a closed set'. This can be derived as follows. The transition matrix shows that if the system at election t is in one of the states $U_R, R \in \{1, 2, \sqrt{5}\}$, then at election $t + 1$ it will be in U_1 and stay there forever. If the system starts at $U_{2\sqrt{2}}$ then, with probability 0.882, in the coming election it will end up in U_1 and never leave that state. The probability that the system will settle in U_1 is given by the first coordinate of $\pi_0 P^I_{10^c}$ and equals 0.781. In the same way one can derive that the system will settle in U_0 with probability 0.219. Furthermore, let the incumbent platform be $y = (2, 3) \in U_1$.[6] Table (1) shows the fraction of votes that the challenger gets if he selects a position $z = (i, j)$, $i, j = 1, \ldots, 5$, ($\times$ refers to fractions of votes smaller than 0.5). Thus, the winning set that corresponds to a position $y \in U_1$ has always at least two elements from U_1 with the highest fraction of votes. Let us now consider the interest group located at position 2 on the second

[6] It should be clear (for reasons of symmetry) that Table 1 for a $y \in U_1$ is the same as the one derived by rotating Table 1 around the center $(3, 3)$ until $(2, 3)$ reaches y.

issue. From the uniformity of the distribution of voters in the space and the homogeneity[7] of voters within types, it follows that the median of the members of this interest group related to the first issue is located at 3. Hence, that interest group will finance a poll at position $(3, 2)$. Note that the altitude at $(3, 2)$ is .59, which is the highest value in Table 1. Thus, the incumbent platform in the coming election will be either $(3, 2)$ or $(3, 4)$. This means that although the incumbent does not leave the U_1 set, a voting cycle appears. Therefore we may conclude that, with probability .781, *(i)* a cycle emerges and *(ii)* the challenger wins with probability 1.

4 Voting cycles driven by interest groups

The 'mean dynamic' analysis from the previous section shows that for the specified parameters of the models, there is only one closed set, $\{U_0\}$ in the benchmark model. However, under conditional polling, there are two closed sets, $\{U_0\}$ and $\{U_1\}$, for the model with interest groups. This raises the question of the dependence of this result on the parameter specification, like the size of the space, the set of strengths, the probability distribution of strengths on that set and so on. The following result provides an answer to that question (for a proof, see Sadiraj, 2002).

Proposition 3 *Assume the distribution of strengths satisfies*

$$\frac{(1-\frac{1}{K})\sum_{s\in\mathcal{S}\setminus\{0\}} p_s^2 + \left(\frac{3}{2}-\frac{1}{K}\right)\sum_{(s_1/s_2)=2} p_{s_1}p_{s_2}}{\sum_{s\in\mathcal{S}\setminus\{0\}} p_s} > \frac{1}{2}. \tag{2}$$

Then, for both models, with and without interest groups, $\{U_0\}$ is a closed set and U_0 is a persistent state. For the model without interest groups, all other states, $U \in \mathcal{U} \setminus U_0$ are transient and in the presence of interest groups and given conditional polling, $\{U_1\}$ is a closed set and U_1 is a persistent state.

It is straightforward to check that (2) holds for the models from the previous section. Hence, the results shown in Section 3 follow directly from Proposition 3. We conclude that voting cycles emerge, once the incumbent visits U_1.

5 Simulations and empirical illustration

The law of large numbers ensures us that the mean-analysis is relevant for populations that are large enough to correct for random deviations. However, the population of voters may not be large enough to cancel out random fluctuations, and therefore, the law of large numbers may not always apply. This may have consequences at the macro-level. That is why in this section we will consider

[7] Voters of some type s and with the same ideal positions on some issue i, make the same decisions to join the relevant interest group.

some simulations for different realizations of voter preferences and investigate whether the predictions of Proposition 3 are valid. Furthermore, we will compare these simulation results to some empirically observed policy outcomes.

Each trial starts with drawing a population of 1000 voters from the uniform distribution on $\mathcal{X}$, where we again assume $K = 5$. The initial position of the incumbent is chosen to be the center, in order to be able to investigate the closeness property of this center for the different models. Each trial was run for 20 elections and we have done 20 different trials. Typical results are represented in panels *(a)-(d)* of Figure 3. Panels *(a)* and *(b)* show that in the basic model the incumbent remains at the center for all elections. This is a robust feature of all trials with the basic model. Panels *(c)* and *(d)* of Figure 3 show that in the interest group model something different occurs: counter to the first statement in Proposition 3, the incumbent leaves the center and positions itself at some other position. This happens in more than half of all trials. From these figures it is apparent that for the basic model, the set that contains the center of the issue space, $\{C\}$, is a closed set even for the stochastic model. However, for the interest group model, the center loses that property for certain realizations of the distribution of voter preferences. For our issue space of 25 positions, simulations show that: for the basic model the property that $\{U_0\}$ is a closed set is maintained if the size of the population is larger than 300; for the model with interest groups, $\{U_0\}$ and $\{U_1\}$ are closed sets if the size of the population is larger than 10000. For populations with size smaller than 1000, neither $\{U_0\}$ nor $\{U_1\}$ are closed sets. Our next step is to relate these simulation results to some empirical data on policy outcomes. An analysis of the policy outcomes for 20 European countries was done in Woldendorp, Keman and Budge (1998). They classified the composition of the government as falling into one of 5 categories, ranging from extreme left (category 1) to extreme right (category 5). The graphs represented in panels *(e)* and *(f)* of Figure 3, respectively, correspond to Iceland data and Finland data, starting with the first time the composition of the government is in the center (position 3) after 1960. We draw attention to two features present in the data from both countries: *(i)* the government composition stays longer at position 3 than at the other positions, that is, the center presents a position which is hard to defeat; *(ii)* although the government composition locates at position 3 it does not stay there forever, that is, the center can be defeated. Comparing these graphs to the graphs generated by the simulations it is clear that the data generated by the interest group model represents the empirical data best. In our view, this provides some support for the model with interest groups presented in Section 2.

6 Concluding remarks

Although simulations provide a valuable aid in characterizing the behavior of the electoral competition model, their power is limited to the domain of the selected parameters. An understanding of the more generic properties of

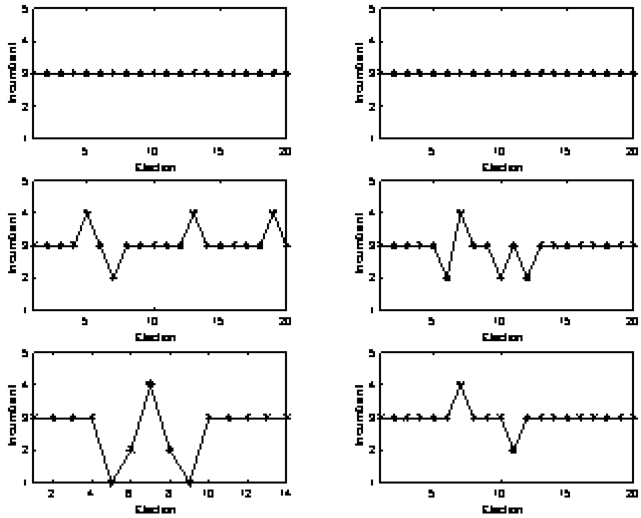

Figure 3: The stability of the center:simulation and empirical data. Panels (a) and (b) show data generated from the benchmark model in simulations 13 and 14, respectively. Panels (c) and (d) show data generated from the model with interest groups in simulations 13 and 14, respectively. Panels (e) and (f) show data generated from the composition of the governments in Finland and Iceland, respectively.

individual-based models requires the use of deterministic approximation models. In this paper we have applied a mean-field approximation to the stochastic models presented in Section 2, by replacing the values of the random variables by their expected values. This leads to deterministic dynamic models of the Markov–type. The main results obtained from the analysis of the deterministic models are as follows. The dynamics of the distance between the policy outcome and the center of the space, and of the probability that the challenger wins an election, replicate qualitatively the respective dynamics generated by the stochastic computational model. For both models, with and without interest groups, the set consisting of the center of the space presents a closed set. For a broad class of probability distributions on a set of strengths $\mathcal{S}$ and under conditional polling, it is shown that *(i)* the set of positions at distance 1 from the center is a closed set for the model with interest groups, and *(ii)* a voting cycle emerges. For the specified model the voting cycle appears with probability .781. To our knowledge, this is the first study pointing at, and providing a micro-foundation for, the possibility of a voting cycle in the presence of a dominant point. A further investigation shows, that if the size of the population is lower than some threshold (1000 for our specified model) voting cycles become frequent phenomena and expand all over the issue space. Our model positions itself in the series of models that point at the electoral instability of voting outcomes.

The inherent property driving our results is that the winning set (i.e., the set of policy platforms that will defeat the current incumbent) increases in the presence of interest groups. This happens in all the stochastic and numerical simulations. Moreover, in Sadiraj *et al.* (2005) it is rigorously shown that, in a slightly different spatial competition framework and under certain mild conditions on the incumbent's position, the winning set for the challenger indeed increases when interest groups are present to coordinate voting behavior.

References

[1] K. Kollman, J.H. Miller and S.E. Page (1992): Adaptive parties in spatial elections. *American Political Science Review* **86**, 929-937.

[2] V. Sadiraj (2002): *Essays in Political and Experimental Economics.* PhD Thesis, University of Amsterdam.

[3] V. Sadiraj, J. Tuinstra and F. van Winden (2005): On the size of the winning set in the presence of interest groups. *CeNDEF Working paper 05-08* University of Amsterdam.

[4] V. Sadiraj, J. Tuinstra and F. van Winden (2006): A computational electoral competition model with social clustering and endogenous interest groups as information brokers. *Public Choice*, **129**, 169-187.

[5] J. Woldendorp, H. Keman and I. Budge (1998): Party government in 20 democracies: an update (1990-1995). *European Journal of Political Research* **33**, 125-164.