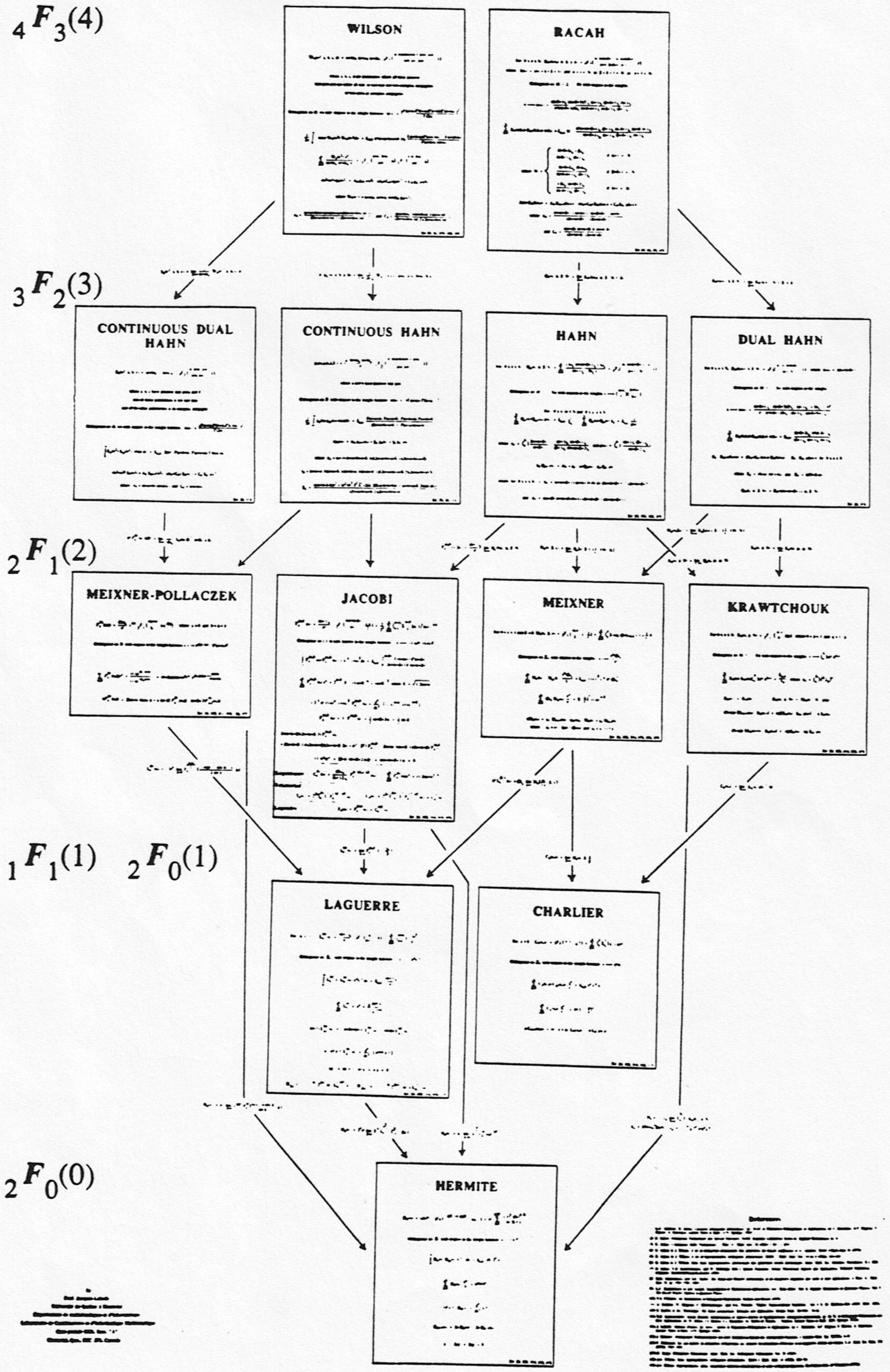
1990

J. Labelle

Askey's scheme
of
hypergeometric orthogonal polynomials



HERMITE

$$H_n(x) = (2x)^n \cdot {}_2F_0\left(\begin{matrix} -n/2,\ (1-n)/2 \\ \_\_\_\_ \end{matrix} ; -1/x^2 \right) = n! \sum_{k=0}^{[n/2]} \frac{(-1)^k (2x)^{n-2k}}{(n-2k)!\, k!}$$

Orthogonal on $\mathbb{R}$ with respect to the weight function $x \longmapsto e^{-x^2}$:

$$\int_{-\infty}^{\infty} H_n(x)\, H_m(x)\, e^{-x^2}\, dx = \delta_{n,m} \sqrt{\pi}\, 2^n\, n!$$

$$\sum_{n=0}^{\infty} H_n(x) \frac{u^n}{n!} = e^{2xu-u^2}$$

$$(-1)^n e^{-x^2} H_n(x) = \frac{d^n}{dx^n} e^{-x^2}$$

$$H_{n+1}(x) = 2x\, H_n(x) - 2n\, H_{n-1}(x)$$

$$y'' - 2xy' + 2ny = 0$$

See [6], [10], [11], [13], [17]

LAGUERRE

For $\alpha > -1$, $\quad L_n^{(\alpha)}(x) = \frac{(\alpha+1)_n}{n!} \cdot {}_1F_1\left({-n \atop \alpha+1}; x\right) = \sum_{k=0}^{n} \binom{n+\alpha}{n-k} \frac{(-x)^k}{k!}$

Orthogonal on $\mathbb{R}_+$ with respect to the weight function $\quad x \longmapsto x^{\alpha} e^{-x}$:

$$\int_0^{\infty} L_n^{(\alpha)}(x)\, L_m^{(\alpha)}(x)\, x^{\alpha} e^{-x}\, dx = \delta_{n,m} \frac{(\alpha+1)_n}{n!} \qquad \Gamma(n+\alpha+1)$$

$$\sum_{n=0}^{\infty} L_n^{(\alpha)}(x)\, u^n = \frac{e^{-xu/1-u}}{(1-u)^{1+\alpha}}$$

$$(n+1)\, L_{n+1}^{(\alpha)}(x) = (-x+2n+\alpha+1)\, L_n^{(\alpha)}(x) - (n+\alpha)\, L_{n-1}^{(\alpha)}(x)$$

$$n!\; x^{\alpha} e^{-x}\, L_n^{(\alpha)}(x) = \frac{d^n}{dx^n}\left\{ x^{n+\alpha} e^{-x} \right\}$$

$$xy'' + (\alpha + 1 - x)\, y' + n\, y = 0$$

$$H_{2m}(x) = (-1)^m\, 2^{2m}\, m!\, L_m^{(-1/2)}(x^2), \quad H_{2m+1}(x) = (-1)^m\, 2^{2m+1}\, m!\, x\, L_m^{(1/2)}(x^2)$$

See [6], [10], [11], [13], [17]

CHARLIER

For $a > 0$, $\quad C_n(x;a) = {}_2F_0\left({-n,-x \atop \text{---}} ; -a^{-1}\right) = \sum_{k=0}^{n} \binom{n}{k}\binom{x}{k} k! \, (-a)^{-k}$

Orthogonal on $\mathbb{Z}_+$ with respect to the weight function $x \longmapsto a^x/x!$:

$$\sum_{k=0}^{\infty} C_n(k;a)\, C_m(k;a)\, \frac{a^k}{k!} = \delta_{n,m}\, e^a\, a^{-n}\, n!$$

$$\sum_{n=0}^{\infty} C_n(x;a)\, \frac{u^n}{n!} = e^u \left(1 - \frac{u}{a}\right)^x$$

$$a\, C_{n+1}(x;a) = (n - x + a)\, C_n(x;a) - n\, C_{n-1}(x;a)$$

See [6], [10], [11], [13], [17]

KRAWTCHOUK

For $0 \le n \le N$, $\mathbf{K}_n(x; p, N) = {}_2F_1\left({-n,-x \atop -N}; 1/p\right)$ where $0 < p < 1$ and $q = 1 - p$

Orthogonal on $\{0, 1, \ldots, N\}$ with respect to the weights $x \longmapsto \binom{N}{x} p^x q^{N-x}$:

$$\sum_{x=0}^{N} \mathbf{K}_n(x)\,\mathbf{K}_m(x)\binom{N}{x} p^x q^{N-x} = \frac{\delta_{n,m}}{\pi_n} \quad \text{where } \pi_n = \binom{N}{n}\left({}^p/_q\right)^n$$

$$\mathbf{K}_n(x) = \mathbf{K}_x(n) \qquad\qquad \mathbf{K}_n(x; p, N) = \mathbf{M}_n(x; -N, -p/q)$$

$$(N-n)p\left(\mathbf{K}_{n+1}(x) - \mathbf{K}_n(x)\right) = nq\left(\mathbf{K}_n(x) - \mathbf{K}_{n-1}(x)\right) - x\,\mathbf{K}_n(x)$$

$$(N-x)p\left(\mathbf{K}_n(x+1) - \mathbf{K}_n(x)\right) = -np\mathbf{K}_n(x) - nq\,\mathbf{K}_{n-1}(x)$$

See [6], [10], [11], [13], [17]

MEIXNER

For $0 < c < 1$ and $\beta > 0$, $\mathbf{M}_n(x; \beta, c) = {}_2F_1\left({-n,-x \atop \beta}; 1 - \frac{1}{c}\right) = \sum_{k=0}^{n} \binom{n}{k} (-x)_k (\beta+k)_{n-k} (-1 + \frac{1}{c})^k$

Orthogonal on $\mathbb{Z}_+$ with respect to the weights $x \longmapsto \dfrac{c^x (\beta)_x}{x!}$:

$$\sum_{k=0}^{\infty} \mathbf{M}_n(k) \cdot \mathbf{M}_m(k) \frac{c^k (\beta)_k}{k!} = \delta_{n,m} (1-c)^{-\beta} c^{-n} n! (\beta)_n$$

$$\sum_{n=0}^{\infty} (\beta)_n \mathbf{M}_n(x) \frac{u^n}{n!} = \left(1 - \frac{u}{c}\right)^x (1-u)^{-x-\beta}$$

$$x\mathbf{M}_n(x) = a_n \mathbf{M}_{n+1}(x) - (a_n+c_n) \mathbf{M}_n(x) + c_n \mathbf{M}_{n-1}(x)$$

where $a_n = (1 - 1/c)^{-1} (\beta+n)$ and $c_n = (c-1)^{-1} n$

See [6], [10], [11], [13], [15], [17]

JACOBI

$$P_n^{(\alpha,\beta)}(x) = \frac{(\alpha+1)_n}{n!} \cdot {}_2F_1\left({-n,n+\alpha+\beta+1 \atop \alpha+1};\frac{1-x}{2}\right) = \frac{1}{2^n}\sum_{k=0}^{n}\binom{n+\alpha}{n-k}\binom{n+\beta}{k}(x-1)^k(x+1)^{n-k}$$

Orthogonal on [-1,1] with respect to the weight function $x \longmapsto (1-x)^\alpha\ (1+x)^\beta$:

$$\int_{-1}^{1} P_n^{(\alpha,\beta)}(x)\, P_m^{(\alpha,\beta)}(x)\, (1-x)^\alpha (1+x)^\beta dx = \delta_{n,m}\, \frac{2^{\alpha+\beta+1}\Gamma(n+\alpha+1)\Gamma(n+\beta+1)}{n!(2n+\alpha+\beta+1)\Gamma(n+\alpha+\beta+1)}$$

$$\sum_{n=0}^{\infty} P_n^{(\alpha,\beta)}(x)\, u^n = 2^{\alpha+\beta}\, R^{-1}\, (1-u+R)^{-\alpha}\, (1+u+R)^{-\beta} \quad \text{where } R = \sqrt{1-2xu+u^2}$$

$$(-2)^n\, n!\, (1-x)^\alpha\, (1+x)^\beta\, P_n^{(\alpha,\beta)}(x) = \frac{d^n}{dx^n}\left\{(1-x)^{n+\alpha}\, (1+x)^{n+\beta}\right\}$$

$$P_0^{(\alpha,\beta)}(x) = 1,\quad P_1^{(\alpha,\beta)}(x) = \frac{1}{2}(\alpha+\beta+2)x + \frac{1}{2}(\alpha-\beta)$$

$$2n(n+\alpha+\beta)(2n+\alpha+\beta-2)\, P_n^{(\alpha,\beta)}(x)$$

$$= (2n+\alpha+\beta-1)\{(2n+\alpha+\beta)(2n+\alpha+\beta-2)x + \alpha^2 - \beta^2\}\, P_{n-1}^{(\alpha,\beta)} - 2(n+\alpha-1)(n+\beta-1)(2n+\alpha+\beta)\, P_{n-2}^{(\alpha,\beta)}$$

$$(1-x^2)y'' + [\beta-\alpha-(\alpha+\beta+2)x]y' + n(n+\alpha+\beta+1)y = 0$$

Gegenbauer : $C_n^{(\lambda)}(x) = \frac{(2\lambda)_n}{(\lambda+1/2)_n} P_n^{(\lambda-1/2,\lambda-1/2)}(x)$ $\qquad \sum_{n=0}^{\infty} C_n^{(\lambda)}(x)u^n = (1-2xu+u^2)^{-\lambda}$

Tchebichef :

$T_n(x) = 2^{2n}\binom{2n}{n}^{-1} P_n^{(-1/2,-1/2)}(x)$ $\qquad U_n(x) = 2^{2n}\binom{2n+1}{n}^{-1} P_n^{(1/2,1/2)}(x) = C_n^{(1)}(x)$

Legendre : $P_n(x) = P_n^{(0,0)}(x) = C_n^{(1/2)}(x)$

See [6], [10], [11], [13], [17]

MEIXNER-POLLACZEK

$$P_n^{(a)}(x;\phi) = \frac{(2a)_n}{n!} e^{in\phi} \, {}_2F_1\left(\begin{matrix}-n, a+ix \\ 2a\end{matrix}; 1-e^{-2i\phi}\right)$$ where $a > 0$ and $0 < \phi < \pi$

Orthogonal on $\mathbb{R}$ with respect to the weight function $x \longmapsto e^{(2\phi-\pi)x} \cdot |\Gamma(a+ix)|^2$

$$\sum_{n\geq 0} P_n^{(a)}(x;\phi)u^n = \frac{(1-ue^{i\phi})^{-a+ix}}{(1-ue^{-i\phi})^{a+ix}} = (1+2u\cos\phi+u^2)^{-a} \, e^{2x \arctan\left(\frac{u\sin\phi}{1+u\cos\phi}\right)}$$

$$nP_n^{(a)}(x;\phi) = 2[(n+a-1)\cos\phi + x\sin\phi] \, P_{n-1}^{(a)}(x;\phi) - (n+2a-2)P_{n-2}^{(a)}(x;\phi)$$

See [6], [10], [11], [13], [16], [17]

DUAL HAHN

For $0 \le n \le N$, $\mathbf{R}_n(\lambda(x); \gamma, \delta, N) \; = \; {}_3F_2\left(\begin{matrix} -n,\ -x,\ x+\gamma+\delta+1 \\ \gamma+1,\ -N \end{matrix} ;\ 1 \right)$ where $\lambda(x) \; = \; x(x+\gamma+\delta+1)$

Orthogonal on $\{0, 1, \ldots, N\}$ with respect to the weights

$$x \longmapsto w(x) \; = \; \frac{(\gamma+\delta+1)_x \cdot (\gamma+\delta+3)/2)_x \cdot (\gamma+1)_x \cdot (-N)_x \cdot (-1)^x}{x! \cdot ((\gamma+\delta+1)/2)_x \cdot (\delta+1)_x \cdot (\gamma+\delta+N+1)_x}$$

$$\sum_{k=0}^{N} \mathbf{R}_n(\lambda(k))\, \mathbf{R}_m(\lambda(k))\, w(k) \; = \; \delta_{n,m}\, \frac{(\gamma+\delta+2)_N\, n!(\alpha-\delta+1)_n}{(\delta+1)_N\, (\alpha+1)_n\, (\gamma+1)_n}$$

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$$B_n \cdot \mathbf{R}_{n+1}(\lambda(x)) \; = \; (D_n+B_n-\lambda(x))\, \mathbf{R}_n(\lambda(x)) \; - \; D_n \cdot \mathbf{R}_{n-1}(\lambda(x)) \quad \text{for } 0 \le n \le N-1$$

where $B_n \; = \; (N-n)\,(\gamma+1+n)$ and $D_n \; = \; n(N+\delta-n+1)$

$$\mathbf{Q}_n(k; \alpha, \beta, N) \; = \; \mathbf{R}_k(n(n+\alpha+\beta+1); \alpha, \beta, N)$$

See [6], [13]

HAHN

For $0 \le n \le N$, $\quad Q_n(x;\alpha,\beta,N) = \sum_{k=0}^{n} \frac{(-n)_k\,(n+\alpha+\beta+1)_k\,(-x)_k}{(\alpha+1)_k\,(-N)_k\,k!} = {}_3F_2\left(\begin{matrix}-n,\ n+\alpha+\beta+1,\ -x\\ \alpha+1,\ -N\end{matrix};1\right)$

Orthogonal on $\{0,1,\ldots,N\}$ with respect to the weights $x \longmapsto \binom{\alpha+x}{x}\binom{\beta+N-x}{N-x}$

For $0 \le n, m \le N$ and $0 \le x, y \le N$,

$$\sum_{x=0}^{N} Q_n(x)\,Q_m(x)\,\rho(x) = \delta_{n,m}\,\frac{1}{\pi_n} \qquad \sum_{n=0}^{N} Q_n(x)\,Q_n(y)\,\pi_n = \delta_{x,y}\,\frac{1}{\rho(x)}$$

where $\pi_n = \binom{N}{n}\frac{2n+\alpha+\beta+1}{\alpha+\beta+1}\;\frac{(\alpha+1)_n\,(\alpha+\beta+1)_n}{(\beta+1)_n\,(N+\alpha+\beta+2)_n}$ and $\rho(x) = \binom{N}{x}\frac{(\alpha+1)_x\,(\beta+1)_{N-x}}{(\alpha+\beta+2)_N}$

$$b_n\,Q_{n+1}(x) = (b_n + d_n - x)\,Q_n(x) - d_n\,Q_{n-1}(x)$$

where, for $0 \le n \le N-1$, $b_n = (n+\alpha+\beta+1)\,(n+\alpha+1)\,(N-n)\,(2n+\alpha+\beta+1)^{-1}\,(2n+\alpha+\beta+2)^{-1}$

and $d_n = n(n+\beta)\,(n+\alpha+\beta+N+1)\,(2n+\alpha+\beta)^{-1}\,(2n+\alpha+\beta+1)^{-1}$

See [6], [10], [12], [13]

CONTINUOUS HAHN

$$p_n(x;a,b,\bar{a},\bar{b}) = i^n \frac{(a+\bar{a})_n (a+\bar{b})_n}{n!} \; {}_3F_2 \left(\begin{matrix} -n,\ n+a+b+\bar{a}+\bar{b}-1,\ a+ix \\ a+\bar{a},\ a+\bar{b} \end{matrix} ; 1 \right)$$

where a and b have positive real part.

Orthogonal on $\mathbb{R}$ with respect to the weight function $w(x) = |\Gamma(a+ix)\,\Gamma(b+ix)|^2$:

$$\frac{1}{2\pi} \int_{\infty}^{\infty} p_n(x)\, p_m(x)\, w(x)dx = \delta_{n,m} \frac{\Gamma(n+a+\bar{a})\,\Gamma(n+a+\bar{b})\,\Gamma(n+b+\bar{a})\,\Gamma(n+b+\bar{b})}{(2n+a+b+\bar{a}+\bar{b}-1)\,\Gamma(n+a+b+\bar{a}+\bar{b}-1)}$$

$$xp_n(x) = \alpha_n\, p_{n+1}(x) + \beta_n\, p_n(x) + \gamma_n\, p_{n-1}(x)$$

where $\alpha_n = (n+1)\,(n+a+b+\bar{a}+\bar{b}-1)/(2n+a+b+\bar{a}+\bar{b}-1)(2n+a+b+\bar{a}+\bar{b})$

$$\gamma_n = (n+a+\bar{a}-1)(n+a+\bar{b}-1)(n+b+\bar{a}-1)(n+b+\bar{b}-1)/(2n+a+b+\bar{a}+\bar{b}-2)(2n+a+b+\bar{a}+\bar{b}-1)$$

$$\beta_n = \frac{i\{(a+b-\bar{a}-\bar{b})n^2 + (a^2+b^2-\bar{a}^2-\bar{b}^2+2ab-2\bar{a}\bar{b}-a-b+\bar{a}+\bar{b})n + (a+b+\bar{a}+\bar{b}-2)(ab-\bar{a}\bar{b})\}}{(2n+a+b+\bar{a}+\bar{b}-2)(2n+a+b+\bar{a}+\bar{b})}$$

See [3], [8], [13]

CONTINUOUS DUAL HAHN

$$S_n(x^2; a, b, c) = (a+b)_n \cdot (a+c)_n \cdot {}_3F_2\left(\begin{matrix} -n, a+ix, a-ix \\ a+b, a+c \end{matrix}; 1\right)$$

where a, b, c have positive real parts and if

one of these parameters is not real then

one of the other parameters is its complex conjugate.

Orthogonal on $(0, \infty)$ with respect to the weight function $w(x) = \left|\frac{\Gamma(a+ix)\Gamma(b+ix)\Gamma(c+ix)}{\Gamma(2ix)}\right|^2$

$$\int_0^\infty S_n(x^2)\, S_m(x^2)\; w(x)\, dx = \delta_{n,m}\; 2\pi n!\; \Gamma(a+b+n)\, \Gamma(a+c+n)\, \Gamma(b+c+n)$$

$$-(x^2+a^2)\, S_n(x^2) = A_n\, S_{n+1}(x^2) - (A_n+C_n)\, S_n(x^2) + C_n\, S_{n-1}(x^2)$$

where $A_n = (n+a+b)\,(n+a+c)$ and $C_n = n(n+b+c-1)$

See [6], [13]

RACAH

For $0 \leq n \leq N$, $\mathbf{R}_n(\lambda(x); \alpha, \beta, \gamma, \delta) = {}_4F_3\left(\begin{matrix} -n,\ n+\alpha+\beta+1,\ -x,\ x+\gamma+\delta+1 \\ \alpha+1,\ \beta+\delta+1,\ \gamma+1 \end{matrix} ; 1\right)$

where $\lambda(x) = x(x+\gamma+\delta+1)$ and $\alpha+1=-N$ or $\beta+\delta+1=-N$ or $\gamma+1=-N$

Orthogonal on $\{0, 1, 2, \ldots, N\}$ with respect to the weights

$$x \longmapsto w(x) = \frac{(\gamma+\delta+1)_x\,((\gamma+\delta+3)/2)_x\,(\alpha+1)_x\,(\beta+\delta+1)_x\,(\gamma+1)_x}{x!((\gamma+\delta+1)/2)_x\,(\gamma+\delta-\alpha+1)_x\,(\gamma-\beta+1)_x\,(\delta+1)_x}$$

$$\sum_{k=0}^{N} \mathbf{R}_n(\lambda(k))\,\mathbf{R}_m(\lambda(k))\,w(k) = \delta_{m,n}\,M \cdot \frac{n!(n+\alpha+\beta+1)_n\,(\beta+1)_n\,(\alpha-\delta+1)_n\,(\alpha+\beta-\gamma+1)_n}{(\alpha+\beta+2)_{2n}\,(\alpha+1)_n\,(\beta+\delta+1)_n\,(\gamma+1)_n}$$

$$\text{where } M = \begin{cases} \dfrac{(\gamma+\delta+2)_N\,(-\beta)_N}{(\gamma-\beta+1)_N\,(\delta+1)_N} & \text{if } \alpha+1 = -N, \\[2ex] \dfrac{(\gamma+\delta+2)_N\,(\delta-\alpha)_N}{(\gamma+\delta-\alpha+1)_N\,(\delta+1)_N} & \text{if } \beta+\delta+1 = -N, \\[2ex] \dfrac{(-\delta)_N\,(\alpha+\beta+2)_N}{(\alpha-\delta+1)_N\,(\beta+1)_N} & \text{if } \gamma+1 = -N. \end{cases}$$

$$\lambda(x)\,\mathbf{R}_n(\lambda(x)) = A_n\,\mathbf{R}_{n+1}(\lambda(x)) - (A_n+C_n)\,\mathbf{R}_n(\lambda(x)) + C_n\,\mathbf{R}_{n-1}(\lambda(x))$$

$$\text{where } A_n = \frac{(n+\alpha+\beta+1)\,(n+\alpha+1)\,(n+\beta+\delta+1)\,(n+\gamma+1)}{(2n+\alpha+\beta+1)\,(2n+\alpha+\beta+2)}$$

$$\text{and } C_n = \frac{n(n+\beta)\,(n+\alpha+\beta-\gamma)\,(n+\alpha-\delta)}{(2n+\alpha+\beta)\,(2n+\alpha+\beta+1)}$$

See [4], [6], [9], [13]

WILSON

$$W_n(x^2; a, b, c, d) = (a+b)_n (a+c)_n (a+d)_n \; {}_4F_3\left(\begin{matrix} -n, n+a+b+c+d-1, a+ix, a-ix \\ a+b, a+c, a+d \end{matrix}; 1\right)$$

where a, b, c, d are parameters which all have positive real part and are either all real, or two real and two complex conjugates, or two pairs of complex conjugates.

Orthogonal on $(0, \infty)$ with respect to the weight function $w(x) = \left| \frac{\Gamma(a+ix)\Gamma(b+ix)\Gamma(c+ix)\Gamma(d+ix)}{\Gamma(2ix)} \right|^2$

$$\frac{1}{2\pi} \int_0^\infty w(x)\, W_n(x^2)\, W_m(x^2)\,dx = \delta_{n,m}\, n!(n+a+b+c+d-1)_n \frac{\Gamma(a+b+n)\Gamma(a+c+n)\ldots\Gamma(c+d+n)}{\Gamma(a+b+c+d+n)}$$

$$\sum_{n=0}^{\infty} \frac{W_n(x^2)\, u^n}{(a+b)_n (c+d)_n\, n!} = {}_2F_1\left(\begin{matrix} a+ix, b+ix \\ a+b \end{matrix}; u\right) {}_2F_1\left(\begin{matrix} c-ix, d-ix \\ c+d \end{matrix}; u\right)$$

$$-(x^2+a^2)\, p_n(x^2) = A_n\, p_{n+1}(x^2) - (A_n+C_n)\, p_n(x^2) + C_n\, p_{n-1}(x^2)$$

where $W_n(x^2) = (a+b)_n (a+c)_n (a+d)_n\, p_n(x^2)$,

$$A_n = \frac{(n+a+b)(n+a+c)(n+a+d)(n+a+b+c+d-1)}{(2n+a+b+c+d-1)\,(2n+a+b+c+d)} \quad \text{and} \quad C_n = \frac{n(n+b+c-1)(n+b+d-1)(n+c+d-1)}{(2n+a+b+c+d-2)(2n+a+b+c+d-1)}$$

See [5], [6], [13], [18], [19]

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