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The Electronic News Net of the SIAM Activity Group on Orthogonal Polynomials and Special Functions

http://math.nist.gov/opsf

OP-SF Net is distributed to OPSF Activity Group members and non-members alike through the OP-SF Talk listserv.

If you are interested in subscribing to the Newsletter and/or OP-SF Talk, or if you would like to submit a topic to the Newsletter or a contribution to OP-SF Talk, please send an email to the OP-SF Net Editors.

Editors:

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Calendar of Events:

April 21-23, 2022

International Conference on Orthogonal Polynomials, Celebrating Francisco Marcellán's 70th birthday

Cádiz, Spain

https://www.marcellanfest.es/

May 23-27, 2022

Baylor Analysis Fest: From Operator Theory to Orthogonal Polynomials, Combinatorics, and Number Theory Baylor University, Waco, TX, USA https://tinyurl.com/BAFconference

May - November, 2022

Symmetries: Algebras and Physics

Thematic Semester, includes the following workshops:

May 23-June 10, 2022 Non-commutative algebras, representation theory and special functions July 25-August 19, 2022

Graph theory, algebraic combinatorics and mathematical physics September 12-October 7, 2022

Integrable systems, exactly solvable models and algebras Centre de Recherches Mathématiques, Montréal, Quebec, Canada http://www.crm.umontreal.ca/2022/Symmetries22/index_e.php

June 13-17, 2022—now virtual due to coronavirus pandemic. OPSFA-16 Centre de Recherches Mathématiques, Montréal, Quebec, Canada http://www.crm.umontreal.ca/2022/OPSFA22/index_e.php

July 5-8, 2022—new dates due to coronavirus pandemic.

Functional Analysis, Approximation Theory and Numerical Analysis (FAATNA) Matera, Italy http://web.unibas.it/faatna20/

August 8-12, 2022

OPSF-S9: Radboud OPSFA Summer School Nijmegen, The Netherlands https://www.ru.nl/radboudsummerschool/courses/2022/opsfa-summer-school/

Topic #1 _____ OP – SF Net 29.2 _____ March 15, 2022

From: Josée Savard josee.savard@umontreal.ca and Luc Vinet (luc.vinet@umontreal.ca) Subject: Announcement: OPSFA-16 to be held online June 13-17, 2022



The 16th International Symposium on Orthogonal polynomials, Special Functions and Applications (OPSFA16), organized by the Centre de Recherches Mathématiques (CRM) will be online from June 13-17, 2022. This conference will be dedicated to the memory of Richard Askey, who died at the age of 86 on October 9, 2019. A special day will be held to remember his legacy.

The registration period is now open until June 17th . The cost of registration is 20\$ CAN. To register, navigate to the following link:

https://www.crm.umontreal.ca/act/form/inscr_opsfa22_e.shtml.

The event will host several internationally renowned plenary speakers as well as minisymposia. The themes of the minisymposia will be posted online shortly as well as the preliminary schedule.

Two contributed talks proposals are currently open. The first is for calls open to all and a second specific to a minisymposium organized for doctoral students and postdoctoral fellows. The deadline to submit your proposal is May 1st.

The activity group also awards the Gábor Szegő Prize every two years to an early-career researcher for outstanding research contributions in the area of orthogonal polynomials and special functions. The Gabor Szegő prize 2021 is awarded to Dr. Atul Dixit for his impressive scientific work solving problems related to number theory using special functions, in particular related to the work of Ramanujan.

Finally, take note that OPSFA16 is part of a thematic semester, Symmetries: Algebras and Physics, organized by the CRM from May to December 2022. It will consist of six 1-month concentration periods devoted to ongoing research in cutting edge topics; each will involve 1- or 2-week work-shops that will be preceded by several preparatory mini-courses.

You can find all the details on this website.

We hope to see many of you! The organizing committee OPSFA16

Topic #2 _____ OP - SF Net 29.2 _____ March 15, 2022

From: Paul Martin (pamartin@mines.edu) Subject: **Paul A. Martin**: One Hundred Years of **Watson**'s treatise on Bessel Functions

One hundred years of Watson's "Bessel Functions"¹ Paul A. Martin (pamartin@mines.edu) Department of Applied Mathematics and Statistics, Colorado School of Mines, Golden, Colorado, USA

George Neville Watson (1886–1965) was born in Westward Ho!, a seaside village in the west of England. He became a student at Trinity College, Cambridge, in 1904; Fellows there included E. T. Whittaker, G. H. Hardy and E. W. Barnes. Watson stayed in Cambridge for ten years. Then, after four years at University College London (UCL), he moved to the University of Birmingham, where he spent the rest of his career, from 1918 until he retired in 1951 as the Mason Professor of Pure Mathematics. The photograph of Watson in Figure 1 is reproduced in several places, including [7, 8, 14].

Watson is perhaps best known as one half of "Whittaker and Watson"; they co-authored what became a standard book on mathematical analysis and special functions [13]. Whit-taker wrote the first edition alone in 1902 (378 pages) but Watson contributed much to later editions, including new material and much more rigour [9, p. 553], [14, p. 524]: the fourth edition [13] has 608 pages. After he retired, Watson envisaged a much expanded

¹This is a slightly edited version of an article published in the *London Mathematical Society Newsletter* in March 2022.



Figure 1: G. N. Watson, with winged collar

edition, but he did not complete it [9, p. 553], [11, p. 256]; after his death, numerous manuscript pages were deposited in the archives of the University of Birmingham.

Watson is well known to applied mathematicians for several contributions:

- The Watson transform, which is a method for replacing a slowly convergent infinite series by another that converges rapidly. He used this method in two papers on the propagation of electromagnetic waves around the Earth, including calculations showing that the postulated Heaviside layer was consistent with experimental results [14, p. 522].
- Watson's lemma, which gives the asymptotic expansion of Laplace-type integrals. He proved this useful result in the context of his study of parabolic cylinder functions [9, p. 554].
- Watson integrals, which are certain triple integrals of trigonometric functions [14, p. 527], [15].

These can be seen as examples of Watson's skill as a "problem solver": he was "most ready to help a colleague in difficulties and was willing to go to great trouble over problems of applied mathematics in which he was not basically interested" [14, p. 527].

However, it is Watson's *Treatise on the Theory of Bessel Functions* (WBF) that stands out as a singular scholarly achievement (Figure 2).

WBF was first published in 1922, with a second edition in 1944 [12]. From the Preface:

This book has been designed with two objects in view. The first is the development of applications of the fundamental processes of the theory of functions of complex variables. ...The second object is the compilation of a collection of results which would be of value to the increasing number of Mathematicians and Physicists who encounter Bessel functions in the course of their researches.

Both objects were achieved: WBF is still in print, and it has been cited about 20,000 times. (Some reviews of both editions have been collected in [8].) Nowadays, one might consult the *NIST Digital Library of Mathematical Functions* for a quick summary of relevant formulae, but WBF remains as the prime resource for serious study of Bessel functions and their many relatives.

Watson is clear that he is aiming for a complete survey of everything known about Bessel functions: there is an extensive bibliography. (For more on the history of Bessel functions, see [6].) However, in many cases, he supplies new proofs of known results. He fixes definitions and notations. For example, he uses $J_{\nu}(z)$ and $Y_{\nu}(z)$ for Bessel functions of the first and second kinds, where z and ν are arbitrary complex variables. "The book contains not only formulae and theoretical investigations, but also extensive tables, some of which Watson had himself calculated. Throughout his life he found relaxation in numerical work, for which he used a Brunsviga calculating machine" [9, p. 555]. That may be true, but we also know that Watson declined a request to do ballistics calculations in Karl Pearson's laboratory at UCL during World War I; see the letter from A. V. Hill to Pearson quoted by Barrow–Green [4, p. 97], where some indication of Watson's character in 1917 may be gleaned: "I have known Watson these last 12 years [Hill and Watson were exact contemporaries at Trinity College], and am interested (and distressed) to hear he is exactly like he used to be. I didn't know that the War, and coming to London, had not made him more humble and human, but from what you say it obviously has not."



Figure 2: Blue plaque at the University of Birmingham

After publication of the first edition of WBF in 1922, Watson completed his work on the fourth edition of Whittaker and Watson in 1927 [13]. The following decade "might be described as Watson's 'Ramanujan period'" [14, p. 525]. "He had received from G. H. Hardy copies of Ramanujan's famous note books" [9, p. 555] and he then endeavoured to supply proofs: this generated about 30 papers. (For a complete list of Watson's papers, up to 1962, see [9]; he also coauthored a long paper 20 years after his death [2].) According to B. C. Berndt, "Watson invested at least ten years to the editing of Ramanujan's notebooks. He never completed his task, but fortunately his efforts have been preserved" and many proofs in Berndt's book are due to Watson [5, p. vi]. Rankin [10] has given

a detailed description of Watson's involvement with Ramanujan's notebooks.

A second edition of WBF was published in 1944: why? Rankin notes that, during World War II, WBF "was in great demand in government scientific establishments, both in [the UK] and abroad. It became difficult to acquire and unofficial copies were circulated in some quarters. It was no doubt largely for this reason that a second edition appeared in 1944" [9, p. 555]. R. A. Askey offers an American perspective, stating that he had "been told that when the work on the first successful atomic pile was being done at the University of Chicago, a copy of [WBF] was chained to a table and always open" [3]. Bessel functions arise in various problems of applied mathematics, and many of these had to be solved quickly. However, in the preface to the second edition, Watson famously admits that his "interest in Bessel functions has waned since 1922", and so his revisions are confined to the "correction of minor errors and misprints and to the emendation of a few assertions". He does not even cite his own occasional papers from the 1930s on Bessel functions.

In his lectures at the University of Manchester on asymptotic methods, Fritz Ursell [1] liked to remark that he knew of only two light-hearted remarks in WBF [12]. One concerns the Stokes phenomenon and its discovery: on p. 202, Watson notes that "it was apparently one of those which are made at three o'clock in the morning". Ursell did not identify the second remark, but the author is fond of this one on p. 523: "The construction of the Neumann series when the Maclaurin series is given is consequently now merely a matter of analytical ingenuity".

Concerning Watson the man, there are some evocative remarks in [9] and [11]; see also [8]. For example, in "manner and appearance (he always wore a wing collar) he recalled the professors of an earlier generation" (see Figure 1) and he "did not like telephones and regarded them as 'an invention of the devil'" [11, p. 256]. He had similar strong aversions to motor-cars and fountain pens [8]. Although he "had a great admiration for his friend and co-author, Sir Edmund Whittaker [who was Professor in Edinburgh], he only

visited Scotland twice, once in June 1939 to receive his Honorary LL. D. from Edinburgh University, and in July 1914 to attend the Napier Tercentenary Congress. He used to say that he feared to make a third visit ...as each of his two previous visits had precipitated a major European catastrophe" [9, p. 552]. Evidently, Watson had a sense of humour!

N. Wahan

Figure 3: George Neville Watson's signature

Acknowledgements. I thank Tony Rawlins for permission to use his picture of the blue plaque in Figure 2 and June Barrow-Green for pointing out the letter about Watson in [4]. I thank them, David Abrahams and Stefan Llewellyn Smith for comments and encouragement.

Bibliography.

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Topic #3 _____ OP – SF Net 29.2 _____ March 15, 2022

From: Tom Koornwinder (thkmath@xs4all.nl) Subject: A U.S.S.R. travel diary of Liz and Dick Askey, September 1987

On February 1, Howard Cohl posted the sad message that Liz Askey, the widow of Dick Askey, had passed away on January 29, 2022, at the age of 85. One can read about her life in the Liz Askey Obituary, which was mostly written by her daughter Suzanne. She also took the accompanying photo of Liz, in May 2007.



Many of us who have been in touch with Dick Askey, will also have met Liz, since she often accompanied Dick during his travels and conference visits. I have met Liz for the first time in the academic year 1979–1980 during Dick's sabbatical in Amsterdam. She was a cordial woman, made funny observations, and was very interested in art and handicraft. She was specialized in children's literature.

Liz was also a good writer. She wrote travel diaries of the longer mathematical trips she made with Dick. During September 1987 – January 1988 they made a world trip to the U.S.S.R., Japan, Australia and India. I own a copy of a typewritten diary written by Liz. I will make scans of these, apply OCR to them, and make the corrected OCR output available on the web. I have done this already for the first part of the diary, about the trip to the U.S.S.R. in September 1987. See https://staff.fnwi.uva.nl/t.h.koornwinder/specfun/AskeyDiary_USSR1987.pdf.

This was a three-week trip based in Moscow, but with rad. Of course, Liz does not write about mathematics, but there

is much about the musueums visited, the walks made, the impression she got of the people and the communist system, the (sometimes delicious but often mean) quality of the food and the bad service, and the Academy Hotel in Moscow where they were terrorized by bedbugs. Some names of well-known mathematicians pass: Gelfand, Zelevinsky and Levitan. They brought a hearing aid for Gelfand, which caused problems at customs. On their first day in Moscow:

"Our guide found us a taxi and took us to the apartment building in which Gelfand and his family lived. We were only going to stay fifteen minutes since Gelfand's wife was ill. He was delighted to see Dick and the conversation went on much longer than 15 minutes. He was interrupted constantly to answer the telephone and we eventually learned that it was his 74th birthday."

About the bed bugs: "Dick was up at 2:30 a.m. with six new bites and he wasn't about to crawl back in there with the beasties who were making a meal out of him. Can't blame him. However he just left to give his presentation at the Gelfand seminar and that's a strenuous evening ahead for him with only three hours of sleep last night. Since talking mathematics is what he's here for, that may be enough to keep him going."

Is anyone reading this able to give a testimony about how Dick performed at the Gelfand seminar?

How could one have predicted in those days that visiting Moscow then during the cold war was easier than right now.

Topic #4 _____ OP – SF Net 29.2 _____ March 15, 2022

From: OP-SF Net Editors Subject: Preprints in arXiv.org

The following preprints related to the fields of orthogonal polynomials and special functions were posted or cross-listed to one of the subcategories of arXiv.org during January and February 2022. This list has been separated into two categories.

OP-SF Net Subscriber E-Prints

http://arxiv.org/abs/2201.00215

On the Parity of the Generalized Frobenius Partition Functions $\varphi_k(n)$ George E. Andrews, James A. Sellers, Fares Soufan

http://arxiv.org/abs/2201.00474

Asymptotics of *k*-nearest neighbor Riesz energies Douglas P. Hardin, Edward B. Saff, Oleksandr Vlasiuk

http://arxiv.org/abs/2201.01352

Turán inequalities for the plane partition function Ken Ono, Sudhir Pujahari, Larry Rolen

http://arxiv.org/abs/2201.01451

On the tau function of the hypergeometric equation Marco Bertola, Dmitry Korotkin

http://arxiv.org/abs/2201.02267

Schwarzian derivative, Painlevé XXV-Ermakov equation and Bäcklund transformations Sandra Carillo, Alexander Chichurin, Galina Filipuk, Federico Zullo

http://arxiv.org/abs/2201.02337

Classical and quantum walks on paths associated with exceptional Krawtchouk polynomials Hiroshi Miki, Satoshi Tsujimoto, Luc Vinet

http://arxiv.org/abs/2201.02399

An extension of an asymptotic result of Tricomi concerning a definite integral R. B. Paris

Weighted cylindric partitions Walter Bridges, Ali Uncu

http://arxiv.org/abs/2201.03086

An elegant Multi-Integral that implies an even more elegant determinant identity of Dougherty and McCammond Tewodros Amdeberhan, Doron Zeilberger

http://arxiv.org/abs/2201.03646

An explanation of the commuting operator "miracle" in time and band limiting Pierre-Antoine Bernard, Nicolas Crampé, Luc Vinet

http://arxiv.org/abs/2201.04589

Time and band limiting operator and Bethe ansatz Pierre-Antoine Bernard, Nicolas Crampé, Luc Vinet

http://arxiv.org/abs/2201.05422

Rational spectral transformation of continued fractions associated to a perturbed R_I type recurrence relations Vinay Shukla, A. Swaminathan

http://arxiv.org/abs/2201.05948

Strict domain monotonicity of the principal eigenvalue and a characterization of lower boundedness for the Friedrichs extension of four-coefficient Sturm-Liouville operators Fritz Gesztesy, Roger Nichols

http://arxiv.org/abs/2201.06630

Distributions of Hook lengths in integer partitions Michael Griffin, Ken Ono, Wei-Lun Tsai

http://arxiv.org/abs/2201.06746

Combinatorial identities associated with a bivariate generating function for overpartition pairs Atul Dixit, Ankush Goswami

http://arxiv.org/abs/2201.06942 Further *q*-supercongruences from a transformation of Rahman Victor J. W. Guo

http://arxiv.org/abs/2201.07049 Gauss *q*-ed from Heine cubed P. L. Robinson

http://arxiv.org/abs/2201.07326

Automated Counting and Statistical Analysis of Labeled Trees with Degree Restrictions Shalosh B. Ekhad, Doron Zeilberger

http://arxiv.org/abs/2201.09301

 ${\cal A}_n$ extensions of some expansion formulas of Liu Bing He

Chain sequences and Zeros of a perturbed ${\it R}_{II}$ type recurrence relation Vinay Shukla, A. Swaminathan

http://arxiv.org/abs/2201.10117

Generalized q-Bernoulli polynomials generated by Jackson q-Bessel functions S. Z. Eweis, Zeinab S. I. Mansour

http://arxiv.org/abs/2201.11820

On the number of p-hypergeometric solutions of KZ equations Alexander Varchenko

http://arxiv.org/abs/2201.12359

The single-indexed exceptional Krawtchouk polynomials Hiroshi Miki, Satoshi Tsujimoto, Luc Vinet

http://arxiv.org/abs/2201.12415

An asymptotic approach to Borwein-type sign pattern theorems Chen Wang, Christian Krattenthaler

http://arxiv.org/abs/2201.12699

The B-B-G Transfer Principle for signature four P. L. Robinson

http://arxiv.org/abs/2201.12871

Investigation of the two-cut phase region in the complex cubic ensemble of random matrices Ahmad Barhoumi, Pavel M. Bleher, Alfredo Deaño, Maxim L. Yattselev

http://arxiv.org/abs/2201.12941

Universality for multiplicative statistics of Hermitian random matrices and the integro-differential Painlevé II equation Promit Ghosal, Guilherme L. F. Silva

http://arxiv.org/abs/2201.13315

Some definite integrals involving Jacobi polynomials Enno Diekema

http://arxiv.org/abs/2201.13337

Higher regularity of homeomorphisms in the Hartman-Grobman theorem for semilinear evolution equations Weijie Lu, Manuel Pinto, Y. H Xia

http://arxiv.org/abs/2202.00214

The combinatorics of hopping particles and positivity in Markov chains Lauren K. Williams

http://arxiv.org/abs/2202.00635

A Simple Proof of Siegel's Theorem Using Mellin Transform Zihao Liu

http://arxiv.org/abs/2202.00800

On $L^2_{\mathbb{R}}$ -best rational approximants to Markov functions on several intervals Maxim L. Yattselev

Laguerre Unitary Ensembles with Jump Discontinuities, PDEs and the Coupled Painlevé V System Shulin Lyu, Yang Chen, Shuai-Xia Xu

http://arxiv.org/abs/2202.01278

Determinantal Formulas for Exceptional Orthogonal Polynomials Brian Simanek

http://arxiv.org/abs/2202.01392

The Madelung Constant in *N* Dimensions Antony Burrows, Shaun Cooper, Peter Schwerdtfeger

http://arxiv.org/abs/2202.02049

The asymptotic expansion of the Humbert hyper-Bessel function R. B. Paris

http://arxiv.org/abs/2202.02485

Sharp regularity of the Hartman-Grobman theorem in ${\it C}^0$ linearization Weijie Lu, Manuel Pinto, Y-H. Xia

http://arxiv.org/abs/2202.02637

Proof of two conjectures on Askey-Wilson polynomials K. Castillo, D. Mbouna

http://arxiv.org/abs/2202.02678

The Existence of Dyon Solutions for Generalized Weinberg-Salam Model Shouxin Chen, Yilu Xu

http://arxiv.org/abs/2202.03139

Free boson realization of the Dunkl intertwining operator in one dimension Luc Vinet, Alexei Zhedanov

http://arxiv.org/abs/2202.03145

Jensen-type inequalities for convex and *m*-convex functions via fractional calculus Yamilet Quintana, José M. Rodríguez, José M. Sigarreta Almira

http://arxiv.org/abs/2202.03340

The q-Lidstone series involving q-Bernoulli and q-Euler polynomials generated by the third Jackson q-Bessel function Z. S. I. Mansour, M. Al-Towailb

http://arxiv.org/abs/2202.03827

Universality for random matrices with equi-spaced external source: a case study of a biorthogonal ensemble Tom Claeys, Dong Wang

http://arxiv.org/abs/2202.04116

Asymptotic spectral properties of the Hilbert *L*-matrix František Štampach

On the algebraic solutions of the Painlevé-III (D7) equation Robert J. Buckingham, Peter D. Miller

http://arxiv.org/abs/2202.04388

Hypergeometric ${}_4F_3(1)$ with integral parameter differences Dmitrii Karp, Elena Prilepkina

http://arxiv.org/abs/2202.04819

Some formulas for fully degenerate Bernoulli numbers and polynomials Taekyun Kim, Dae San Kim

http://arxiv.org/abs/2202.04843

A Stieltjes algorithm for generating multivariate orthogonal polynomials Zexin Liu, Akil Narayan

http://arxiv.org/abs/2202.05925

Bispectrality and biorthogonality of the rational functions of q-Hahn type Ismaël Bussière, Julien Gaboriaud, Luc Vinet, Alexei Zhedanov

http://arxiv.org/abs/2202.06020

Colored vertex models and *k*-tilings of the Aztec diamond Sylvie Corteel, Andrew Gitlin, David Keating

http://arxiv.org/abs/2202.06456

Discrete orthogonality of hypergeometric polynomial sequences on linear and quadratic lattices Luis Verde-Star

http://arxiv.org/abs/2202.07298

Some observations about Hankel determinants of the columns of Pascal triangle and related topics

Johann Cigler

http://arxiv.org/abs/2202.07763

Combinatorial formula for arithmetic density Robert Schneider, Andrew V. Sills

http://arxiv.org/abs/2202.07769

Bohemian Matrix Geometry Robert M. Corless, George Labahn, Dan Piponi, Leili Rafiee Sevyeri

http://arxiv.org/abs/2202.08911

Symmetry of terminating series representations of the Askey-Wilson polynomials Howard S. Cohl, Roberto S. Costas-Santos

http://arxiv.org/abs/2202.08918

Expansion for a fundamental solution of Laplace's equation in flat-ring cyclide coordinates Lijuan Bi, Howard S. Cohl, Hans Volkmer

http://arxiv.org/abs/2202.08923

Peanut harmonic expansion for a fundamental solution of Laplace's equation in flat-ring coordinates Lijuan Bi, Howard S. Cohl, Hans Volkmer

Quadratic decomposition of bivariate orthogonal polynomials Amílcar Branquinho, Ana Foulquié Moreno, Teresa E. Pérez

http://arxiv.org/abs/2202.09900

A Linear Time, and Constant Space, Algorithm to Compute the Mixed Moments of the Multivariate Normal Distributions Shalosh B. Ekhad, Doron Zeilberger

http://arxiv.org/abs/2202.10167

On another characterization of Askey-Wilson polynomials D. Mbouna, A. Suzuki

http://arxiv.org/abs/2202.10374

On Smooth Perturbations of Chebyshëv Polynomials and $\bar\partial\text{-Riemann-Hilbert}$ Method Maxim L. Yattselev

http://arxiv.org/abs/2202.10597

On the monodromy manifold of *q*-Painlevé VI and its Riemann-Hilbert problem Nalini Joshi, Pieter Roffelsen

http://arxiv.org/abs/2202.11017

Orthogonal polynomials, Toda lattices and Painlevé equations Walter Van Assche

http://arxiv.org/abs/2202.12164

The Dunkl-Laplace transform and Macdonald's hypergeometric series Dominik Brennecken, Margit Rösler

http://arxiv.org/abs/2202.12857

Asymptotic expansions of Kummer hypergeometric functions with three asymptotic parameters a, b and z N. M. Temme, E. J. M. Veling

http://arxiv.org/abs/2202.13281

Polynomial algebras of superintegrable systems separating in Cartesian coordinates from higher order ladder operators Danilo Latini, Ian Marquette, Yao-Zhong Zhang

Other Relevant OP-SF E-Prints

http://arxiv.org/abs/2201.00127

On a weighted zero-sum constant related to the Jacobi symbol Santanu Mondal, Krishnendu Paul, Shameek Paul

http://arxiv.org/abs/2201.00342

High Precision Computation of Riemann's Zeta Function by the Riemann-Siegel Formula, II Juan Arias de Reyna

A Generalized Lerche-Newberger Formula Parker Kuklinski, Michael Warnock, David A. Hague

http://arxiv.org/abs/2201.00867

Modified Special Functions Defined by Generalized M-Series and Their Properties Enes Ata

http://arxiv.org/abs/2201.01124

Euler's integral, multiple cosine function and zeta values Su Hu, Min-Soo Kim

http://arxiv.org/abs/2201.01189

Olsson.wl : a Mathematica package for the computation of linear transformations of multivariable hypergeometric functions B. Ananthanarayan, Souvik Bera, S. Friot, Tanay Pathak

http://arxiv.org/abs/2201.01402

Shuffle product formula of the Schur multiple zeta values of hook type Maki Nakasuji, Wataru Takeda

http://arxiv.org/abs/2201.01676

Iterated Integrals and Special Values of Multiple Polylogarithm at Algebraic Arguments Kam Cheong Au

http://arxiv.org/abs/2201.01678

Comparison of methods for the calculation of the real dilogarithm regarding instruction-level parallelism Alexander Voigt

http://arxiv.org/abs/2201.02027

Weighted sum formulas for finite alternating multiple zeta values with some parameters Takumi Anzawa

http://arxiv.org/abs/2201.02371

The Green's function of the Lax-Wendroff and Beam-Warming schemes Jean-François Coulombel

http://arxiv.org/abs/2201.02376

Proving some conjectures on Kekulé numbers for certain benzenoids by using Chebyshev polynomials Guoce Xin, Yueming Zhong

http://arxiv.org/abs/2201.02617

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Remainder in Modified Mertens Formula and Ramanujan Inequality Gennadiy Kalyabin

Smooth and polyhedral norms via fundamental biorthogonal systems Sheldon Dantas, Petr Hájek, Tommaso Russo

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A Green's function for the source-free Maxwell equations on $AdS^5 \times S^2 \times S^3$ Damien Gobin, Niky Kamran

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Non-commutative Hermite-Padé approximation and integrability Adam Doliwa

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Incompressible active phases at an interface. I. Formulation and axisymmetric odd flows Leroy L. Jia, William T. M. Irvine, Michael J. Shelley

Topic #5 _____ OP – SF Net 29.2 _____ March 15, 2022

From: OP-SF Net Editors Subject: Submitting contributions to OP-SF Talk

To contribute a news item to OP-SF NET, send e-mail to one of the OP-SF Editors howard.cohl@nist.gov, or spost@hawaii.edu.

Contributions to OP-SF NET 29.3 should be sent by May 1, 2022.

OP-SF NET is the electronic newsletter of the SIAM Activity Group on Special Functions and Orthogonal Polynomials (SIAG/OPSF). We disseminate your contributions on anything of interest to the special functions and orthogonal polynomials community. This includes announcements of conferences, forthcoming books, new software, electronic archives, research questions, and job openings as well as news about new appointments, promotions, research visitors, awards and prizes. OP-SF Net is transmitted periodically through a post to OP-SF Talk which is currently managed and moderated by Howard Cohl (howard.cohl@nist.gov). Anyone wishing to be included in the mailing list (SIAG/OPSF members and non-members alike) should send an email expressing interest to him.

OP-SF Talk is a listserv associated with SIAG/OPSF which facilitates communication among members, non-members and friends of the Activity Group. To post an item to the listserv, send email to howard.cohl@nist.gov.

WWW home page of this Activity Group: http://math.nist.gov/opsf Information on joining SIAM and this activity group: service@siam.org

The elected Officers of the Activity Group (2020-2022) are: Peter Alan Clarkson, Chair Luc Vinet, Vice Chair Andrei Martínez-Finkelshtein, Program Director Teresa E. Pérez, Secretary

The appointed officers are:

Howard Cohl, OP-SF NET co-editor Sarah Post, OP-SF NET co-editor Bonita Saunders, Webmaster Topic #6 _____ OP - SF Net 29.2 _____ March 15, 2022

From: OP-SF Net Editors Subject: Thought of the Month by **Dick Askey**

"Part of the secret of success in studying and using special functions is to try to remember exactly what is necessary, and nothing more."

Dick Askey (1933–2019), in p. 9, Orthogonal Polynomials and Special Functions, Society for Industrial and Applied Mathematics, Philadelphia, 1975.

Contributed by Paul A. Martin

Comment by Charles Dunkl (03/11/2022)

This quote by Dick Askey is amusing—but Dick had the reputation "Askey knows the literature" which was said at an analysis seminar at Wisconsin (by Walter Rudin (1921-2010), my Ph.D. advisor, when I was a grad student 63–65). Maybe he didn't memorize every important formula but he evidently knew where to find stuff he needed.