Errata and comments for my 1985 paper *Special orthogonal polynomial systems mapped onto each other by the Fourier–Jacobi transform*

collected by Tom Koornwinder, thkmath@xs4all.nl

last modified: April 26, 2021

These are errata and comments for the paper

T. H. Koornwinder, *Special orthogonal polynomial systems mapped onto each other by the Fourier–Jacobi transform*, in *Polynômes orthogonaux et applications*, Lecture Notes in Math. 1171, Springer-Verlag, 1985, pp. 174–183.

p.177, (3.3): For $\alpha=\beta=\pm\frac12$, $\delta>-1$, $\mu\in\mathrm{i}\mathbb{R}$, $\lambda\in\mathbb{R}$ and taking in view that $\phi_\lambda^{(-\frac12,-\frac12)}(t)=\cos(\lambda t)$, $\phi_\lambda^{(\frac12,\frac12)}(t)=2\lambda^{-1}\sin(\lambda t)/\sinh(2t)$, we can apply quadratic transformations for, on the one hand Jacobi polynomials and on the other hand Wilson polynomials together with continuous Hahn polynomials (see [R4, (2.29), (2.30)]), in order to rewrite (3.3) for $\alpha=\beta=-\frac12$,

$$(-1)^n\,\frac{(\delta+1)_n}{(\delta+1)_{2n}}\,\frac{(2n)!}{n!}\int_0^\infty(\cosh t)^{-\delta-\mu-1}\,P_{2n}^{(\delta,\delta)}(\tanh t)\,\cos(\lambda t)\,\mathrm{d}t=\frac{2^{\delta+\mu+2n-1}(-1)^n}{n!}$$
$$\times\frac{\Gamma\big(\frac12(\delta+\mu+1+\mathrm{i}\lambda)\big)\Gamma\big(\frac12(\delta+\mu+1-\mathrm{i}\lambda)\big)}{\Gamma(\delta+\mu+1+2n)}\,W_n\big(\tfrac14\lambda^2;\tfrac12(\delta+\mu+1),\tfrac12(\delta-\mu+1),0,\tfrac12\big),$$

respectively for $\alpha=\beta=\frac12$,

$$\frac{16(-1)^n}{\lambda}\,\frac{(\delta+1)_n}{(\delta+1)_{2n+1}}\,\frac{(2n+1)!}{n!}\int_0^\infty(\cosh t)^{-\delta-\mu-1}\,P_{2n+1}^{(\delta,\delta)}(\tanh t)\,\sin(\lambda t)\,\mathrm{d}t$$
$$=\frac{2^{\delta+\mu+2n+3}(-1)^n}{n!}\,\frac{\Gamma\big(\frac12(\delta+\mu+1+\mathrm{i}\lambda)\big)\Gamma\big(\frac12(\delta+\mu+1-\mathrm{i}\lambda)\big)}{\Gamma(\delta+\mu+2+2n)}$$
$$\times W_n\big(\tfrac14\lambda^2;\tfrac12(\delta+\mu+1),\tfrac12(\delta-\mu+1),\tfrac12,1\big),$$

as the case $\alpha=\beta=\frac12(\delta+\mu+1)$, $\gamma=\delta$ of Koelink [R3, Lemma 2.1] (n even respectively odd):

$$\int_{-\infty}^\infty(\cosh t)^{-\delta-\mu-1}\,P_n^{(\delta,\delta)}(\tanh t)\,\mathrm{e}^{-\mathrm{i}\lambda t}\,\mathrm{d}t=\frac{\Gamma\big(\frac12(\delta+\mu+1+\mathrm{i}\lambda)\big)\Gamma\big(\frac12(\delta+\mu+1-\mathrm{i}\lambda)\big)}{\Gamma(\delta+\mu+1+n)}$$
$$\times\,2^{\delta+\mu}\mathrm{i}^{-n}p_n\big(\tfrac12\lambda;\tfrac12(\delta+\mu+1),\tfrac12(\delta-\mu+1),\tfrac12(\delta-\mu+1)\big).$$

p.179, (4.4): Replace the second upper parameter $\alpha+\delta+1$ in the ${}_4F_3$ by $n+\alpha+\delta+1$.

p.179, last line: (error observed in [R1, Remark 3.3]) The expression for B_n should be

$$B_n=\frac{(n+\alpha+1)(2n+\alpha+\beta+\delta-\mu+2)}{(n+1)(2n+\alpha-\beta+\delta+\mu+2)}\,A_n+\frac{n(2n+\alpha-\beta+\delta+\mu)}{(n+\alpha)(2n+\alpha+\beta+\delta-\mu)}\,C_n\,.$$

p.181, (5.11): This is the inversion pair as in Faraut [8, §IV], but without the discrete terms in the inversion formula, and with Faraut's $\phi(x,s)$ expressed in terms of Whittaker functions by $\phi(x,s) = (2x)^{-\frac{1}{2}} W_{k,s}(2x)$ (our curly W is just the usual Whittaker W-function). Earlier than Faraut this inversion pair was given by Wimp. [R5, (4.9), (4.10)]. Slightly later than Faraut this was also treated by Carroll [R2].

References

[R1] N. Ben Abdallah and F. Chouchene, *New recurrence relations for Wilson polynomials via a system of Jacobi type orthogonal functions*, J. Math. Anal. Appl. 498 (2021), 124978, 17 pp.

[R2] R. Carroll, *On a transform theory involving Whittaker functions*, Applicable Anal. 16 (1983), 85–90.

[R3] H. T. Koelink, *On Jacobi and continuous Hahn polynomials*, Proc. Amer. Math. Soc. 124 (1996), 887–898.

[R4] T. H. Koornwinder, *Quadratic transformations for orthogonal polynomials in one and two variables*, in: *Representation theory, special functions and Painlevé equations*, Adv. Stud. Pure Math., Vol. 76, Math. Soc. Japan, Tokyo, 2018, pp. 418–447; `arXiv:1512.09294`.

[R5] J. Wimp, *A class of integral transforms*, Proc. Edinburgh Math. Soc. (2) 14 (1964), 33–40.