

Holographic anatomy and its applications

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Based on:

- Determining the dual field theory data from an *asymptotically $AdS \times X$* geometry and vice versa:

Kaluza-Klein holography with K. Skenderis, 0603016;

Coulomb branch vevs with K. Skenderis, 0604169;

Anatomy of bubbling solutions with K. Skenderis, 0706.0216.

- Applications to the fuzzball proposal:

Fuzzball solutions for black holes and D1-brane–D5-brane microstates with K. Skenderis, 0609154;

Holographic anatomy of fuzzballs with I. Kanitscheider, K. S., 0611171;

Fuzzballs with internal excitations with I. Kanitscheider, K. S., 0704.0690.

Introduction

This year will be the 10th anniversary of the AdS/CFT conjecture, and by now there are many known examples of gravity/gauge theory dualities.

Typically **tests** of the conjecture have been made at the **conformal point**, from the first work on matching non-renormalized quantities (protected operators, their correlation functions) through to the more recent work on integrability, whose ultimate aim is to prove the conjecture **at the conformal point, in the planar limit**.

Many interesting applications of the conjecture however involve **breaking conformal invariance**.

On the one hand one wants to use **holographic engineering** to open a window into gauge theories at strong coupling. For example, there have been many attempts to construct geometries realizing duals of **confining gauge theories with chiral symmetry breaking**.

Going in the other direction, from gauge theory data to geometry, there is a fundamental question at stake: how is the **geometry reconstructed** from gauge theory data?

Of course one already needs to answer this question to find the geometric dual of a given gauge theory, where symmetries and supersymmetry do not uniquely fix it.

The first part of this talk will concern developments in addressing these issues at a **precise, quantitative** level.

Outline

1. Holographic anatomy

A. Holographic renormalization

B. KK holography: recovering the compact part of the geometry

C. Applications

2. The fuzzball program

1. Holographic anatomy

So, given a geometry conjectured to be dual to a certain gauge theory, what would one like to compute?

Let us pose this question in the context of an **asymptotically** $AdS_5 \times X_5$ geometry, dual to a QFT that in the UV flows to a non-trivial CFT. And assume one has identified the conformal theory, or at least its chiral primaries.

Then one would first like to deduce the **vacuum structure** corresponding to the geometry, namely **the vevs and parameters of deformations**. For example, in the dual of a confining theory one would like to extract the vev of the **gluino condensate** $\langle \lambda \bar{\lambda} \rangle$.

Computing such vevs **quantitatively** is quite subtle. In a given geometry one can often see the existence of a gluino condensate **qualitatively** as follows.

The basic AdS/CFT dictionary relates the gluino bilinear operator to a scalar supergravity field and the **linearized dictionary** [Witten, 98] relates the **normalizable** mode in the near (AdS) boundary asymptotics of this field to the condensate vev. This relationship is however imprecise - vevs are actually **non-linear** in the sugra asymptotics.

Whilst in some cases the linearized approximation gives an answer which is qualitatively correct, there are equally many known cases where it gives **qualitatively incorrect** results, e.g. typically one finds $E \neq 0$ for a susy vacuum.

A. Holographic renormalization

In the context of asymptotically AdS spacetimes, i.e. suppressing the compact space X , the precise relationship between spacetime asymptotics and field theory data is by now well understood.

The defining AdS/QFT formulae of [GKP, Witten 1998] are formal, because of infinite volume divergences in the bulk and corresponding UV divergences in the field theory.

Deriving the vevs therefore requires **holographic renormalization**, which is the precise gravitational analogue of QFT renormalization.

[Henningson, Skenderis (1998) Balasubramanian, Kraus (1999) de Haro, Skenderis, Solodukhin (2000) Bianchi, Freedman, Skenderis (2001) Papadimitriou, Skenderis (2004) Review: Skenderis (2002)]

Asymptotically (locally) AdS_{d+1} spacetimes have the following asymptotic (Fefferman-Graham) form

$$ds^2 = \frac{dr^2}{r^2} + \frac{1}{r^2} g_{ij}(x, r) dx^i dx^j$$

where

$$g_{ij}(x, r) = g_{(0)ij} + r^2 g_{(2)ij} + \dots + r^d \left(\log r^2 h_{(d)ij} + g_{(d)ij} \right) + \dots$$

This is an expansion in r with the conformal boundary of the spacetime located at $r = 0$.

One can hence obtain a **renormalized supergravity action** as follows:

- **Regulate** divergences by restricting radial coordinate $r \geq \epsilon$.
- **Evaluate** the action on the **asymptotic solution**.
- Remove infinite terms with **local covariant counterterms**.

The precise relation between correlation functions and asymptotics then follows from applying GKPW prescription to this renormalized action:

$$\langle T_{ij} \rangle = \frac{2}{\sqrt{g_{(0)}}} \frac{\delta S_{SUGRA}^{ren}}{\delta g_{(0)}^{ij}} = \frac{dl}{16\pi G_N} [g^{(d)ij} + X_{ij}^{(d)}(g_{(0)})].$$

where $X_{ij}^{(d)}(g_{(0)})$ are **local (non-linear) functions** of $g_{(0)}$.

The underlying structure of the renormalized correlators is best exhibited in a **radial Hamiltonian formalism**, where the radial coordinate plays the role of time. [Papadimitriou, Skenderis, (2004)]

The renormalized 1-point functions are simply the **part** of the **radial canonical momentum** that **scales properly**, i.e. with weight equal to the dimension of the dual operator. In the example given here

$$\langle T_{ij} \rangle = \pi_{(d)ij}$$

So 1-point functions are expressed **linearly** in terms of (renormalized) radial canonical momenta, which in general depend **non-linearly** on coefficients in the asymptotic expansions of supergravity fields.

B. Kaluza-Klein holography [Skenderis, MT (2006)]

The previous discussion relates to the **asymptotically AdS** part of the geometry, or equivalently to operators dual to fields in the corresponding $(d+1)$ -dimensional **gauged supergravity**.

However, solutions of gauged supergravity only give information about the lowest dimension operators, which does not characterize the vacuum structure uniquely, and can be singular where the $10d$ solution is not. Moreover, many known $10d$ solutions cannot be reduced to gauged supergravity solutions.

An important question is **how is the compact part of the geometry reconstructed?** And equivalently **how does one obtain vevs of operators dual to supergravity fields not contained in gauged supergravity?**

Kaluza-Klein holography is a method to extract vevs of all gauge invariant operators from any **asymptotically** $AdS_{d+1} \times X$ solution.

The basic idea is to reduce to $(d+1)$ dimensions using a **non-linear gauge invariant Kaluza-Klein map** without truncating and then apply standard $(d+1)$ -dimensional holographic renormalization formulae.

The vevs of all chiral primary operators can thence be expressed non-linearly in terms of the near boundary asymptotics of the $10d$ supergravity solution.

The algorithmic steps are the following:

1. Expand **perturbations** of $10d$ fields about the $AdS_{d+1} \times X$ background such that $\Phi = \phi_B + \phi$ where

$$\phi(x, y) = \sum \psi^I(x) Y^I(y)$$

and Φ denotes all $10d$ fields; ϕ_B is the $AdS_{d+1} \times X$ solution; ϕ are the fluctuations and $Y^I(y)$ denotes collectively all harmonics of the compact manifold.

2. Insert this expansion into the $10d$ equations and find the equations that **gauge invariant** combinations satisfy. These equations reduce upon use of **a non-linear KK map** to $(d+1)$ -**dimensional field equations** which can be integrated into an action.

3. Now standard holographic renormalization can be carried out.

The precise relationship between vevs and supergravity asymptotics can be summarized most efficiently as follows. Let \mathcal{O}_k^I label the set of operators of dimension k . Then

$$\langle \mathcal{O}_k^I \rangle = \pi_k^I + \sum c_{I_1 \dots I_j}^I \pi_{k_1}^{I_1} \dots \pi_{k_j}^{I_j}.$$

Here π_k^I is again the renormalized canonical momentum of the field dual to the operator \mathcal{O}_k^I . (The terms non-linear in canonical momenta arise whenever $k = \sum k_i$; the constants $c_{I_1 \dots I_j}^I$ are related to **extremal correlators**.)

Each momentum term π_k^I can be rewritten explicitly and non-linearly in terms of the **asymptotics of the 10d solution**.

Remarks

- This method allows one to extract **vevs and deformation parameters** from **any supergravity solution** which is asymptotically $AdS \times X$. Conversely given field theory data **asymptotic solutions** can be reconstructed and the full solution is in principle **uniquely** determined.
- In particular, by purely **algebraic manipulations**, one can obtain not just the mass and R-charge (angular momentum) but the vevs of all chiral primary operators. These capture in a very precise way the duals to **multipole moments** of the supergravity solution.
- One point functions are the easiest data to extract, since higher point functions involve solving fluctuation equations on the geometry. Moreover, vevs typically satisfy stronger **non-renormalization theorems**.

- The holographic formulae can be **derived in generality** and then applied to specific examples. E.g. formulae to analyse LLM bubbling solutions are equally applicable to 1/4 and 1/8 BPS bubbling solutions.
- Any precise computation in gravity/gauge duality **requires** analogous techniques; this holds also for non-local operators and corresponding bulk branes/strings.
- Given an asymptotically $(AdS \times X)$ geometry, it is now possible to extract from the geometry **sufficient QFT data to distinguish the holographic dual**. Except in highly symmetric situations, the data extracted from the $(d+1)$ -dimensional gauged supergravity solution could not uniquely identify the field theory dual.

C. Applications

Our method allows **strong tests** of gravity/gauge theory duality beyond the conformal point.

Examples: $\mathcal{N} = 4$ SYM on the Coulomb branch (multi-center D3-branes), and on $R \times S^3$ (LLM bubbling solutions): exact matching of non-renormalized vevs for generic distributions. [\[0604169, 0706.0216\]](#)

More generally, these techniques can be used to **identify** the field theory duals of previously known solutions...

And can be combined with supersymmetric classification tools to develop **holographic engineering**, the systematic construction of holographic duals.

2. The fuzzball proposal

A. General **2-charge fuzzball** solutions

C. **Field theory dual**

D. **Implications** for the fuzzball program

2. The fuzzball proposal

Associated with any black hole there are an exponential number of **horizon-free** solutions that look like the black hole asymptotically but generically differ from it up to the horizon scale. [Lunin, Mathur (2001)]

These solutions represent the **microstates** of the black hole; the original black hole provides only the **average** description of the system.

The AdS/CFT correspondence **motivates** this picture, at least for supersymmetric black holes. [\[0609154\]](#)

More precisely, the gravity/gauge theory dictionary relates a given asymptotically AdS geometry to either a **deformation** of the CFT or the **CFT in a non-trivial vacuum** characterized by the expectation values of gauge invariant operators. Conversely, one expects that for any **stable state** of the CFT (such as the BPS states) there exists an asymptotically AdS solution, whose **asymptotics encode the vevs** of gauge invariant operators **in that state**.

If the field theory is in a **pure state**, there is no entropy and one expects the corresponding geometry to be **horizon-free**.

There is however **no guarantee** that the geometry should be **well-described by supergravity alone**, i.e. weakly curved everywhere.

The key questions in this scenario are:

- Can one find **enough** such geometries for each black hole?
- What **properties** should such geometries have, to be associated with black hole microstates?
 - Can one show quantitatively how black hole properties emerge upon **coarse graining**?

Answering these questions in generality is currently out of reach.

One approach is to look for **candidate typical geometries** for supersymmetric 3-charge and 4-charge black holes (which have macroscopic horizons).

However, one can also focus on the simplest possible example, the **2-charge D1-D5 system on T^4 and $K3$** . In this case one can obtain general horizon-free non-singular solutions via dualities from known **F1-P solutions** [Callan et al], [Dabholkar et al] (1995).

This system thus provides an ideal test case for a detailed understanding of the correspondence between geometries and microstates.

A. General 2-charge fuzzball solutions [KST], 0704.0690

The solutions are characterized by a number of arbitrary **curves**.

- **24** curves for the **K3** solutions; **most general such solution**.
- **8** curves for the T^4 solutions; **most general solution carrying only bosonic excitations on T^4** .

The curves represent the profile of the string in the original F1-P system as well as the profile of the charge waves in the case of the heterotic F1-P system.

The solutions of Lunin-Mathur (2001) are a subset of these solutions, characterized by **4 curves** representing the blow-up of the naive geometry to a supertube in the transverse space.

$$ds^2 = \frac{f_1^{1/2}}{\tilde{f}_1 f_5^{1/2}} [-(dt - A_i dx^i)^2 + (dy - B_i dx^i)^2]$$

$$+ f_1^{1/2} f_5^{1/2} dx_i dx^i + f_1^{1/2} f_5^{-1/2} ds_{M^4}^2,$$

$$e^{2\Phi} = \frac{f_1^2}{f_5 \tilde{f}_1}, \quad B_{ty}^{(2)} = \frac{\mathcal{A}}{f_5 \tilde{f}_1}, \quad B_{\bar{\mu}i}^{(2)} = \frac{\mathcal{A} \mathcal{B}_i^{\bar{\mu}}}{f_5 \tilde{f}_1},$$

$$B_{ij}^{(2)} = \lambda_{ij} + \frac{2\mathcal{A} A_{[i} B_{j]}}{f_5 \tilde{f}_1}, \quad B_{\rho\sigma}^{(2)} = f_5^{-1} k^\gamma \omega_{\rho\sigma}^\gamma, \quad C^{(0)} = -f_1^{-1} \mathcal{A},$$

$$C_{ty}^{(2)} = 1 - \tilde{f}_1^{-1}, \quad C_{\bar{\mu}i}^{(2)} = -\tilde{f}_1^{-1} \mathcal{B}_i^{\bar{\mu}}, \quad C_{ij}^{(2)} = c_{ij} - 2\tilde{f}_1^{-1} A_{[i} B_{j]},$$

$$C_{tyij}^{(4)} = \lambda_{ij} + \frac{\mathcal{A}}{f_5 \tilde{f}_1} (c_{ij} + 2A_{[i} B_{j]}), \quad C_{\bar{\mu}ijk}^{(4)} = \frac{3\mathcal{A}}{f_5 \tilde{f}_1} \mathcal{B}_{[i}^{\bar{\mu}} c_{jk]},$$

$$C_{ty\rho\sigma}^{(4)} = f_5^{-1} k^\gamma \omega_{\rho\sigma}^\gamma, \quad C_{ij\rho\sigma}^{(4)} = (\lambda_{ij}^\gamma + f_5^{-1} k^\gamma c_{ij}) \omega_{\rho\sigma}^\gamma,$$

$$C_{\rho\sigma\tau\pi}^{(4)} = f_5^{-1} \mathcal{A} \epsilon_{\rho\sigma\tau\pi},$$

- The solution is determined in terms of **26** (K3)/**10** (T^4) harmonic functions, $(K, f_5, A_i, \mathcal{A}, \mathcal{A}^{\alpha-})$, which in turn are determined by the **24** (K3)/**8** (T^4) curves $F^i(v), \mathcal{F}(v), \mathcal{F}^{\alpha-}(v)$. For example,

$$f_5 = \frac{Q_5}{L} \int_0^L \frac{dv}{(x_i - F_i(v))^2}; \quad \mathcal{A} = -\frac{Q_5}{L} \int_0^L \frac{\partial_v \mathcal{F} dv}{(x_i - F_i(v))^2}.$$

- F^i are associated with transverse excitations, $\mathcal{F}, \mathcal{F}^{\alpha-}$ with excitations in the internal manifold.

- The solution has the **same mass and conserved charges** as the naive (singular) D1-D5 system but has **non-zero angular momentum** provided the curves F^i are non-zero, and carries **multipole** moments.

In the decoupling limit the solutions become asymptotically $AdS_3 \times S^3 \times T^3/K3$ so one may use KK holography to identify the field theory dual.

The vev of the stress energy tensor is (non-trivially) zero,

$$\langle T_{ij} \rangle = 0$$

in agreement with the fact that the fuzzballs are conjectured to be dual to Ramond ground states in the CFT. The solutions also have R charges (angular momenta) $\langle J^{ij} \rangle \sim \int dv F^i \partial_v F^j$.

People often use only this data in identifying the dual but there is an infinite amount of other data, namely vevs of scalar chiral primary operators which can be expressed in terms of the field asymptotics near the AdS boundary, and hence in terms of the coefficients in the expansions of the harmonic functions.

B. Field theory dual

The dual CFT is a deformation of a sigma model with target space the symmetric orbifold of the compactification manifold.

- The holographic vevs invalidate naive proposals that associate each of the fuzzball solutions (specified by a curve F^i) to a **single R-ground state**.
- We proposed a **precise map** that associates a given supergravity solution to a **specific superposition** of R-ground states. The exact form of the superposition depends on the curves determining the solution.
- This proposal passes **all kinematical tests** and **all accessible dynamical tests**: computing the vevs of gauge invariant operators in these superpositions gives **exact agreement** with the gravity computations, **within the approximations used**.

In general however one must **go beyond supergravity** to accurately describe the physics of the system.

- Most of the geometries contain small regions of **high curvature**, so they are not well-described by supergravity.

- When one includes geometries with **internal excitations**, the situation is even worse: geometries with only internal excitations $F^i(v) = 0$ collapse to the naive singular geometry. These should **account for a finite fraction** of the black hole entropy but are **not visible in sugra approximation** at all!

- Even the geometries that only involve macroscopic scales generically can only be resolved from each other by **effects beyond supergravity**; their vevs differ from each other by **a very small amount** ($\sim 1/N$ effects).

C. Implications for the fuzzball proposal

One could argue (or hope) that these problems are confined to the two charge system, which after all is not a macroscopic black hole.

However, on rather general grounds one can argue that these issues will persist in other cases. The typical **scale** of the fuzzball geometry is **linearly related** to the angular momenta/R charges: the greater the angular momentum, the more the supertube blows up in the transverse R^4 .

Many **3-charge** D1-D5-P microstates also have very small R charges and the corresponding geometric duals most likely have **small scales**. As in the 2 charge case, a **finite fraction** of the entropy would be associated with geometries carrying only excitations on the internal manifold, **not blown up at all in R^4** .

Put another way, the natural basis for microstates in the CFT is **not** the natural basis for geometries.

One might hope to find a good **geometric basis**, in which all geometries have a finite scale on the R^4 , comparable with the horizon scale of the original black hole.

However, again on general grounds, one will not avoid having **small quantum numbers**: in such a basis, large numbers of geometries will **not be distinguishable** in supergravity.

What have we learned?

The success in constructing a map between **horizon-free** solutions and **microstates** consistently with the AdS/CFT correspondence provides the most stringent test of the fuzzball proposal to date.

Generically however one needs to go **beyond the leading supergravity approximation** to extract the physics.

Given that many of the fuzzball geometries are not likely well-described in supergravity, and explicitly constructing typical geometries for other black holes is difficult, perhaps we need a different approach to the problem.

We argued that it is likely there do not exist enough geometries, well-described and distinguishable in supergravity, to span the entire set of black hole microstates.

However a **sufficiently representative** basis may still exist. That is, suppose one chooses a single representative of the indistinguishable geometries, and **assigns a measure to this geometry**.

Thus to make progress within supergravity, one should understand **quantitatively** how **typical** is the state associated with any given fuzzball solution, which in turn requires understanding the precise map between geometries and states.

Our methods could be used to **identify** candidate fuzzball geometries [Bena, Warner et al], to see how representative they are. Also to **holographically engineer** missing geometries.

Conclusions

We have seen how holography can increasingly be promoted to a **very precise framework** that can be used to compute gauge theory properties from asymptotically $AdS \times X$ geometries and vice versa.

Future directions mentioned during the talk include using KK holography as a tool in developing **systematic holographic engineering**....

Also **pushing the fuzzball proposal** further, understanding how black hole properties emerge upon coarse-graining.

Equally important, and not emphasised so far, is developing our understanding of the **holographic reconstruction of spacetime**. Our results show how the **compact** part of the geometry is reconstructed; work is underway on seeing how **global structure** such as horizons are encoded in gauge theory data and hence boundary asymptotics.

Thank you!