

Prior on an *insolubiliūm* of Jean Buridan*

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Abstract

We present Prior's discussion of a puzzle about validity found in the writings of the 14th-century French logician Jean Buridan and show how Prior's study of this puzzle may have provided the conceptual inspiration for Prior's development of hybrid logic.

1 Introduction

Elsewhere in this volume, we give a historical overview of Arthur Prior's work on medieval logic, focusing on the unpublished material in his archives. In this paper we take a more conceptual approach towards showing his debt to the medieval logicians. In [Uc-] we saw how the works of the Stoic logician Diodoros Chronos and medieval logicians such as Jean Buridan and Peter de Rivo provided inspiration for Prior's development of temporal logic. In this paper we show that an *insolubiliūm* (puzzle) of Jean Buridan may also have provided the original inspiration for hybrid logic.

2 A *insolubiliūm* about validity

2.1 Medieval conceptions of validity

Jean Buridan was a French logician working at the University of Paris in the early 14th century.¹ Buridan, like many medieval logicians, defined 'proposition' in a fashion antithetical to the standard modern definition. By the term *propositio*, Buridan meant not an abstract entity, always existing and eternally true or eternally false, but rather a specific mental, spoken, or written token declarative sentence. On such a view, propositions are transient objects, which come into and go out of existence and are not necessarily sharable.²

Such a view naturally has consequences for how validity is to be defined. According to standard modern definitions of validity, an argument is valid if

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¹For further discussion of Buridan's works and Prior's research on Buridan, see [Uc-, §3.3].

²For more details on ancient and medieval theories of the proposition, see [Nu73].

whenever propositions expressed by the premises are true, the propositions expressed by the conclusion must be true, or that the truth of the premises forces the truth of the conclusion. Since truth or falsity, and hence possible truth and possible falsity, can only be ascribed to things which exist, this definition builds in the assumption that the propositions expressed by the premises and the conclusion always exist. On the medieval conception of propositions that we’re considering now, validity can no longer be defined as a relationship between necessarily existing and unchanging entities, because truth values do not attach to necessarily existing abstract objects but instead attach to contingently existing transient concrete tokens. In order for a proposition to be true, it must first exist. However, we can easily imagine circumstances in which true premises are written down, but where the conclusion is not written down (or otherwise spoken or thought), making the premises true and the conclusion not true. So, the modern definition of validity is obviously not suitable.

Medieval definitions of validity are also couched in terms of necessity and truth and a certain relation between the truth of the premises and the truth of the conclusion holding in some necessary fashion.³ However, these definitions when combined with the contingent nature of propositions on the medieval view give rise to some paradoxical results. Two of these paradoxes, or *insolubilia*, Prior investigated in detail, in [Pr69f]. In this paper, we present the results of his research, and show a natural extension of these results to basic hybrid logic. We conclude that, though there is no explicit textual evidence for this, Prior’s attempt to provide a semantics for Buridan’s distinction between ‘possible’ and ‘possibly-true’ may have provided a conceptual grounding and inspiration for the development of hybrid logic.

We make one note on terminology before turning to the details of Prior’s research on Buridan. Because the modern conception of propositions is so strongly entrenched, and because the medieval view of propositions presented above is so foreign and antithetical to the modern view, we attempt to lessen any confusion which might arise because of the differences of these views by calling token propositions by their Latin name, *propositio* (pl. *propositiones*). Throughout the following we use *propositio* to refer to a specific spoken, mental, or written token, and we use the English word ‘proposition’ in its modern sense.⁴

2.2 The *insolubium*

In [Pr69f], Prior considers an *insolubilia* or sophism of Buridan’s. An insoluble is a “proposition arrived at by apparently valid forms of reasoning, which nonetheless implies its own contradictory” [Bu66, p. 5]. The standard presentation of an insoluble in a medieval logical text is to first state the insoluble, then give an argument for its truth, and another argument for its falsity, and then, an argument for the correctness of one of these arguments and the incorrectness of the other. The specific insoluble discussed by Prior is the first one of the eighth chapter of Buridan’s *Sophismata*:

SOPHISM. (1) Every proposition is affirmative, so none is negative.

³In [Br93, pp. 88–90], Brodie presents three definitions of validity which are discussed in medieval literature, each of which have different problems connected with the fact that the validity relation must hold between contingent entities.

⁴Note, however, that this convention is not followed in the texts that we are quoting. We trust that context will make it clear which sense is being used.

It is proved, first, by the argument from contraries, for just as it follows that if every man is ill, then no man is healthy, because it is impossible for the same person to be both healthy and ill, so it follows in the proposed [case] that it is impossible for the same proposition to be both affirmative and negative at once.

...

The opposite is argued, because from a possible proposition there does not follow an impossible. And yet the first proposition is possible, namely, “Every proposition is affirmative.” For God could destroy all negatives, leaving affirmatives. Thus, every proposition would be affirmative. But the other is impossible, namely, “None is negative”, for in no case could it be true. For whenever it is not, it is neither true nor false, and whenever it is, then some [proposition] is negative, namely, it. Hence, it is false to say that none is negative [Bu66, pp. 180–181].

Both arguments are intuitively plausible. The first one turns on an equivalence representable in modern notation as

$$\forall x(Px \rightarrow Ax) \equiv \neg \exists (Px \wedge \neg Ax)$$

This is just the interdefinability of the quantifiers.

The second argument is more interesting because it turns on properties specific to Buridan’s *propositiones*. The *propositio* “No *propositio* is negative” can never be true, because in order to be true, it must first exist (i.e., be thought, written, or spoken), and as it is itself negative, as soon as it exists, it contradicts itself. (A bit later on in Ch. VIII, Buridan follows this line of effect in considering the argument from “No *propositio* is negative” to its contradictory “Some *propositio* is negative”.)

Buridan accepts the argument in favor of the inference, and rejects the argument against it. He concludes that if one allows there to be premises which are possible even though they are never true, then we are not dealing with a case of an impossible conclusion following from a possible one. He says that

it is manifest that a proposition is not called possible because it can be true, nor impossible because it cannot be true [Bu66, p. 182].⁵

This conclusion leads him to make a distinction between sentences which are *possible* and those which are *possibly-true*. It is this distinction which Prior formalizes in [Pr69f].

When Prior discusses the inference from “No *propositio* is negative” to “Some *propositio* is negative”, he notes that such examples may appear to be evincing “some sort of confusion between use and mention, or between object-language and metalanguage” [Pr69f, p. 481]. The thought is that there is merely some fundamental confusion going on and that somewhere Buridan is doing something illicit. Prior goes on to note that this is not the case, and that “there is nothing against a language containing its own syntax, though there may be plenty against its containing its own semantics” [Pr69f, p. 481]. Indeed, we have

⁵The diametric opposition of this view to Diodoros’s, discussed in [Uc-, §3.1], will hopefully have struck the reader forcefully.

examples of languages containing their own syntax, namely, Peano arithmetic, which is expressive enough to express syntactical notions such as ‘proof’, ‘provable’, etc. To show that there is no problem with a language containing its own syntax, and that the move which Buridan is making by having truth attach to individual tokens of sentences is not illicit, Prior constructs a language containing some of its own syntax, where the object-language and the metalanguage are sharply distinguished. He uses this to give a semi-formalization of the paradox, and we now present this semi-formalization.

3 Prior’s semi-formalization

In this section we present a slightly modified version of the semi-formalization which Prior used to analyze Buridan’s insoluble. We have two languages, an object language \mathcal{L} and a metalanguage \mathcal{M} . Our metalanguage \mathcal{M} is standard English. We define \mathcal{L} syntactically. \mathcal{L} is composed of three types of strings of marks: terms, signs of quantity, and copulae.

Definition 3.1. The *terms* of \mathcal{L} are the strings

propositio
affirmativa
negativa

We shall, as needed, use capital Roman letters, A, B, C, \dots , as variables for these terms.

Definition 3.2. The *signs of quantity* of \mathcal{L} are the strings

omnis
quaedam
nulla

Definition 3.3. The *copulae* of \mathcal{L} are the strings

est
non est

Definition 3.4. A string of characters is a *sentence* of \mathcal{L} or an *\mathcal{L} -sentence*⁶ if and only if it is a sign of quantity followed by a term followed by a copulae (which can be *non est* only if the sign of quantity is *quaedam*) followed by another (not necessarily distinct) term.

Thus, all \mathcal{L} -sentences will be of one of the following four forms:

- *omnis A est B*
- *nulla A est B*
- *quaedam A est B*
- *quaedam A non est B*

⁶When clear, we will drop ‘ \mathcal{L} ’ and refer to these strings simply as ‘sentences’.

Strictly speaking, these strings are syntactic entities which, as yet, have no meaning. However, the choice for \mathcal{L} -strings of these particular standard Latin terms was meant to be transparent. When we give the truth conditions for these \mathcal{L} -sentences, their meanings will correspond to the meanings of the Latin sentences.

Before we can present the truth definitions for \mathcal{L} -sentences, we need to introduce a notion which will allow the distinctive nature of Buridan's *propositiones* as concrete, existing objects to be reflected. To do so, we assume we have at our disposal an unlimited number of sheets of paper. On these sheets, certain \mathcal{L} -sentences may be written. These inscriptions on sheets of paper are our tokens. We will define two types of truth with respect to tokens on a sheet of paper: a sentence may be *true on a sheet of paper* and a sentence may be *true of a sheet of a paper*. In order to define these two types of truth, we must first define what each of the \mathcal{L} -terms refers to, or, in Prior's words, *connotes*. Prior says:

Each term is associated with a particular group of shapes, which it may be said to *connote*, though this means no more than that the presence on a sheet of marks of certain shapes will determine. . . whether or not sentences containing certain terms are to be counted as "true on their sheets" [Pr69f, p. 483].

Definition 3.5. The *connotation* of a term is defined as follows:

- The term *propositio* *connotes* all \mathcal{L} -sentences.
- The term *negativa* *connotes* all \mathcal{L} -sentences whose sign of quantity is *nulla* or whose copula is *non est*.
- The term *affirmativa* *connotes* all \mathcal{L} -sentences which are not connoted by the term *negativa*.

We often say that a sentence is of type A rather than that it is connoted by term A .

We now give the truth conditions for *true on a sheet* for each type of sentence.

Definition 3.6 (Truth on a sheet).

- A sentence of the form *Omnis A est B* is true on a sheet iff
 1. It is written on the sheet.
 2. There is at least one sentence on the sheet which is of type A .
 3. There is no sentence on the sheet which is of type A which is not of type B .
- A sentence of the form *Nulla A est B* is true on a sheet iff
 1. It is written on the sheet.
 2. There is no sentence on the sheet which is of both type A and type B .
- A sentence of the form *Quaedam A est B* is true on a sheet iff
 1. It is written on the sheet.

2. There is at least one sentence on the sheet which is of both type *A* and type *B*.
- A sentence of the form *Quaedam A non est B* is true on a sheet iff
 1. It is written on the sheet, and either
 2. there is at least one sentence on the sheet which is of type *A* and is not of type *B*, or
 3. there is no sentence on the sheet of type *A*.⁷

Before we turn to examples of this definition, we make a few notes on the relationships between the four types of sentences. These four types of sentences are the four found in the traditional Aristotelian Square of Opposition. The truth definition given above satisfies the standard relationships in this square, particularly that sentences of the form *Omnis A est B* and *Quaedam A non est B* are contradictories of each other and sentences of the form *Nulla A est B* and *Quaedam A est B* are also contradictories of each other. Hence, if both of a pair of contradictories occur on a sheet, then one will be false on the sheet and the other will be true on the sheet.⁸

We now give examples of this truth definition.

<p>Omnis negativa est affirmativa</p> <p>Quaedam propositio est negativa</p>	<p>Omnis propositio est affirmativa</p> <p>Quaedam propositio non est negativa</p>	<p>Nulla affirmativa est negativa</p> <p>Quaedam affirmativa non est negativa</p>
Sheet 1	Sheet 2	Sheet 3

On Sheet 1, both sentences are false: They are both false because there are no negative *propositiones* on the sheet.

On Sheet 2, the first sentence is false and the second is true. The second is true because the first *propositio* is affirmative and the first is false because the second violates the truth conditions of the first.

And on Sheet 3, the first sentence is true and the second false. The first is true by the connotations of the terms; the affirmative sentences are defined as those which are not negative. The second is false because neither sentence is affirmative.

⁷Note that in this definition, one of the conditions of a sentence of the form *Omnis A est B* being true is that at least one sentence of form both *A* and *B* occurs on the sheet. Since *Quaedam A non est B* is the contradictory of *Omnis A est B*, when it is present on a sheet, its truth can be triggered merely by there being no sentences of type *A*. This phenomenon, ‘existential import’, is another widely-discussed difference between medieval and modern logical theories, but one which will not occupy us here.

⁸The proofs of these claims are straightforward and are not proved here.

It is now clear how we can use \mathcal{L} -sentences to make claims about the syntax of other \mathcal{L} -sentences, in a completely unproblematic manner which involves no confusion of metalanguage and object language. Because we can do so, we can make a distinction between sentences which are possible and those which are possibly true.

First, note that this language contains some sentences such that any time they are present on a sheet, they are true on that sheet, and some sentences such that any time they are present on a sheet, they are false on that sheet. Consider the following sentences:

Nulla affirmativa est negativa
Quaedam affirmativa est negativa

The first says that no affirmative sentence is negative. Given the connotations of *affirmativa* and *negativa*, the first sentence is clearly tautological, and the second sentence, being its contradictory, will equally clearly be false on any sheet on which it appears. This is nothing surprising, as we expect any standard logical system to contain tautologies and contradictions. But contrast these two sentences with the following:

Quaedam propositio est affirmativa
Quaedam propositio non est affirmativa
Nulla propositio est negativa

The first two will be true on any sheet on which they occur, and the last will be false on any sheet on which it occurs. But these cases differ relevantly from the first two. Consider the first sentence and its contradictory, *Nulla propositio est affirmativa*. While the first is true on any sheet it is written on, its contradictory is not false on every sheet it is written on. For example, if it is the only sentence written on a sheet, then it is true on that sheet.

If we move into the metalanguage, we can see another difference between the two groups of statements. We cannot imagine any sheet of paper which has a sentence which is both negative and affirmative. The sentence *Quaedam affirmativa est negativa* is a logical contradiction, because as a sentence is affirmative if and only if it is not negative, this is simply the assertion

$$\exists x(\neg Nx \wedge \neg Nx)$$

likewise, its contradictory is the logical necessity

$$\forall x(Nx \rightarrow Nx)$$

But we can easily imagine a sheet of paper which is correctly described by “No proposition is negative” (to adapt a reason of Buridan’s given later in the same text, God could easily have annihilated all negative propositions, so that the meta-claim “No proposition is negative” would be true), and ones where “Some proposition is affirmative” is an incorrect description. As Buridan points out, there is nothing logically contradictory about the claim $\neg \exists x Nx$, nor anything logically necessary about the claims $\exists x Ax$ and $\exists x \neg Ax$.

Sentences of the type of the second there are the ones which underpin Buridan’s conclusion that there are some sentences which are possible, but not possibly-true, and some which are impossible but not necessarily-false.

4 A hybrid logic approach

The presentation in the previous section corresponds, with few changes of notation, to Prior's original consideration of the *sophisma*. However, his discussion of the difference between 'possible' and 'possibly-true' is done in an informal, meta-language setting. It is possible to give a very natural and straightforward extension of the previous semantics to a hybrid semantics in order to give a formal presentation of the distinction in the object language, which we do now.

We begin by extending \mathcal{L} to a new syntax \mathcal{L}' . \mathcal{L}' contains all the terms, signs of quantity, and copulae of \mathcal{L} , plus an infinite set of numerals \mathbb{N} and the modalities $@_n$, \diamond , \heartsuit , \square , and \boxplus . We call elements of \mathbb{N} *nominals*, and let the variables n, m, ℓ, k, \dots range over the nominals.

Definition 4.1. A string of characters is a *propositio* of \mathcal{L}' (or an \mathcal{L}' -*propositio*) iff it is a sentence of \mathcal{L} .⁹ We correspondingly modify def. 3.5 so that *propositio* connotes all \mathcal{L}' -*propositiones*.

Definition 4.2. A string of characters is a *sentence* of \mathcal{L}' (or an \mathcal{L}' -*sentence*) iff it is one of the following:

- It is an \mathcal{L}' -*propositio*.
- It is $@_n$ followed by an \mathcal{L}' -*propositio*.
- It is \diamond followed by an \mathcal{L}' -*propositio*.
- It is \heartsuit followed by an \mathcal{L}' -*propositio*.
- It is \square followed by an \mathcal{L}' -*propositio*.
- It is \boxplus followed by an \mathcal{L}' -*propositio*.

A model \mathfrak{S} is formally defined as a tuple $\langle S, V, N \rangle$, where S is a (possibly infinite) set of sheets s_1, s_2, s_3, \dots , V is a function assigning \mathcal{L}' -*propositiones* to subsets of S , and N is a function from \mathbb{N} to S such that $N(n) = s_n$. We abuse notation and say that $V(s_n)$ is the set of *propositiones* which are written on s_n . We define two notions of truth. The first applies to *propositiones* only, and is a simplification of the truth definition presented in the previous section.

Definition 4.3 (Truth of a sheet).

- A *propositio* of the form *Omnis A est B* is true of a sheet s_n iff
 1. There is at least one *propositio* on s_n of type A .
 2. There is no *propositio* on s_n which is of type A which is not of type B .
- A *propositio* of the form *Nulla A est B* is true of a sheet s_n iff
 1. There is no *propositio* on s_n which is of both type A and type B .
- A *propositio* of the form *Quaedam A est B* is true of a sheet s_n iff
 1. There is at least one *propositio* on s_n which is of both type A and type B .

⁹As before, we will omit the \mathcal{L}' when ambiguity will not occur.

- A *propositio* of the form *Quaedam A non est B* is true of a sheet s_n iff
 1. Either there is at least one *propositio* on s_n which is of type A and is not of type B ,
 2. or there is no *propositio* on s_n of type A .

Let φ be an arbitrary *propositio*. We then say that $s_n \Vdash \varphi$ iff φ is true of s_n .

With this definition of truth of a sheet, we can give a complete truth definition for arbitrary \mathcal{L}' formulas. Let φ be an arbitrary \mathcal{L}' -*propositio*.

Definition 4.4 (Truth).

- $s_n \models @_m \varphi$ iff $\varphi \in V(s_n)$ and $s_m \Vdash \varphi$
- $s_n \models \varphi$ iff $s_n \models @_n \varphi$

The addition of the $@_n$ operator allows us to distinguish between possibility and possible truth. Formally, possibility and possible truth, like truth, are evaluated with respect to a sheet of paper:

Definition 4.5 (Possibility & Possible Truth).

- $s_n \models \diamond \varphi$ iff $\varphi \in V(s_n)$ and there exists an m s.t. $s_n \models @_m \varphi$
- $s_n \models \heartsuit \varphi$ iff $\varphi \in V(s_n)$ and there exists m s.t. $s_m \models @_m \varphi$
- $s_n \models \square \varphi$ iff $\varphi \in V(s_n)$ and for all m , $s_n \models @_m \varphi$
- $s_n \models \boxplus \varphi$ iff $\varphi \in V(s_n)$ and for all m , $\varphi \in V(s_m)$ implies $s_m \Vdash \varphi$

Like \mathcal{L} -sentences above, strings like $\diamond \varphi$ are strictly speaking syntactic entities which have no meaning. However, it should be intuitively clear that the truth conditions for these four operators will allow us to treat them as object-language renderings of standard meta-language notions, and read $\diamond \varphi$ as ‘ φ is possible’; $\heartsuit \varphi$ as ‘ φ is possibly-true’; $\square \varphi$ as ‘ φ is necessary’; and $\boxplus \varphi$ as ‘ φ is necessarily true’. We can then conclude that φ is possible but not possibly-true if $\forall m, n$ s.t. $s_m \models @_n \varphi$, $m \neq n$, and there are m, n s.t. $s_m \models @_n \varphi$, as well as the following:

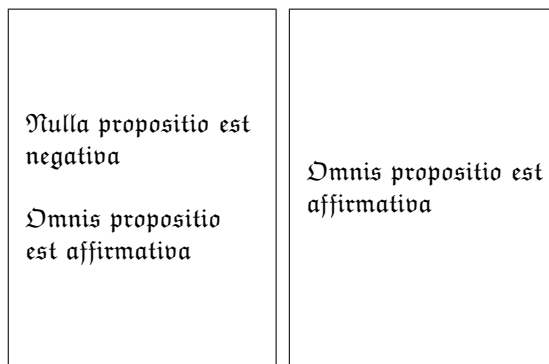
Lemma 4.6.

- (a) $s_n \models \varphi$ implies $s_n \models \diamond \varphi$ implies $s_n \models \heartsuit \varphi$, and not vice versa.
 (b) $s_n \models \square \varphi$ implies $s_n \models \boxplus \varphi$ implies $s_n \models \varphi$, and not vice versa.

Proof.

- (a) Assume $s_n \models \varphi$. This means that $s_n \models @_n \varphi$, that is, $\varphi \in V(s_n)$ and $s_n \Vdash \varphi$. Hence $\exists m$ s.t. $s_m \models @_m \varphi$, namely n . So $s_n \models \diamond \varphi$. Further $\exists m$ s.t. $s_n \models @_m \varphi$, namely n again. So $s_n \models \heartsuit \varphi$.

For the reverse direction, consider the following model: Let $\varphi = \text{Nulla propositio est negativa}$. $\varphi \in V(s_4)$, and $s_5 \Vdash \varphi$. Hence, $s_4 \models @_5 \varphi$, so $s_4 \models \diamond \varphi$. However, $s_4 \not\models \heartsuit \varphi$ because neither $s_4 \models @_4 \varphi$ nor $s_5 \models @_5 \varphi$. Let $\psi = \text{Omnis propositio est affirmativa}$. $s_5 \models @_5 \psi$, and $\psi \in V(s_4)$, so $s_4 \models \diamond \psi$. However, $s_4 \not\models \psi$, since $s_4 \not\models \psi$.

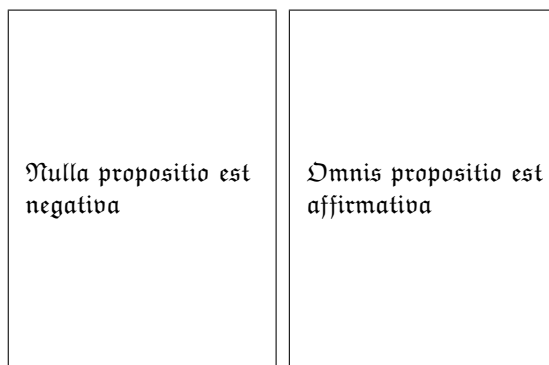


Sheet 4

Sheet 5

- (b) Let φ be arbitrary and assume $s_n \models \Box\varphi$. Then $\varphi \in V(s_n)$ and $\forall m, s_n \models @_m\varphi$. Suppose $s_n \not\models \Box\varphi$. Then either $\varphi \notin V(s_n)$ or $\exists m, \varphi \in V(s_m)$ but $s_m \not\models \varphi$. The first disjunct results in an immediate contradiction. So, fix m . Then $s_n \models @_m\varphi$. But then $s_m \Vdash \varphi$, which is also a contradiction. That $s_n \models \varphi$ is obvious from the preceding.

For the reverse direction, let ψ be as above. Since $\psi \in V(s_5)$ and $s_5 \Vdash \psi$, $s_5 \models \psi$. However, $s_5 \not\models @_4\psi$, so $s_5 \not\models \Box\psi$. To show that $s_n \models \Box\varphi$ doesn't imply $s_n \models$, consider the following model:



Sheet 6

Sheet 7

Let $\varphi = \text{Omnis propositio est affirmativa}$. Then $s_7 \Vdash \varphi$, and $\varphi \in V(s_7)$. Since $\varphi \notin V(s_6)$, $s_7 \models \Box\varphi$. However, $s_6 \not\models \varphi$, so $s_7 \not\models @_6\varphi$. Hence, $s_7 \not\models \Box\varphi$.

□

Throughout his paper, Prior is scrupulous about keeping meta-language notions such as truth and possibility strictly within the meta-theory and outside of his syntax, so that one could not make semantic claims such as “Every propositio is possible” or “Some negative is not true” within the object language. With the introduction of the possibility operators into the syntax, there is a natural worry that we have somehow violated this strict division, and introduced something illicit and potentially problematic into our logic. This is not the case, because

in \mathcal{L}' we distinguish between *propositiones* and sentences. *Propositiones* can refer to themselves; they cannot refer to sentences, and sentences do not refer to *propositiones*. In this fashion, we are able to provide a higher layer of formalization extending the formalization originally presented by Prior without introducing any of the problematic issues he wished to avoid.

This hybridization of the system presented in [Pr69f] as a way to model Buridan’s distinction is eminently natural, which makes it noteworthy that there is no evidence in his paper that Prior ever considered such a development himself. Prior was translating and reading Buridan, and writing this paper and another on similar sophisms, at the same time that he was exploring the four grades of tense logical involvement which led to his hybridization of tense logic.¹⁰ Given how natural it is to extend the notion of sentences on sheets to propositions in possible worlds, and to formalize how *propositiones* can make claims about *propositiones* on other sheets by means of sheet-indices and operators on these indices, it is plausible that Prior’s work with medieval discussions of medieval problems provided some inspiration for his later development of hybrid logic.

Regardless of what the actual relationship between Prior’s work with Buridan and his development of hybrid logic, the hybridization of Prior’s framework is interesting not only as a model of a perhaps antiquated medieval theory, but also in its own right. At the end of his paper, Prior notes that

Part of the interest of these results is that relations between the truth, falsehood, possible-truth, necessary-truth, possibility and necessity of sentences on sheets may be thought of as “mirroring” certain relations between features of what is or is not the case in the world. . . If there can be a somewhat more sophisticated semantics than some of the stock ones, there can also be a more sophisticated modal logic [Pr69f, pp. 491–492].

Hybridization of the sheets-of-paper models is one natural next step towards this more sophisticated semantics.¹¹

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¹⁰See [Pr68a] and [Blo6].

¹¹Hybrid logic is not the only semantics that can make this distinction. An alternative semantics, called double-index semantics, which could be used to model this phenomenon was developed and published a few years after Prior’s introduction of hybridization; Creswell says that “[double-indexing] seems to have been first investigated by Frank Vach in his UCLA Ph.D., but the first published use is by Kamp (1971). It was later used in Åqvist (1973) and Segerberg (1973), who called it *two-dimensional modal logic*” [Cr85, p. 154]. (The references are [Ka71], [Åq73], and [Se73].) However, given Prior’s close connection with hybrid logic and lack of evidence that he was familiar with the double-indexing or two-dimensional approach, we feel that the hybrid semantics is more natural for extending his account of Buridan’s *sophisma*. My thanks to the anonymous referee for pointing me towards these references.

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