The age of regular variation: tales on tails

Symposium on the occasion of Guus Balkema's 65th birthday

Date:	November 8, 2002
Location:	B.C.P. Jansen Instituut Universiteit van Amsterdam Plantage Muidergracht 12 Room C3

Programme

11.00-11.30	Coffee and Welcome
11.30-11.35	Opening
11.35-12.20	Paul Embrechts (ETH Zürich): *Ruin, operational risk and how fast stochastic processes mix*
12.20-14.00	Lunch break
14.00-14.45	Claudia Klüppelberg (TU München): *Limit laws for exponential families - theoretical results and applications*
14.45-15.30	Sidney Resnick (Cornell University): *Limits of On/Off Hierarchical Product Models for Data Transmission*
15.30-16.00	Tea
16.00-16.45	Laurens de Haan (Erasmus Universiteit): *Approximations for the tail (empirical) distribution function*
16.45-17.00	Closing by Tom Koornwinder (director of the Korteweg-de Vries Institute for Mathematics)
17.00	Reception

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Abstracts

Paul Embrechts: *Ruin, operational risk and how fast stochastic processes mix*

Due to the new guidelines on banking supervision, operational risk has become a focus of attention for risk managers. This talk is based on joint work with Gennady Samorodnitsky. A robustness type of result for heavy-tailed ruin esti- mates in a risk process under a broad class of time change models will be presented. These results will be motivated by questions concerning the quan- titative modelling of Operational Risk.

Laurens de Haan: *Approximations for the tail (empirical) distribution function*

Extreme value conditions have been with us for a long time. Somewhat more recently second order extreme value conditions have been studied and used. The former and the latter can be expressed both in terms of the distribution function and in terms of the quantile function. Recently (1998) Holger Drees derived sharp uniform inequalities connected with the limit relations for the quantile function and showed that these lead to a useful approximation for the tail empirical quantile function. I want to present analogous inequalities connected with the limit relation for the distribution function and show that these lead to a useful approximation for the tail empirical distribution. I shall mention two probabilistic applications but shall not mention any statistical application...
(joint work with Holger Drees and Deyuan Li)

Claudia Klüppelberg: *Limit laws for exponential families - theoretical results and applications*

A random vector X generates a natural exponential family of vectors X^λ, $\lambda \in \Lambda$, where Λ is the set, where the moment generating function $Ee^{\lambda X}$ is finite. Assume that Λ is open and X non-degenerate. Suppose there exist affine transformations $\alpha_\lambda(x) = A_\lambda x + a_\lambda$ depending continuously on the parameter λ and a non-degenerate vector Y so that

$$\alpha_\lambda^{-1}(X^\lambda) \quad \stackrel{d}{\to} \quad Y \tag{1}$$

when λ diverges. The limit laws in the univariate case are the gamma and normal laws, whereas in the multivariate case a larger variety of limit laws occurs.
In a series of papers, under the scientific leadership of Guus Balkema, various analytic results in this context were proven. We summarize some univariate results in the context of a normal limit law.
1) A density f with a Gaussian tail (a generalization of a log-concave density) is in the domain of attraction (in the sense of (1)) of the normal law. This has been proved by a local limit theorem for the exponential family generated by f.
2) The normal domain of attraction is closed with respect to convolutions, leading to precise asymptotics for convolutions of densities with Gaussian tails.
3) The limit theorem for the exponential family yields Abel-Tauber theorems for densities with Gaussian tails.

These results can be used to solve various applied problems:
• Saddlepoint approximations of densities with Gaussian tails become exact in the tail.
• In an GI/G/1 queue, where $(N(t))_{t\geq 0}$ is the stationary renewal arrival process, W the steady-state sojourn time and L the queue length, the tail behaviour of $L \stackrel{d}{=} N(W)$ can be determined.
• The Value-at-Risk (α-quantile) of a high-dimensional portfolio is often approximated by the Delta-Gamma-Normal model, which is a stochastic second order Taylor expansion based on a multivariate normal vector. An approximation can be derived (for $\alpha \to 0$).
• Extreme value theory for MA processes with light-tailed innovations can be derived.

References

S. Asmussen, C. Klüppelberg and K. Sigman (1998) Sampling at subexponential times, with queueing applications. *Stoch. Proc. Appl.* **79**, 265-286.

A.A. Balkema, C. Klüppelberg and S.I. Resnick (1993) Densities with Gaussian tails. *Proc. London Math. Soc.* **66**, 568-588.

A.A. Balkema, C. Klüppelberg and S.I. Resnick (1999) Limit laws for exponential families. *Bernoulli* **5**, 951-968.

A.A. Balkema, C. Klüppelberg and S.I. Resnick (1999) Domains of attraction for exponential families and asymptotic behaviour of Laplace transforms. *Stoch. Proc. Appl.* To appear.

A.A. Balkema, C. Klüppelberg and S.I. Resnick (1999) Stability for multivariate exponential families. *J. Math. Sciences.* To appear.

A.A. Balkema, C. Klüppelberg and S.I. Resnick (2002) Abel-Tauber theory: a unifying approach. In preparation.

A.A. Balkema, C. Klüppelberg and U. Stadtmüller (1995) Tauberian results for densities with Gaussian tails. *J. London Math. Soc.* **51**, 383-400.

O.E. Barndorff-Nielsen and C. Klüppelberg (1992) A note on the tail accuracy of the univariate saddlepoint approximation *Annales de Toulouse (Série 6)* **1**, 5-14.

O.E. Barndorff-Nielsen and C. Klüppelberg (1999) Tail exactness of multivariate saddlepoint approximations. *Scand. J. Stat.* **26** 253-264.

S. Jaschke, C. Klüppelberg and A. Lindner (2002) Asymptotic behavior of tails and quantiles of quadratic forms of Gaussian vectors. Submitted for publication.

C. Klüppelberg and A. Lindner (2002) Extreme value theory for moving average processes with light-tailed innovations. In preparation.

Sidney Resnick: *Limits of On/Off Hierarchical Product Models for Data Transmission*

A hierarchical product model seeks to model network traffic as a product of independent on/off processes. Previous studies have assumed a Markovian structure for component processes amounting to assuming that exponential distributions govern *on* and *off* periods but this is not in good agreement with traffic measurements. However, if the number of factor processes grows and input rates are stabilized by allowing the *on* period distribution to change suitably, a limiting on/off process can be obtained which has exponentially distributed *on* periods and whose *off* periods are equal in distribution to the busy period of an $M/G/\infty$ queue. We give a fairly complete study of the possible limits of the product process as the number of factors grow and offer various characterizations of the approximating processes. We also study the dependence structure of the approximations. (joint work with Gennady Samorodnitsky)