Problem Set 3

February 16, 2009

Please hand in Problem 1 by February 23, 2pm.

1 Problem 1

a) [2 pts] Consider a closed string with the sigma model action in conformal gauge:

$$S = \frac{T}{2}\int d^2\sigma(\dot{X}^2 - X'^2) \tag{1}$$

Consider world-sheet light-cone coordinates $\sigma^{\pm} = \tau \pm \sigma$ and show that in these coordinates the action takes the form:

$$S = T\int d\sigma^+ d\sigma^- \partial_+ X^\mu \partial_- X_\mu \tag{2}$$

b) [2 pts] Show that this action is invariant under the infinitesimal transformation:

$$\delta X^\mu = a_n e^{2in\sigma^-}\partial_- X^\mu \tag{3}$$

where a_n is a constant infinitesimal parameter.

c) [2 pts] As we showed in class for any continuous symmetry of the Lagrangian there is a corresponding conserved current, called the Noether current. Starting from (2), use the Noether method to show that the Noether current corresponding to the symmetry (3) is given by

$$j^+ = T(\partial_- X^\mu \partial_- X_\mu)e^{2in\sigma^-}, \qquad j^- = 0. \tag{4}$$

In other words, make the parameter a_n in (3) local and show that the variation of the action (2) is proportional to $\partial_\pm a_n$. The coefficients of $\partial_\pm a_n$ are

the components $j^{\pm}$ of the Noether current in light cone coordinates. Verify that the current (4) in conserved when the field equations hold.

d) [2 pts] The Noether current transforms as a vector under coordinate changes. Use this to write the component $j^0 = j^\tau$ in terms of j^+, j^-.

e) [2 pts] The Noether charge is defined by:

$$Q = \int d\sigma \, j^0. \tag{5}$$

Using your result from d), show that the Noether charge corresponding to the symmetry (3) is equal to the Virasoro generator L_n :

$$\int d\sigma \, j^0 = L_n. \tag{6}$$

In deriving this relation, you can use the fact that the current j^+ in (4) is related to the stress-energy tensor T_{--} in BBS (2.36), and the fact thtat T_{--} can be expanded in L_m as in BBS (2.73).

2 Problem 2

Problem 2.3 of BBS.
Clarification: in part (i), you are supposed to compute spacetime momentum current $P^{25}_\alpha = T\partial_\alpha X^{25}$ defined in BBS (2.67) and see if it is conserved or not because of the modified boundary condition.

3 Problem 3

1. Prove equations (2.73) and (2.74) in BBS, starting from the expressions for the energy-momentum tensor in terms of the field X^μ (2.36)–(2.37) and using the mode expansion (2.40)–(2.41).

2. Using the Poisson brackets (2.51)–(2.52) derive the Virasoro algebra (2.84).