Chasing electric flux in hot QCD

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work (?) with:
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motivation

- 4d Yang-Mills (YM) theory undergoes transition
  - *cold and confined phase / hot and deconfined phase*
  - *critical temperature* T_c
- widely used order parameter: Polyakov loop
  - $\langle P \rangle = \langle \frac{1}{N} Tr \mathcal{P} \exp(ig \int_0^{1/T} dx_0 \, A_0) \rangle$
  - $\langle P \rangle \neq 0$ *in deconfined phase*
- there are some problems
  - P *creates single fundamental static color source*
  - *not gi; does not belong to physical Hilbert space*
  - *cannot be defined at* $T = 0$
  - *UV divergences in the continuum limit*
- alternative: 't Hooft flux loop operator (see rest of talk)
  - *in some sense dual to Wilson loops*
  - *NB spatial Wilson loop shows only area law at* $T \neq 0$
    while at $T = 0$ *area/perimeter law for broken/unbroken Z(N) symmetry*

hot YM

measure color-electric flux through a large surface S

- take e.g. rectangular $L \times L$ loop in xy plane, $L \gg l_D$
- $(\mathbf{E})_i = F_{0i}$ is in Lie Algebra of SU(N)
  - ▷ *ask for flux projected onto some special direction Y in SU(N)*
  - ▷ *flux* $\Phi = \frac{1}{g}\int_S d\mathbf{S} \cdot Tr\mathbf{E}Y$
  - ▷ *note that* $\langle\Phi\rangle = 0$
- define 't Hooft loop (or E-flux) operator
  - ▷ $V = \exp(4\pi i\,\Phi)$ ['t Hooft 78; Kovner 92]
  - ▷ *note that* $\langle V\rangle$ *behaves non-trivially*
- gauge invariance?
  - ▷ $F^{\Omega}_{\mu\nu} = \Omega F_{\mu\nu}\Omega^{-1}$ *transforms as adjoint*
  - ▷ *choose special Y's: generators of center of SU(N)* [CKA/Kovner 00]
  - ▷ $\exp(2\pi i Y_k) = \exp(2\pi i \frac{k}{N})1\!\!1_{N\times N}$ *for* $k = 0, \ldots, N-1$
  - ▷ *not unique. simple choice:* $Y_k = \frac{1}{N}\, diag(\{k\}^{N-k}, \{k-N\}^{k})$

get set of gi operators $V_k = \exp(\frac{4\pi i}{g}\int_S d\mathbf{S} \cdot Tr\mathbf{E}Y_k)$

- for a detailed proof see CKA, lecture notes Bielefeld Graduate School, Sept 2007

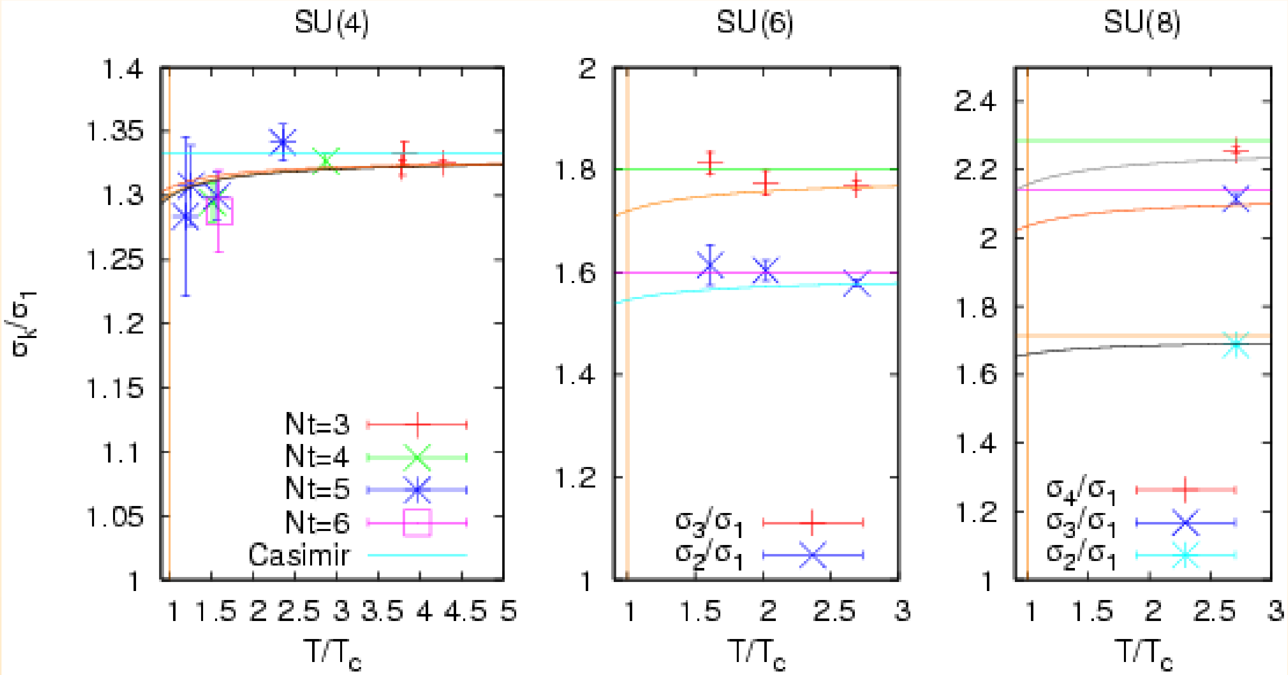
hot YM

can now define the thermal averages

- $\langle V_k \rangle \sim \exp(-\sigma_k(T) L^2)$

and can compute these

- with analytic methods: weak-coupling expansion (see rest of the talk)
- on the lattice [de Forcrand/Lucini/Noth 05]

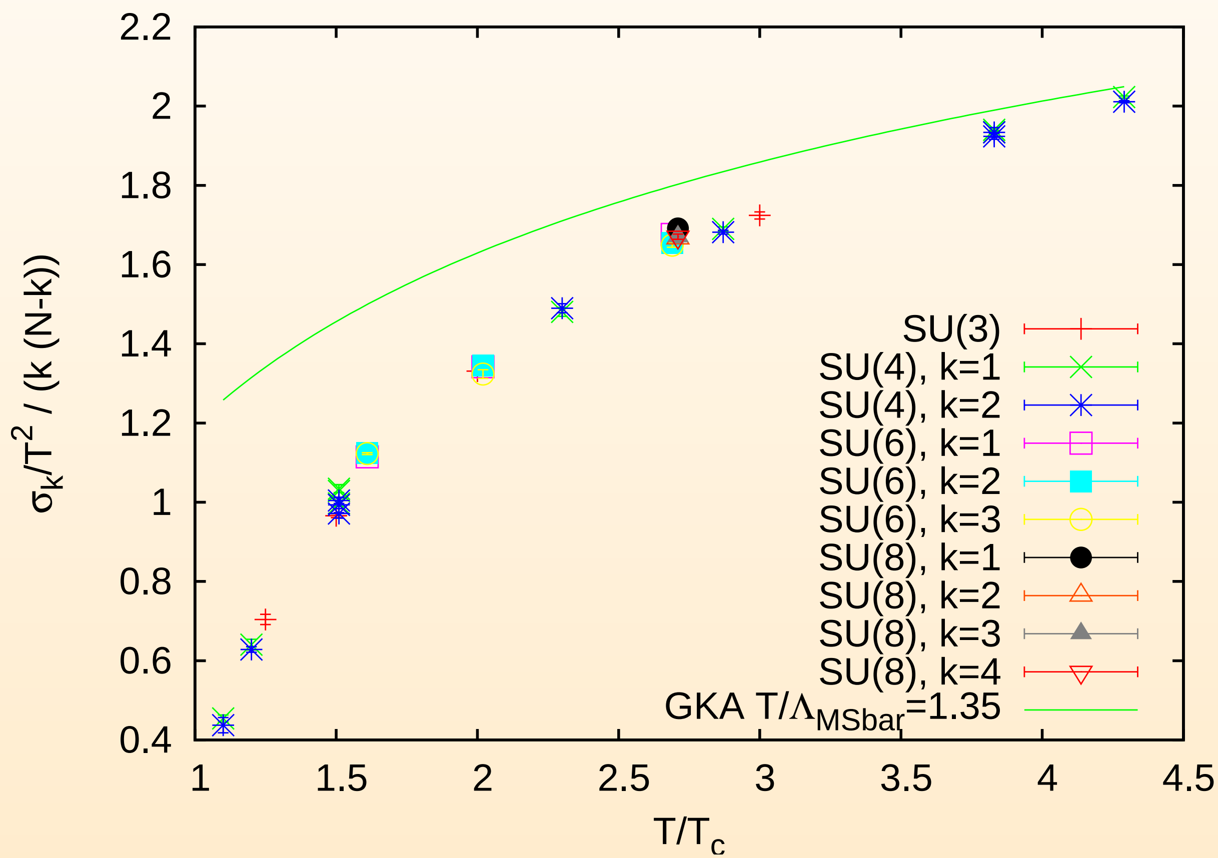


▷ *hor. lines: LO* $\frac{k(N-k)}{(N-1)}$. *other lines: NNLO with* $T_c/\Lambda_{\overline{MS}} = (1.1, 1.35)$

hot YM

... BUT ...

- lattice sees more violation of ''Casimir scaling'' at low temperature [dF/L/N 05]



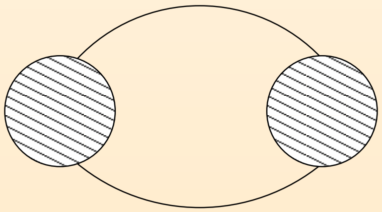
▷ *line: NLO σ_k with 2-loop running coupling, at $T_c/\Lambda_{\overline{MS}} = 1.35$*

- lattice data falls on universal curve

analytic results

perturbation theory has been driven to NNLO

- $\frac{\sigma_k}{T^2} = \frac{4\pi^2}{3\sqrt{3g^2N}} k(N-k) \left(1 - 15.27853..\frac{g^2N}{(4\pi)^2} + c_3(N,k)\frac{g^3N^{\frac{3}{2}}}{(4\pi)^3} + c_4(N,k)\frac{g^4N^2}{(4\pi)^4}\right)$
  - ▷ *LO: Casimir scaling*
  - ▷ *NLO: Bhattacharya/Goksch/CKA/Pisarski 91,92; shows same scaling!*
  - ▷ *NNLO: Giovannangeli/CKA 03,04*
    c_3 *small, contributes (as in pressure) with sign opposite* g^2
    c_3 *given as 2d integral + inf sum*
  - ▷ *NNNLO: unknown. doable? useful?!*
- NNLO term (relative g^3) nonanalytic in α_s ?!
  - ▷ *like in pressure: originated from IR sensitive* g^4 *graph*
  - ▷ *and all-orders resummation of this specific sector*



setup

- compute σ_k as tunneling effect through perturbatively calculable potential barrier
  - ▷ *so first have to compute an effective potential*

- presence of V_k breaks center group symmetry
  - ▷ *chooses specific directions* Y_k
  - ▷ *hence the Polyakov loop* P *gets multiplied by phase* $\exp(2\pi i \frac{k}{N})$ *when moving through surface* S *(at, say,* $z = 0$*)*
  - ▷ *so the field component* $A_0(z)$ *is discontinuous at* S

- parametrize this behavior with (diagonal, traceless, N×N) matrices C
  - ▷ $P \equiv Tr\mathcal{P}\, \exp(i \int_0^{1/T} d\tau\, A_0(\tau, z)) = Tr\, \exp(iC(z))$

- constrain path integral to these allowed A_0

- average over all of them
  - ▷ $\langle V_k \rangle = \int [\mathcal{D}C(z)] \int [\mathcal{D}A_0][\mathcal{D}\mathbf{A}]\delta\left(P - Tr\, \exp(iC(z))\right) \exp(-S[A])$
    $= \int [\mathcal{D}C(z)] \exp(-L_x L_y\, U[C(z)])$
  - ▷ *for large loops this defines the minimum value of the effective potential* U
    $= \exp(-L_x L_y\, \sigma_k)$
    with $\sigma_k = min_{\{C(z)\}}\, U[C(z)]$

setup

- minimal profile of eff.Pot. U is realized along simplest path
  - *path in space of matrix* $C \sim 2\pi T q Y_k$, *with* $0 < q < 1$
  - *proof? [CKA: large N, N=3, N=4; no general one yet.]*
- compute U in perturbative expansion around Polyakov loop constraint
  - $A_0 = C(z) + gQ_0$ *and* $A_i = gQ_i$
- choose background field gauge
  - $S_{gf} = \frac{1}{\xi} Tr[D_\mu(C) Q_\mu]^2$
- ''do loop expansion, minimize wrt q$\Rightarrow$ get LO, NLO terms''
  - *e.g. 1-loop term* $\int [\mathcal{D}Q] \exp(-Q\, S''(C)\, Q)$
  - $S''(C)_{\mu\nu} = -D^2(C)\, g_{\mu\nu} + (1 - \frac{1}{\xi}) D_\mu D_\nu$
  - *hence* $U(C)_{1-loop} = \frac{1}{2} Tr \ln(-D^2(C)\, g_{\mu\nu}) + ghost$
  - $C = 2\pi T q Y_k$ *in* D_0, *inverse propagator is* $(2\pi T(n+q))^2 + \mathbf{p}^2$
  - *need* $\sum\!\!\!\!\!\!\int \ln[(2\pi T(n+q))^2 + \mathbf{p}^2]$
- ''recognize IR enhanced g^4 diagrams, resum $\Rightarrow$ get NNLO''
- ''magnetic sector does not contribute before g^5 $\Rightarrow$ relative g^4 computable''

q-integrals

what is a q-integral?

- smooth interpolation between bosonic and fermionic Matsubara sums
  - ▷ *here no quarks: chemical potential(s)* $\mu = 0$
  - ▷ *zero-components of 4-momenta are* $P_0 = 2\pi T(n+q)$
  - ▷ *because of overall* $\sum_{n\in\mathbb{Z}}$*, take* $q \in [0,1)$
  - ▷ *additional reflection symmetry of sum-integrals under* $q \to 1-q$
  - ▷ $q = \{0, \frac{1}{2}\}$ *corresponds to* {*bosonic,fermionic*} *sum-integrals*
- some nice properties of known bosonic computations lost
  - ▷ *2-loop sunset ''master'' sum-integral does not vanish anymore*
  - ▷ *hence 2-loop computation does not factorize into 1-loop structures*
  - ▷ *numerator structure in sum-integrals more persistent than usual*
- a simple example: 1-loop tadpole
  - ▷ $$\Lambda^{2\epsilon} \sum\!\!\!\!\!\!\int_P \frac{(P_0)^m}{(P^2)^n} = \frac{\left(\pi T^2\right)^{\frac{d+1+2\epsilon}{2}}}{(2\pi T)^{2n-m}} \left(\frac{\Lambda^2}{\pi T^2}\right)^{\epsilon} \frac{\Gamma(n-d/2)}{\Gamma(n)\Gamma(1/2)} \times$$ $$\times \Big[\zeta(2n-m-d,q) + (-1)^m \zeta(2n-m-d,1-q)\Big]$$
  - ▷ *generalized Zeta fct* $\zeta(z,q) = \sum_{n=0}^{\infty} \frac{1}{(n+q)^z} = q^{-z} + \zeta(z,q+1)$

lattice

definition of ‘t Hooft loop

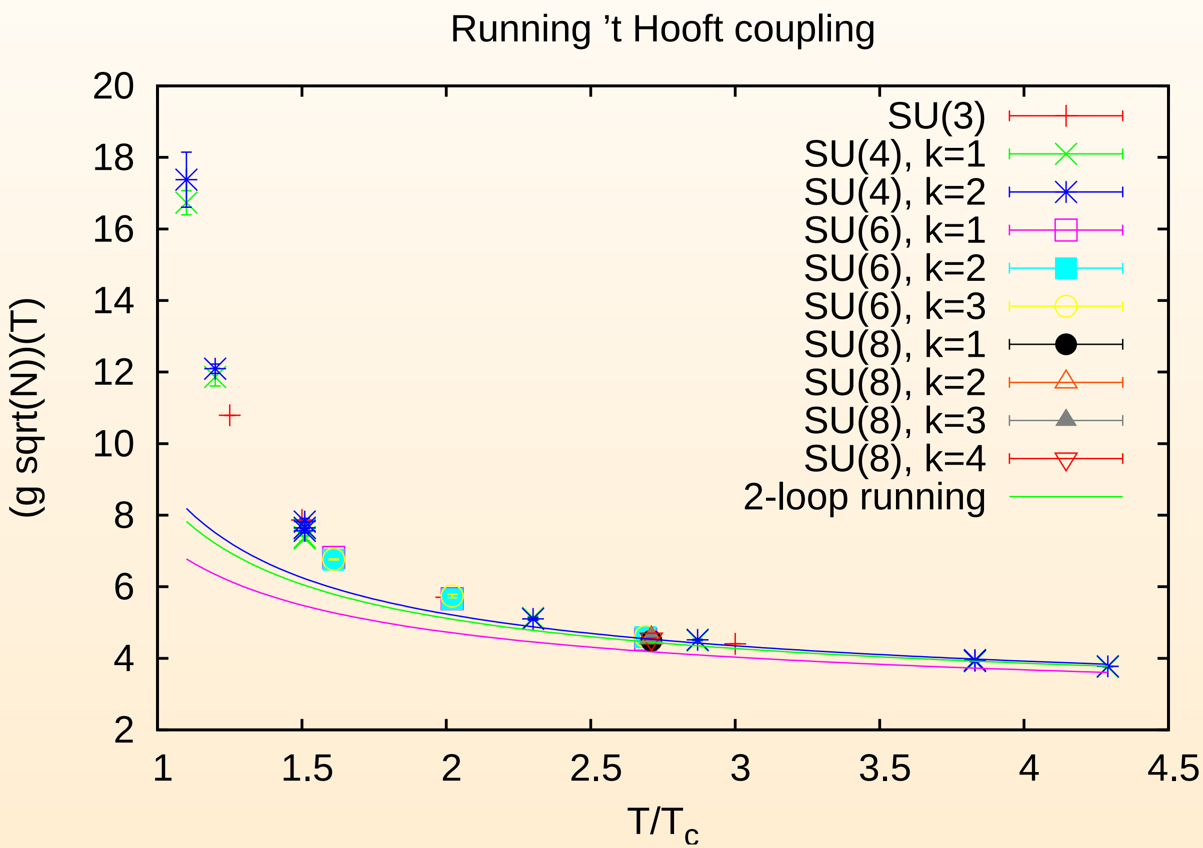
- take usual partition fct with Wilson plaquette action
  - ▷ $Z = \int dU \, \exp[-\beta \sum_P (1 - \frac{1}{N} Tr U_P)]$
- switch on ‘‘by hand’’ magnetic flux along closed contour C
  - ▷ *C is on dual lattice*
  - ▷ *multiply some U_P by non-trivial element of center group z_k*
  - ▷ *pick those belonging a P piercing the surface spanned by C*
  - ▷ *independent of surface $\rightarrow$ take simplest one*
  - ▷ *call the partition fct measured with changed action Z_C*
- ‘t Hooft loop is cost in free E to create this magnetic flux
  - ▷ $\langle V_k \rangle = \frac{Z_C}{Z}$
  - ▷ *e.g. for SU(2), $z_1 = -1$: rewrite $\langle V_1 \rangle = \langle \exp(-\beta \sum_{P_{flipped}} Tr\, U_P) \rangle$ where average is taken with standard Wilson action*

difficult measurement

- observable exponentially suppressed
  - ▷ *sampling problem; increases with loop size*

‘t Hooft scheme coupling

- from data, can even attempt to extract non-perturbative coupling [dF/L/N 05]



  - ▷ *extracted from LO* $\sigma_k \sim 1/\sqrt{g^2 N}$
  - ▷ *lines: 2loop running coupling,* $T_c/\Lambda_{\overline{MS}} = (1.10, 1.25, 1.35)$

- not entirely universal: fails for e.g.pressure

Conclusions

- there is an object V_k that serves as order-parameter in hot YM
- tensions $\sigma_k \sim \ln V_k$ hard to measure on the lattice
- lattice measurements indicate universal behavior
- weak-coupling expansion seems to get ratios correct
  but not normalization close to T_c
- so go ahead and compute the NNNLO term
- σ_k/σ_1 are tests for formulations of YM derived from e.g. string theory