Plasma instabilities in non-abelian gauge theories

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SEWM 2008, Amsterdam, August 27, 2008

J. Berges, S. Scheffler, D. Sexty, PRD 77 (2008), 034504
(arXiv:0712.3514 [hep-ph])

Outline of the talk

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Introduction

- Hydrodynamic calculations assuming $\tau_0 \simeq 0.5 - 2\,\mathrm{fm}/c$ can describe RHIC data well (Luzum, Romatschke, arXiv:0804.4015 [nucl-th])
- Rapid isotropization before τ_0 is an essential ingredient for the applicability of (almost) ideal hydrodynamics (Arnold et al., PRL 94)

Numerical approaches:

1. Soft classical gauge fields + hard classical particles (Arnold, Moore, Yaffe; Rebhan, Romatschke Strickland; Dumitru, Nara, Strickland; Bödeker, Rummukainen)
2. Classical statistical gauge field evolution (Romatschke, Venugopalan; this work)

Outline of the talk

Approach

Setup:

- Classical statistical limit of pure SU(2)- gauge theory
- Static geometry, i. e. no expansion
- Lattice discretization of the fields
- Anisotropic initial conditions

Classical statistical approximation reliable for high occupation numbers (Aarts, Berges, PRL 88; Arrizabalaga, Smit, Tranberg, JHEP 0410; Berges, Gasenzer, PRA 76)

Implementation

Use common lattice discretization scheme

Link variables: $U_{x,\mu} := e^{igaA_\mu(x)}$
Plaquette variables: $U_{x,\mu\nu} := U_{x,\mu} U_{(x+\hat{\mu}),\nu} U^{-1}_{(x+\hat{\nu}),\mu} U^{-1}_{x,\nu}$

Dynamics from Wilson- lattice action in Minkowski- spacetime:

$$S = \beta_s \sum_x \sum_{\substack{i,j \\ i<j}} \left\{ \frac{1}{2\mathrm{tr}\mathbb{1}} \mathrm{tr}\left(U_{x,ij} + U^\dagger_{x,ij}\right) - 1 \right\} - \beta_0 \sum_x \sum_i \left\{ \frac{1}{2\mathrm{tr}\mathbb{1}} \mathrm{tr}\left(U_{x,0i} + U^\dagger_{x,0i}\right) - 1 \right\}$$

where

$$\beta_0 := \frac{2\gamma \mathrm{tr}\mathbb{1}}{g_0^2}\ ,\ \beta_s := \frac{2\mathrm{tr}\mathbb{1}}{\gamma g_s^2}\ ,\ \gamma := \frac{a_s}{a_t}$$

We use temporal axial gauge $A_0 \equiv 0$ and $g_0 = g_s = 1$.
Variation w. r. t. spatial links $\Rightarrow$ Equations of motion

Initial conditions

Compute physical quantities (e. g. correlators) according to

$$\langle\, A(t,\mathbf{x})A(t',\mathbf{y})\,\rangle = \int \mathcal{D}A(t_0)\,\mathcal{D}\dot{A}(t_0)\,P[A(t_0),\dot{A}(t_0)]\,A(t,\mathbf{x})A(t',\mathbf{y})$$

Initial probability functional $P[A(t_0),\dot{A}(t_0)]$:

$$\langle\, A^a_j(0,\mathbf{p})A^b_k(0,-\mathbf{p})\,\rangle \sim C\delta^{ab}\delta_{jk}\exp\{-\frac{p_x^2+p_y^2}{2\Delta_x^2}-\frac{p_z^2}{2\Delta_z^2}\}\,\delta(\dot{A}(t_0))$$

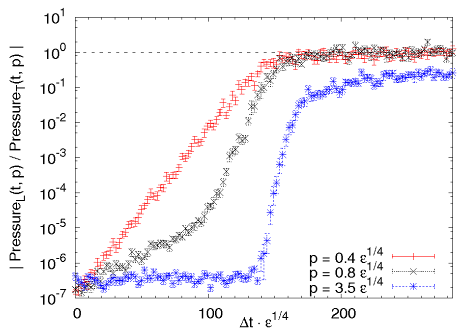
N.B.:

- $\Delta_x \gg \Delta_z$ (distribution $\delta(p_z)-$ like on the lattice)
- $\partial_t\mathbf{A}(t=0)\equiv 0 \;\Rightarrow$ Gauss constraint fulfilled
- Amplitude C determined from fixed average energy density ϵ

Outline of the talk

1. Introduction
2. Classical statistical gauge theory on the lattice
3. Fast dynamics
4. Slow dynamics
5. Conclusions & outlook

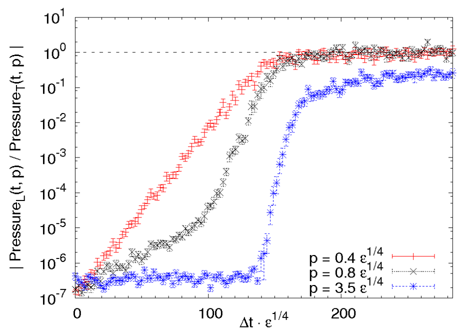
Fast dynamics: Instabilities



$$T_{33}(t,\mathbf{x}) = \frac{1}{2a_s^4 g^2}\Big\{\gamma^2\Big(\big(1-\frac{1}{2}\,\mathrm{tr}\,U_{x,01}\big) + \big(1-\frac{1}{2}\,\mathrm{tr}\,U_{x,02}\big) - \big(1-\frac{1}{2}\,\mathrm{tr}\,U_{x,03}\big)\Big) + \big(1-\frac{1}{2}\,\mathrm{tr}\,U_{x,23}\big) + \big(1-\frac{1}{2}\,\mathrm{tr}\,U_{x,31}\big) - \big(1-\frac{1}{2}\,\mathrm{tr}\,U_{x,12}\big)\Big\}$$

(Analogous formula for $T_{11}(t,\mathbf{x})$ and $T_{22}(t,\mathbf{x})$)

Fast dynamics: Instabilities



⇝ Fourier transform w. r. t. $\mathbf{x}$ and obtain $T_{11}(t, \mathbf{p})$, $T_{22}(t, \mathbf{p})$, $T_{33}(t, \mathbf{p})$.

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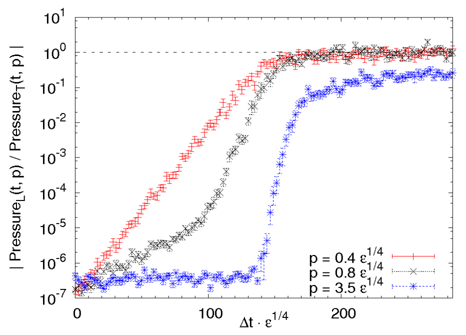

Plotted is

$$\left|\frac{T_{33}(t,\mathbf{p}_{||})}{T_{\perp}(t,\mathbf{p}_{\perp})}\right|_{|\mathbf{p}_{||}|=|\mathbf{p}_{\perp}|}$$

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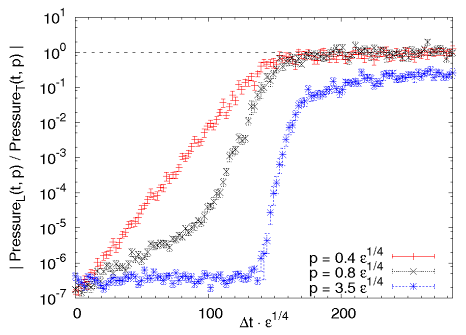

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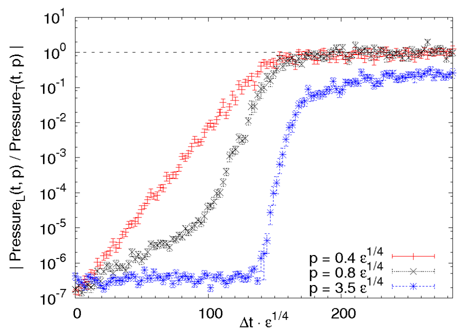
N. B.:

1. Narrow band of low momentum modes unstable initially: *primary instabilities*
2. Later, fast growth in a broad range in the UV: *secondary instabilities*
3. Diagrammatic analysis of secondary instabilities (cf. PRD 77)
4. Findings qualitatively similar to parametric resonance in scalar field theory

Fast dynamics: Instabilities



N. B.:

5. Instabilities drive the system towards an isotropic state
6. IR- regime $(0 < p \lesssim \epsilon^{1/4} \simeq 1\,\mathrm{GeV})$ becomes quickly isotropic
7. $1/\gamma^{(\mathrm{pr})}_{\mathrm{max}} \sim \mathcal{O}(1\ \mathrm{fm/c})$ for primary instabilities, assuming $\epsilon \sim 30\,\mathrm{GeV/fm}^3$
8. The UV only isotropizes at very late times

Plotted is

$$\left|\frac{T_{33}(t,\mathbf{p}_{||})}{T_{\perp}(t,\mathbf{p}_{\perp})}\right|_{|\mathbf{p}_{||}|=|\mathbf{p}_{\perp}|}$$

Outline of the talk

Slow dynamics: Non-thermal scaling behaviour (I)

Consider the correlation function:

$$F(t,t,\mathbf{p}) := \int d^3x\, e^{-i\mathbf{p}\cdot\mathbf{x}} \langle\, A(t,\mathbf{x})A(t,0)\,\rangle \;\sim (n(\mathbf{p}) + \frac{1}{2})/\omega(\mathbf{p})$$

Want to know:

- Is there a stationary solution such that $F(t,t,\mathbf{k}) \sim |\,\mathbf{k}\,|^{-(1+\alpha)}$?
- If yes, what is the value of the exponent α?

UV- power law spectra in F / in the particle number $n(\mathbf{p})$ are well-known in early universe cosmology (Micha, Tkachev, PRL 90; Felder, Kofman, PRD 63; Berges, Rothkopf, Schmidt, PRL 101)

For non-Abelian plasmas, UV spectra with an exponent $\alpha = 2$ have been found by other groups (e. g. Arnold, Moore, PRD 73; Strickland, J. Phys. G. 34)

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Slow dynamics: Non-thermal scaling behaviour (II)

What do we expect for α from field theory?

Dropping colour indices, split up the full propagator according to

$$G_{\mu\nu}(x,y) \equiv F_{\mu\nu}(x,y) - \frac{i}{2}\rho_{\mu\nu}(x,y)\,\mathrm{sign}(x^0 - y^0)$$

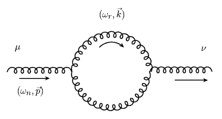
where

$$F_{\mu\nu}(x,y) \equiv \frac{1}{2}\langle\,\{A_\mu(x), A_\nu(y)\}\,\rangle \text{ and } \rho_{\mu\nu}(x,y) \equiv i\langle\,[A_\mu(x), A_\nu(y)]\,\rangle\;.$$

The evolution equation for F (Berges, PRD 70) reads:

$$\{(D_0^{-1})_\mu^{\ \kappa} + M^2(x)\}\,F_{\kappa\nu}(x,y) = \int_{t_0}^{z_0} d^3z \Big\{\Sigma^{(\rho)}_{\mu\kappa}(x,z)F^{\kappa}_{\ \nu}(z,y) - \Sigma^{(F)}_{\mu\kappa}(x,z)\rho^{\kappa}_{\ \nu}(z,y)\Big\}$$

The leading order contribution to the self-energies on the right hand side arises from this diagram:



Slow dynamics: Non-thermal scaling behaviour (III)

We are interested in a stationary solution satisfying

$$\int d^3p \left\{ \Sigma^{(\rho)}_{\mu\kappa}(p) F^{\kappa}{}_{\nu}(p) - \Sigma^{(F)}_{\mu\kappa}(p) \rho^{\kappa}{}_{\nu}(p) \right\} \overset{!}{=} 0 \tag{1}$$

and showing scaling behaviour according to

$$F(sp) = s^{-(2+\alpha)} F(p) \text{ and } \rho(sp) = s^{-2} \rho(p) \; . \tag{2}$$

N. B.: This implies $F(t, s\mathbf{p}) = s^{-(1+\alpha)} F(t, \mathbf{p})$.

Assuming (2) and interchanging momenta by means of a Zakharov transformation similarly as in (Berges et al., PRL 101) shows that (1) is fulfilled if

$$\alpha = \frac{3}{2} \; .$$

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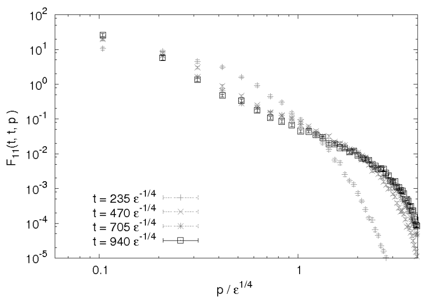
Slow dynamics: Non-thermal scaling behaviour (IV)

Compute $F(t, t, \mathbf{p})$:

- on 128^3- lattice
- after the instabilities have saturated
- in Coulomb gauge using the stochastic overrelaxation algorithm (Cucchieri, Mendes, NPB 471)
- and fit to $const. \times k^{-(1+\alpha)}$



$$\begin{aligned} 1+\alpha &= (1+1.52) \pm 0.07 \\ \chi^2/\text{d.o.f.} &= 1.1 \\ \text{Fit range:} &\quad [0.6\,\epsilon^{0.25}, 2.0\,\epsilon^{0.25}] \end{aligned}$$

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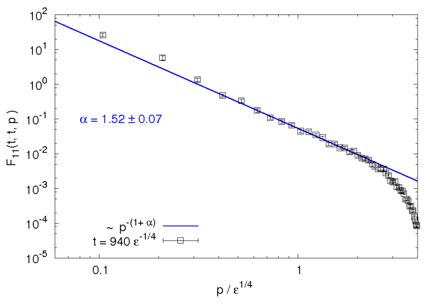

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Outline of the talk

Conclusions & outlook

Main results:

- Characteristic time scales of plasma instabilities:
$$1/\gamma_{\mathrm{max}} \sim 1\ \mathrm{fm/c}$$
- Early isotropization at low characteristic momenta
$$p \lesssim 1\ \mathrm{GeV}$$
- UV- power law spectrum with perturbative index
$$\alpha = 1.5$$

Open questions:

- Is this sufficient for hydrodynamics?
- Quantum corrections for the UV- modes?
- IR- scaling behaviour?

Thanks for your attention.

Appendix: Diagrammatic analysis of secondaries (I)

Correlation function $F^{ab}_{\mu\nu}(x,y) := \langle A^a_\mu(x) A^b_\nu(y) \rangle$ obeys a 2PI-evolution equation:

$$\{(D_0^{-1})_\mu^{\ \kappa} + M^2(x)\}\, F_{\kappa\nu}(x,y) = \int_{t_0}^{z_0} d^3z \Big\{ \Sigma^{(\rho)}_{\mu\kappa}(x,z) F^{\kappa}_{\ \nu}(z,y) - \Sigma^{(F)}_{\mu\kappa}(x,z) \rho^{\kappa}_{\ \nu}(z,y) \Big\}$$

Try to identify times when certain diagrams make $\mathcal{O}(1)$- contributions to the correlation function $F^{ab}_{\mu\nu}(x_0, y_0, \mathbf{p})$ in analogy to parametric resonance in scalar theories (Berges and Serreau, PRL 91) .

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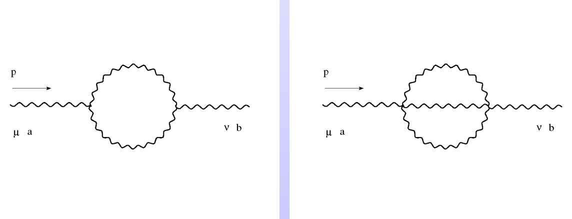
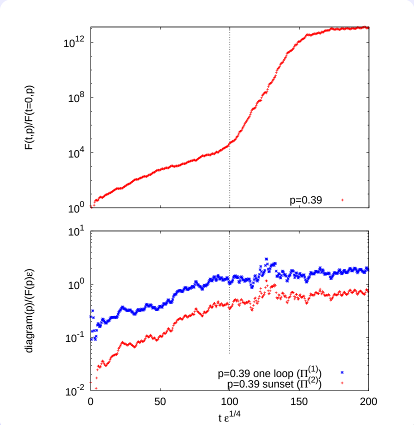


Appendix: Diagrammatic analysis of secondaries (II)



Upper panel:

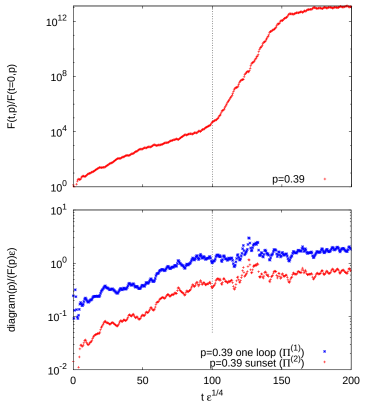
$$F(t,t,\mathbf{p})/F(t=0,t'=0,\mathbf{p})$$

Lower panel:

$$\left|\frac{\text{diagram}(t,t,\mathbf{p})}{F(t,t,\mathbf{p})\cdot\epsilon}\right|$$

Same $\mathbf{p}\,||\,\hat{z}$ chosen in both panels.

Appendix: Diagrammatic analysis of secondaries (II)



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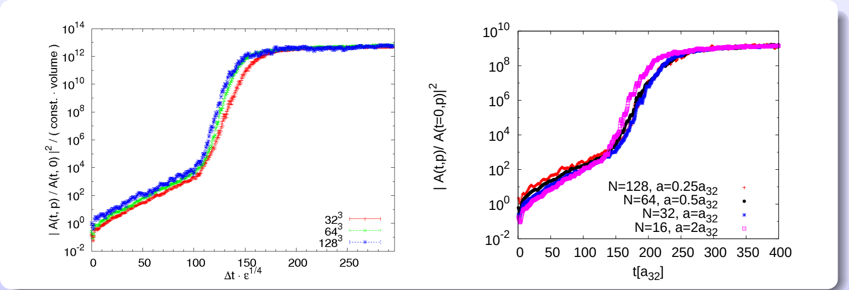
Lower panel:

$$\left|\frac{\text{diagram}(t,t,\mathbf{p})}{F(t,t,\mathbf{p})\cdot\epsilon}\right|$$

Onset of secondaries coincides with fluctuation effects becoming large, analogous to parametric resonance in scalar theories.

Appendix: Volume and cutoff independence



Checking for possible volume and cutoff sensitivities of the results.