Hydrodynamics and Heavy-Ion Collisions

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Outline

What is Hydrodynamics

Hydrodynamics $=$ Energy-Momentum Conservation

$$\partial_\mu T^{\mu\nu} = 0$$

Ideal Hydro

- Ideal hydro Energy-Momentum Tensor

$$T_0^{\mu\nu} = \epsilon u^\mu u^\nu - p\Delta^{\mu\nu},$$

$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$

- Continuity Equation:

$$u_\nu \partial_\mu T_0^{\mu\nu} = 0$$

- Euler Equation:

$$\Delta^\alpha_\nu \partial_\mu T_0^{\mu\nu} = 0$$

Viscous Hydro

- Viscous Hydro: Departures from Equilibrium
- Viscous hydro Energy-Momentum Tensor

$$T^{\mu\nu} = T_0^{\mu\nu} + \Pi^{\mu\nu}$$

- $\Pi^{\mu\nu}$ contains first, second, ...spatial gradients

Gradient Expansion Hierachy

1. Zeroth Order: Ideal Hydrodynamics (“Euler equation”)
2. First-Order: Viscous Hydrodynamics (“Navier-Stokes equation”)
3. Second-Order: Viscous Hydrodynamics (e.g. “Müller-Israel-Stewart theory”)

Important Parameters for Hydro

- Mean free path λ_{mfp} (should be smaller than system size L)
- Reynolds number

$$Re \sim \frac{\epsilon + p}{|\Pi^{\mu\nu}|} \sim \frac{sT}{\eta/L}$$

Zeroth Order Hydro (Ideal Hydro)

- Ideal Hydro has no viscosity

$$\Pi^{\mu\nu} = 0$$

- Small *Re* correspond to laminar (“smooth”) flow
- Large *Re* correspond to turbulent flow
- Ideal Hydro: $\Pi^{\mu\nu} = 0$, $Re = \infty$
- Ideal Hydro is (strictly speaking) a sick theory!

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First Order Viscous Hydro (Navier-Stokes)

- For simplicity, consider only shear viscosity η
- Navier-Stokes equation:

$$\Pi^{\mu\nu} = \eta\langle\nabla^\mu u^\nu\rangle,$$

  where

$$\langle\nabla^\mu u^\nu\rangle = \nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\nabla_\alpha u^\alpha$$

- *Re* finite, can get laminar flow

First Order Viscous Hydro (Navier-Stokes)

- Consider small perturbations $\delta\epsilon(t,x)$, $\delta u^i(t,x)$
- Hydro Equations $\Delta^y_\nu \partial_\mu T^{\mu\nu} = 0$ give

$$\partial_t \, \delta u^y(t,x) - \frac{\eta}{\epsilon+p}\partial_x \, \delta u^y(t,x) = 0$$

- In Fourier-Space,

$$\left(i\omega - \frac{\eta}{\epsilon+p}k^2\right)\delta\tilde{u}^y(\omega,k) = 0$$

- “Dispersion relation”

$$|\omega| = k^2\frac{\eta}{\epsilon+p}$$

First Order Viscous Hydro (Navier-Stokes)

- Diffusion "group velocity"

$$v_T \equiv \frac{d|\omega|}{dk} = 2k \frac{\eta}{\epsilon + p}$$

- For large $k \to \infty$,

$$v_T \to \infty$$

- Perturbations propagate at superluminal speed!
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Second Order Viscous Hydro

- Problematic modes are $k \gg 1$: outside of hydrodynamic regime
- Nevertheless problematic in numeric problems (almost always for hydro!)
- Regulate by hand?
- Look for regulator from microscopic physics: 2nd order gradients!

Second Order Viscous Hydro

History: Müller-Israel-Stewart theory

- Change the Navier-Stokes relation

$$\Pi^{\mu\nu} = \eta \langle \nabla^\mu u^\nu \rangle,$$

to

$$\underbrace{\tau_\Pi \Delta^\mu_\alpha \Delta^\nu_\beta D \Pi^{\alpha\beta}}_{\sim \tau_\Pi \partial_t \Pi^{\mu\nu}} + \Pi^{\mu\nu} = \eta \langle \nabla^\mu u^\nu \rangle + \ldots$$

- Differential equation for $\Pi^{\mu\nu}$ with new parameter τ_Π (=second order transport coefficient)

History: Müller-Israel-Stewart theory

- For weak-coupling QCD, $\tau_\Pi = 6\eta/(\epsilon + p)$
- Consequence: transverse perturbations move with

$$\lim_{k\to\infty} v_T^2 = \frac{\eta}{(\epsilon + p)\tau_\Pi} = \frac{\eta}{s}\frac{1}{T\tau_\Pi}$$

so that $v_T^2 \simeq \frac{1}{6} < 1$!

History: Müller-Israel-Stewart theory

- MIS: no ordering principle (many 2nd order terms, which to keep?)
- Matching of MIS to strongly coupled field theories?
- Regime of applicability of MIS (large gradients?)

Recently clarified:

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Recently clarified:

Hydrodynamics, conformal invariance, and holography

- Bulk viscosity $\zeta \sim T_\mu^\mu$ (e.g. Kharzeev, Tuchin 07)
- Considering $\zeta = 0$ means considering $T_\mu^\mu = 0$ ("conformal fluids")
- Consider hydrodynamics of conformal fluids, e.g. fluids with
$$T^{\mu\nu} \to e^{6\omega} T^{\mu\nu}$$
when $g_{\mu\nu} \to e^{-2\omega} g_{\mu\nu}$.

Hydrodynamics, conformal invariance, and holography

Write down all terms to 2nd order in gradients in d dimensions (c.f. Andrei's and Paul's talk?):

$$\begin{aligned}
\Pi^{\mu\nu} &= -\eta\sigma^{\mu\nu} - \tau_\Pi\left[{}^{<}D\Pi^{\mu\nu>} + \frac{d}{d-1}\Pi^{\mu\nu}(\nabla\cdot u)\right] \\
&\quad + \kappa\left[R^{<\mu\nu>} - (d-2)u_\alpha R^{\alpha<\mu\nu>\beta}u_\beta\right] \\
&\quad + \frac{\lambda_1}{\eta^2}\Pi^{<\mu}{}_\lambda\Pi^{\nu>\lambda} - \frac{\lambda_2}{\eta}\Pi^{<\mu}{}_\lambda\Omega^{\nu>\lambda} + \lambda_3\Omega^{<\mu}{}_\lambda\Omega^{\nu>\lambda}\,.
\end{aligned}$$

[Baier, Romatschke, Son, Starinets, Stephanov 07]

Second Order Conformal Viscous Hydro

- Five allowed second-order transport coefficients: $\tau_\Pi, \kappa, \lambda_1, \lambda_2, \lambda_3$
- More general than Müller-Israel-Stewart (which has $\kappa = 0$)
- Weakly coupled plasmas (Boltzmann equation):

$$\tau_\Pi = 6\frac{\eta}{\epsilon + p}, \kappa = 0, \lambda_1 = ?, \lambda_2 = -2\tau_\Pi \eta, \lambda_3 = 0$$

- Strongly coupled $\mathcal{N} = 4$ SYM:

$$\tau_\Pi = \frac{2(2 - \ln 2)\eta}{\epsilon + p}, \kappa = \frac{\eta}{\pi T}, \lambda_1 = \frac{\eta}{2\pi T}, \lambda_2 = -\frac{\eta \ln 2}{\pi T}, \lambda_3 = 0$$

[Baier, Romatschke, Son, Starinets, Stephanov 07], [Bhattacharyya, Hubeny, Minwalla, Rangamani 07], [Natsuume, Okamura, 07]

Second Order Conformal Viscous Hydro

- For weakly coupled plasmas

$$\lim_{k\to\infty} v_L, v_T < 1$$

- For stronlgy coupled $\mathcal{N} = 4$ SYM

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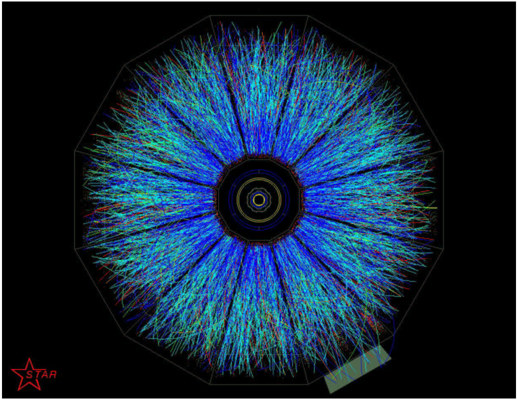
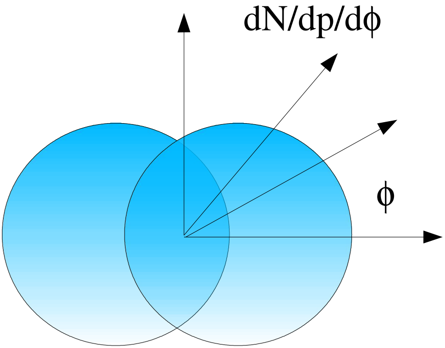
Let’s use viscous hydro for Heavy-Ion Collisions!

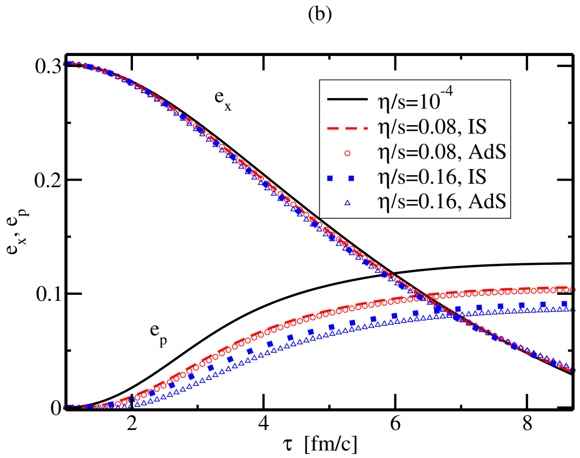


Outline

Can we apply hydro at RHIC?



Can we apply hydro at RHIC?
Change 2nd order vs. 1st order



[Luzum, Romatschke 08]

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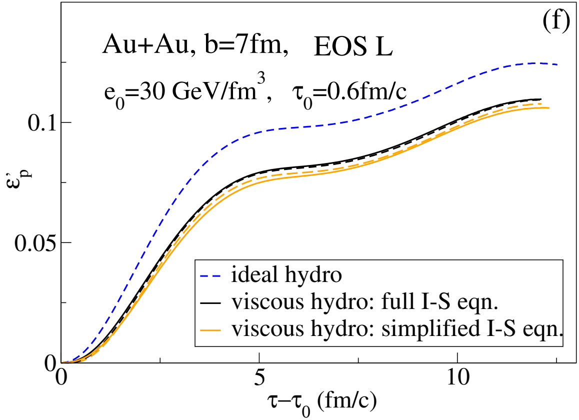

[Song, Heinz 08]

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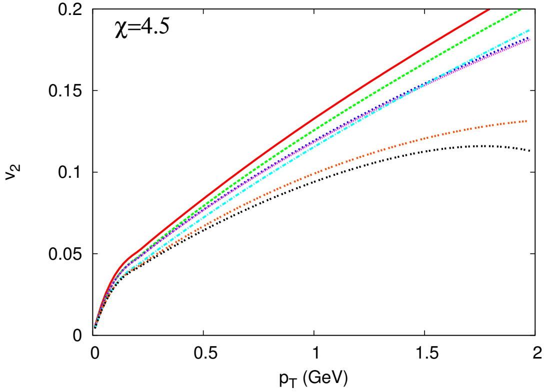
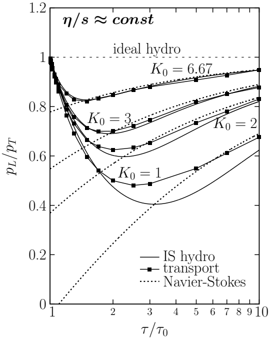


[Dusling, Teaney 07]

Can we apply hydro at RHIC?

0+1d hydro vs. transport

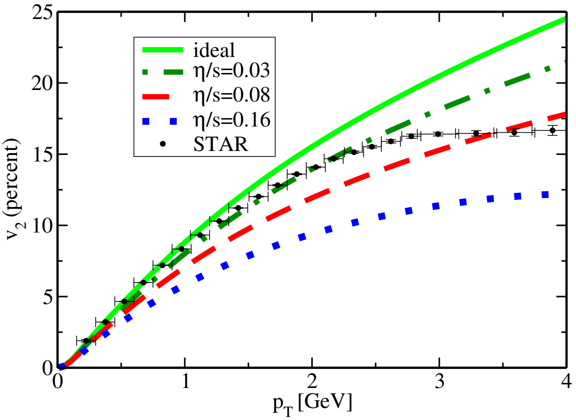


[Huovinen, Molnar 08]

Can we apply hydro at RHIC?

Answer: Yes, if $\eta/s < 0.5$

Do we agree on size of viscous effects?



[Romatschke[2] 07]: $\sim$ 25% at $p_T = 1.5$ GeV

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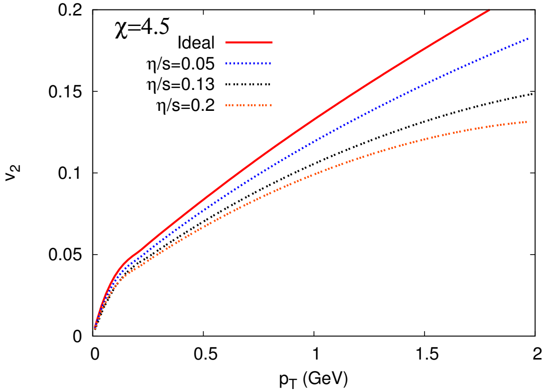

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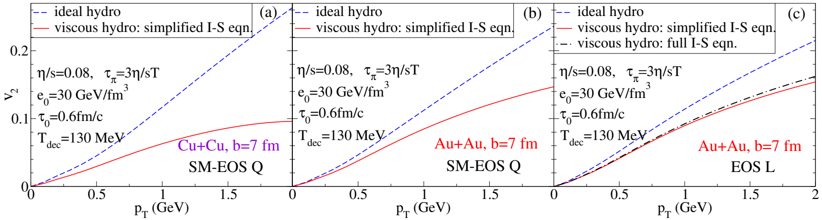

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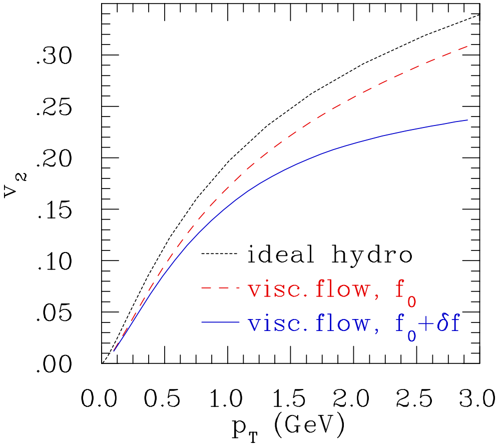
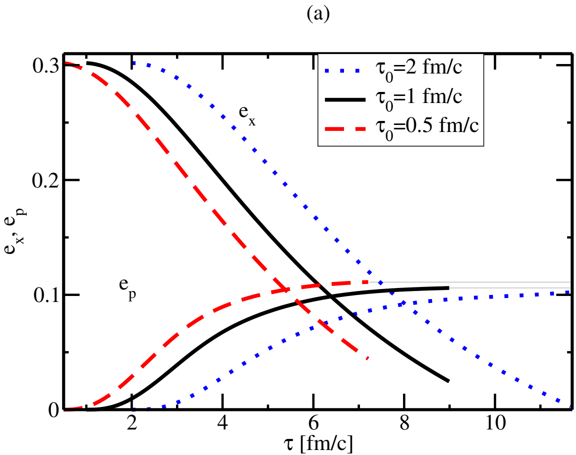


[Molnar, Huovinen 08]: $\sim$ 20% at $p_T = 1.5$ GeV

Do we agree on size of viscous effects?

Answer: Yes, we (now!) do

Does hydro imply early thermalization?



[Luzum, Romatschke 08]

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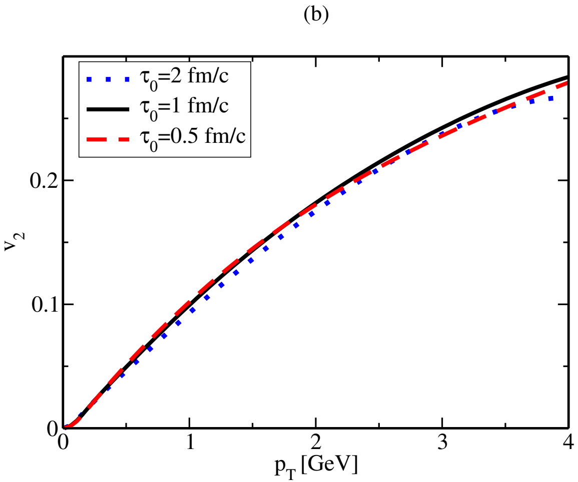

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Does hydro imply early thermalization?

Answer: No

Answer assuming pre-hydro is free-streaming: Maybe

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Things to know about Hydro @ RHIC

For any hydrodynamic model of a heavy-ion collision

- Hydrodynamics = differential equations. Need to fix initial/boundary conditions!
- the time when to start the hydrodynamic evolution
- the initial distribution of energy density (Glauber? CGC?)
- the equation of state for QCD (lattice!)
- the freeze-out procedure (Cooper-Frye?)
- There is much more to RHIC hydro than just fluid dynamics!

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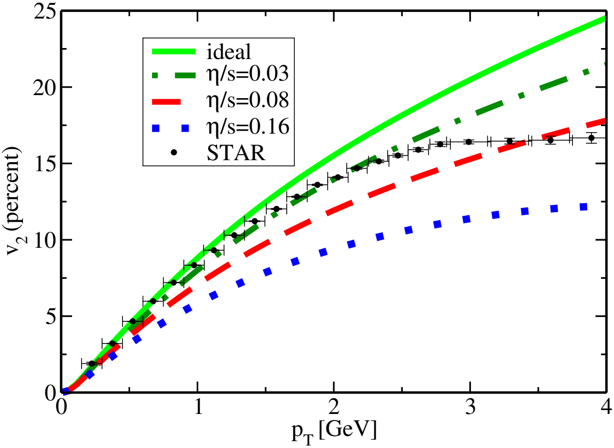
Hydro @ RHIC: mode d'emploi

Use (some) RHIC data to fix freedom:

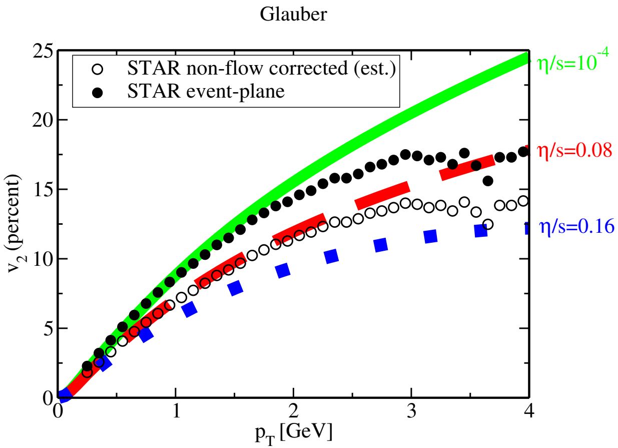
- use centrality dependence of multiplicity to fix the initial distribution of energy density
- use centrality dependence of $< p_T >$ to fix hydro starting/stopping time/temperature
- Once this is done, v_2 in the hydro model is fixed and can be compared to data ("prediction")

Min. Bias v2 (Glauber)



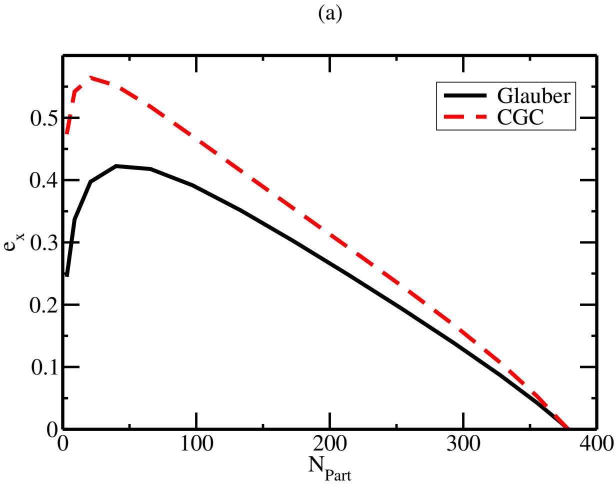
[Romatschke[2] 07]

New Min Bias v2 data



[Luzum, Romatschke 08]

Eccentricity: Glauber vs CGC



[Luzum, Romatschke 08], adaption of [Drescher *et* al. 06]

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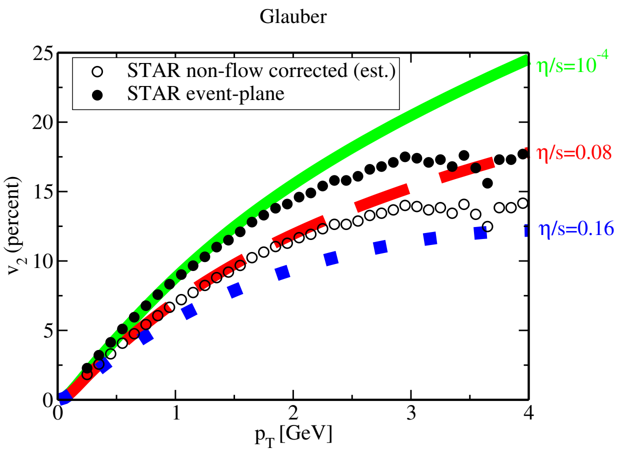
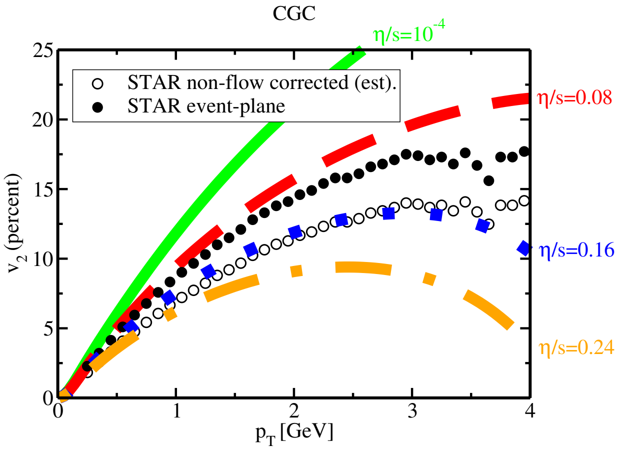


[Luzum, Romatschke 08]

Min. Bias v2 (CGC)



[Luzum, Romatschke 08]

What is η/s at RHIC

Answer: $\eta/s < 0.5$ (all groups)

Answer: $\eta/s = 0.1 \pm 0.1(th.) \pm 0.08(exp)$ for *our* model

Summary

- Viscous hydro theory is solid
- Viscous hydro groups agree on size of v_2 reduction
- Extraction of η/s dominated by uncertainties from ICs and non-flow
- Open Problem: above which Reynolds number ($\propto s/\eta$) onset of turbulence?

Bonus Material

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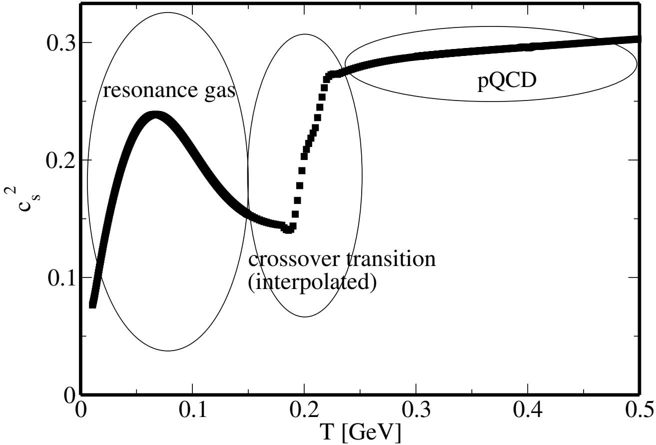
Speed of Sound from Laine and Schröder, PRD73

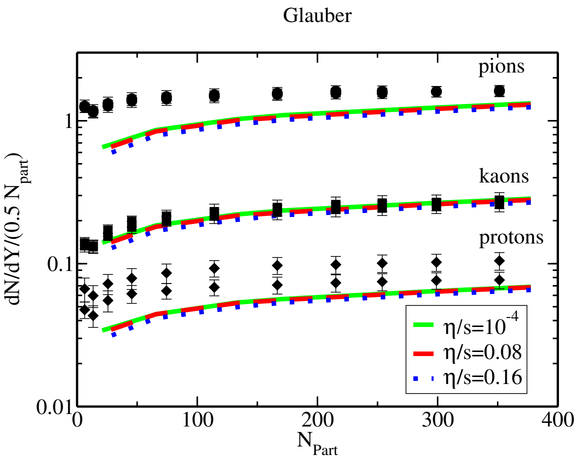


Multiplicity in Viscous Hydro

	$\frac{dN_{\pi,\text{visc}}}{dy} / \frac{dN_{\pi,\text{ideal}}}{dy}$	$\frac{dN_{K,\text{visc}}}{dy} / \frac{dN_{K,\text{ideal}}}{dy}$
$\eta/s = 0.08$	1.06	1.06
$\eta/s = 0.16$	1.12	1.12
$\eta/s = 0.24$	1.18	1.19
$\eta/s = 0.32$	1.23	1.23
$\eta/s = 0.40$	1.28	1.28

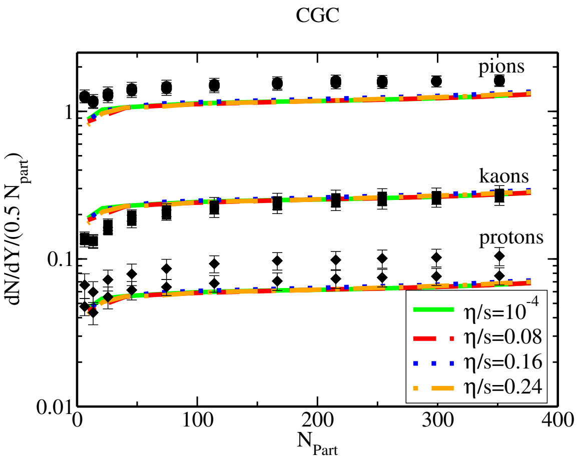
For $\tau_0 = 1$ fm/c, viscous hydro creates $\sim 0.75\,\eta/s$ more final multiplicity!

Multiplicity (Glauber)



[Luzum, Romatschke 08]

Multiplicity (CGC)



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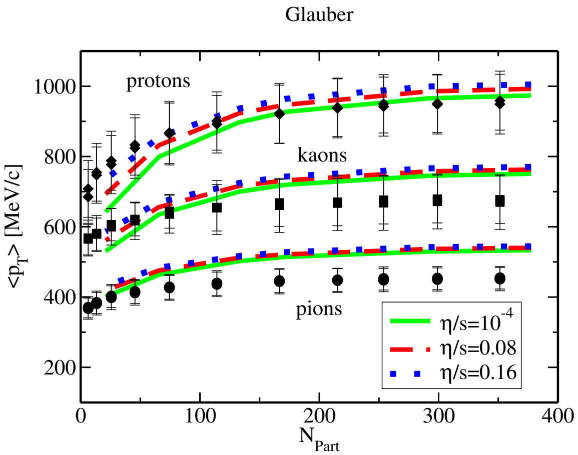
Mean momentum (Glauber)



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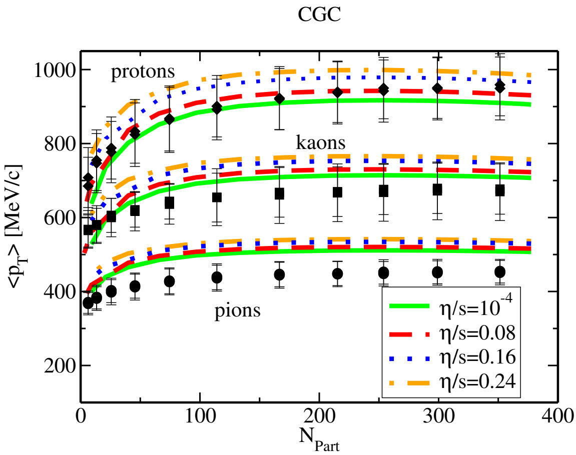

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