Nonabelian plasma instabilities in Bjorken expansion

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work in collaboration with:

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Quark-gluon plasma at RHIC

Surprises at RHIC (judging from hydro simulations):

- Very early thermalization/isotropization
- Very low shear viscosity

New paradigm: **sQGP**

(very successful toy model:

maximally supersymmetric large-N_c YM theory at infinite 't Hooft coupling from AdS/CFT)

But:

wQGP (thermal pQCD)

not yet fully understood, especially far from equilibrium!

furthermore: RHIC close to phase transition (reason for sQGP?)

LHC will reach $\gtrsim 3T_c$ — wQGP to be discovered there?

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clean theoretical laboratory: $g \ll 1$ (**s**uper-**e**xtra-**w**eak-**QGP**)

Scales of wQGP

- T: energy of hard particles
- gT: thermal masses, Debye screening mass,
  Landau damping
- g^2T: magnetic confinement, color relaxation, rate for small angle scattering
- g^4T: rate for large angle scattering, $\eta^{-1}T^4$

Effective theory at scale gT: Hard-(Thermal-)Loop Effective Action

[Frenkel, Taylor & Wong; Braaten & Pisarski 1991

equivalent to: gauge-covariant Boltzmann-Vlasov

[Blaizot & Iancu 1993, Kelly, Liu, Lucchesi & Manuel 1994

in particular required for:

- Bottom-up thermalization [Baier, Mueller, Schiff & Son 2000]

$$t_{eq} \propto g^{-13/5}$$

- Shear viscosity [Arnold, Moore & Yaffe]

$$(\eta/s)^{-1} = g^4 \ln(1/g) f(\ln(1/g))$$

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  $$(\eta/s)^{-1} = g^4 \ln(1/g) f(\ln(1/g)) + (\eta/s)^{-1}_{\text{anomalous}}$$

  [Asakawa, Bass & Müller, PRL 96 ('06) 252301

Hard (Thermal) Loops — Boltzmann-Vlasov

With color-neutral background distribution $v \cdot \partial\, f_0(\mathbf{p},\mathbf{x},t) = 0$, $v^\mu = p^\mu/p^0$

gauge covariant Boltzmann-Vlasov (collisionless):

$$v \cdot D\, \delta f_a(\mathbf{p},\mathbf{x},t) = g v_\mu F_a^{\mu\nu} \partial_\nu^{(p)} f_0(\mathbf{p},\mathbf{x},t) = -g(\mathbf{E}_a + \mathbf{v} \times \mathbf{B}_a) \cdot \nabla_{\mathbf{p}} f_0,$$

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So far: mostly stationary $f_0(\mathbf{p})$ with $\partial_\mu f_0 \equiv 0$

- isotropic: $f_0(\mathbf{p}) = f_0(|\mathbf{p}|)$, $\nabla_{\mathbf{p}} f_0 \propto \mathbf{v}$

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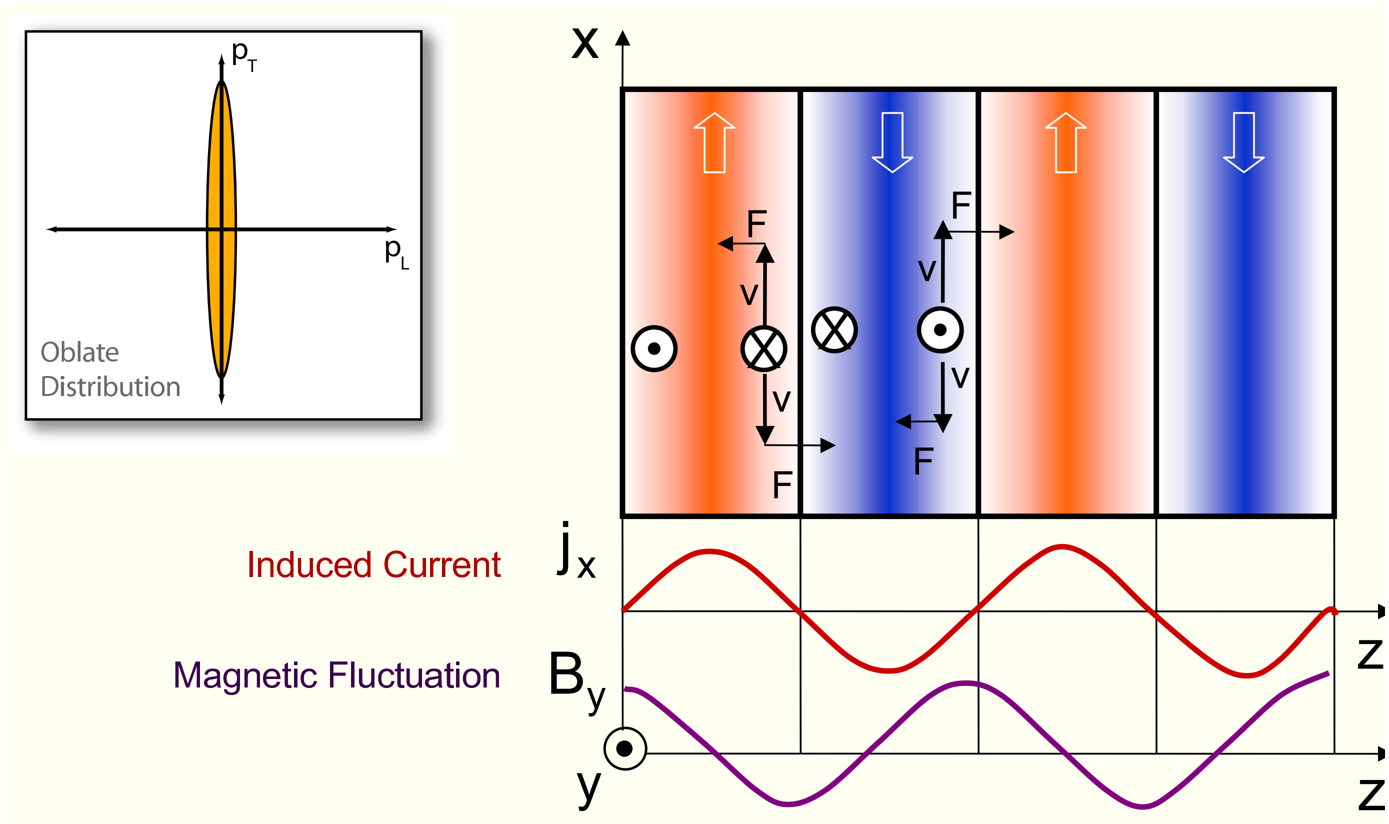
- anisotropic $f_0(\mathbf{p})$, $\nabla_\mathbf{p} f_0 \not\propto \mathbf{v}$

  $v \cdot D\, \delta f_a(\mathbf{p}, \mathbf{x}, t) = -g(\mathbf{E}_a + \mathbf{v} \times \mathbf{B}_a) \cdot \nabla_\mathbf{p} f_0$ unstable!

Filamentation (Weibel) instabilities



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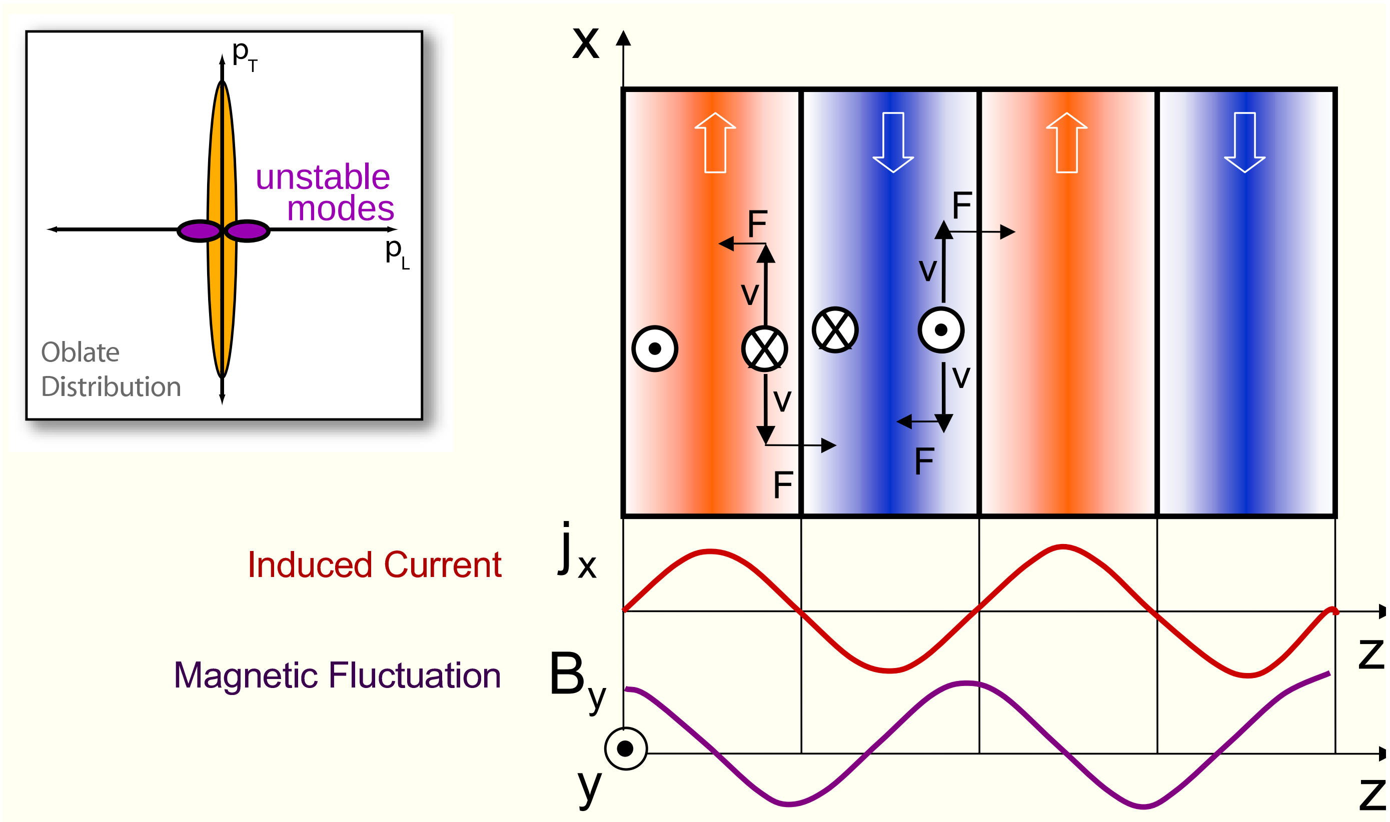

Hard Loop Effective Theory

Auxiliary field formulation: [Nair; Blaizot & Iancu 1994; Mrówczyński, AR & Strickland 2004]

$$\delta f^a(x;p) = -gW^a_\mu(t,\mathbf{x};\mathbf{v})\partial^\mu_{(p)} f_0(\mathbf{p})$$

$$[v \cdot D(A)]W_\mu(x;\mathbf{v}) = F_{\mu\gamma}(A)v^\gamma$$

$$v^\mu \equiv p^\mu/|\mathbf{p}| = (1,\mathbf{v}$$

$$j^\mu(x) = -g^2 \int \frac{d^3p}{(2\pi)^3}\frac{1}{2|\mathbf{p}|}\, p^\mu \,\frac{\partial f(\mathbf{p})}{\partial p^\nu} W^\nu(x;\mathbf{v}),$$

Hard Loop effective theory: (hard) scale $|\mathbf{p}|$ integrated out

Discretized Hard Loop Effective Theory

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for real-time lattice simulation: discretize also velocity space

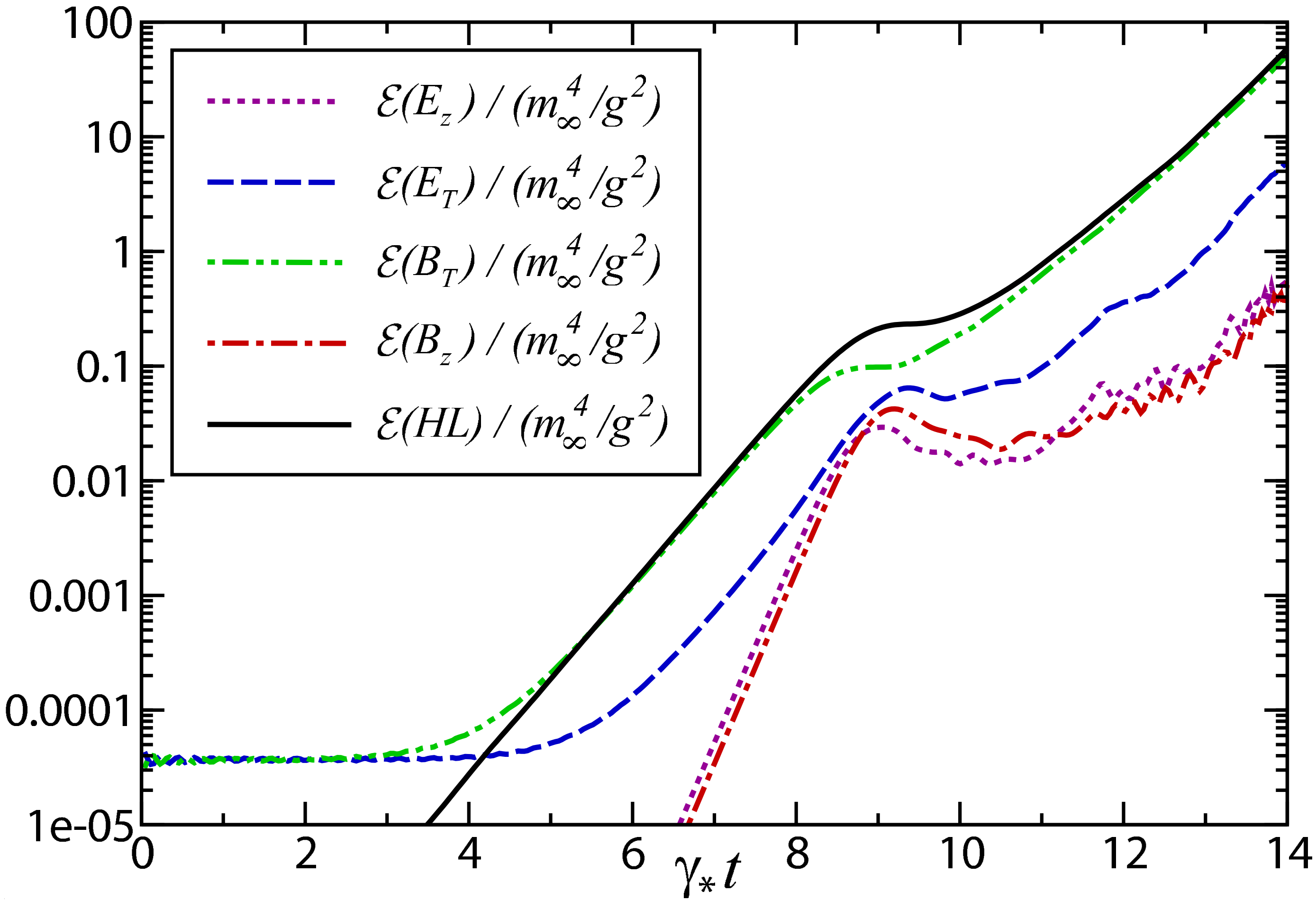
$$D_\rho(A)F^{\rho\mu} = j^\mu(x) = \frac{1}{\mathcal{N}}\sum_{\mathbf{v}} v^\mu \mathcal{W}_{\mathbf{v}}(x)$$

“disco balls” 

Transversely constant modes: 1D+3V

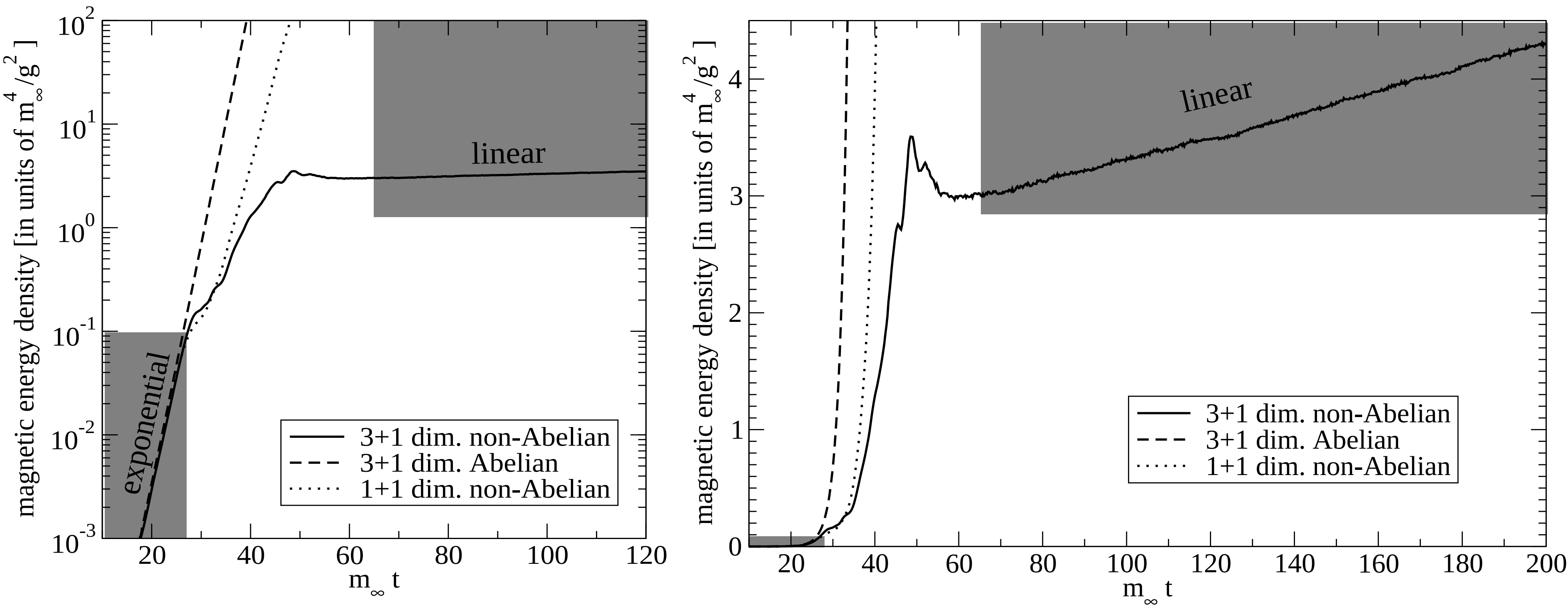
i.e., modes with momentum exactly in anisotropy direction:
(includes most unstable modes)

[AR, Romatschke & Strickland, PRL 94 ('05) 102303



3D+3V

[Arnold, Moore & Yaffe, PRD72 ('05) 054003

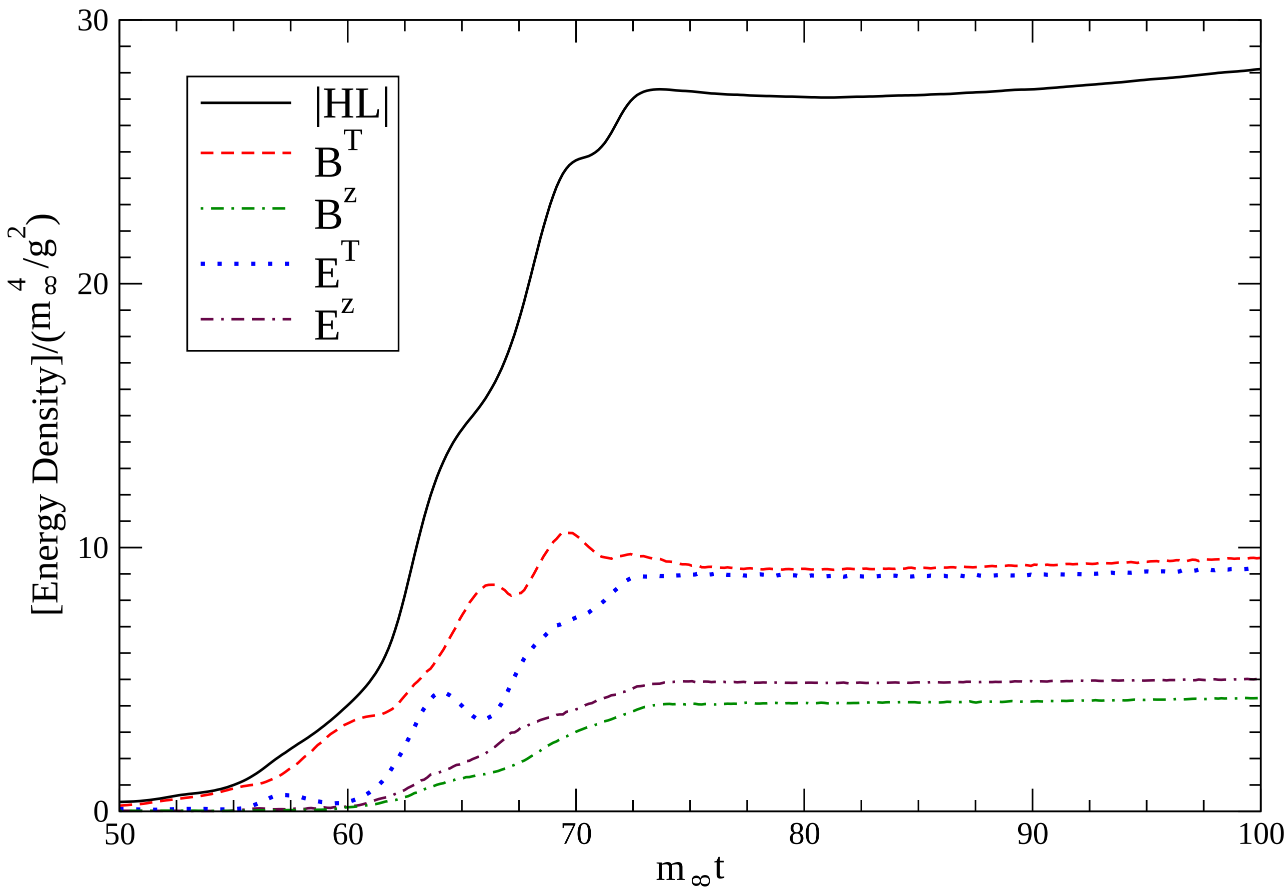


(btw different discretization method: finite number of spherical harmonics W_{lm})

3D+3V

Discoball discretization - somewhat larger anisotropy

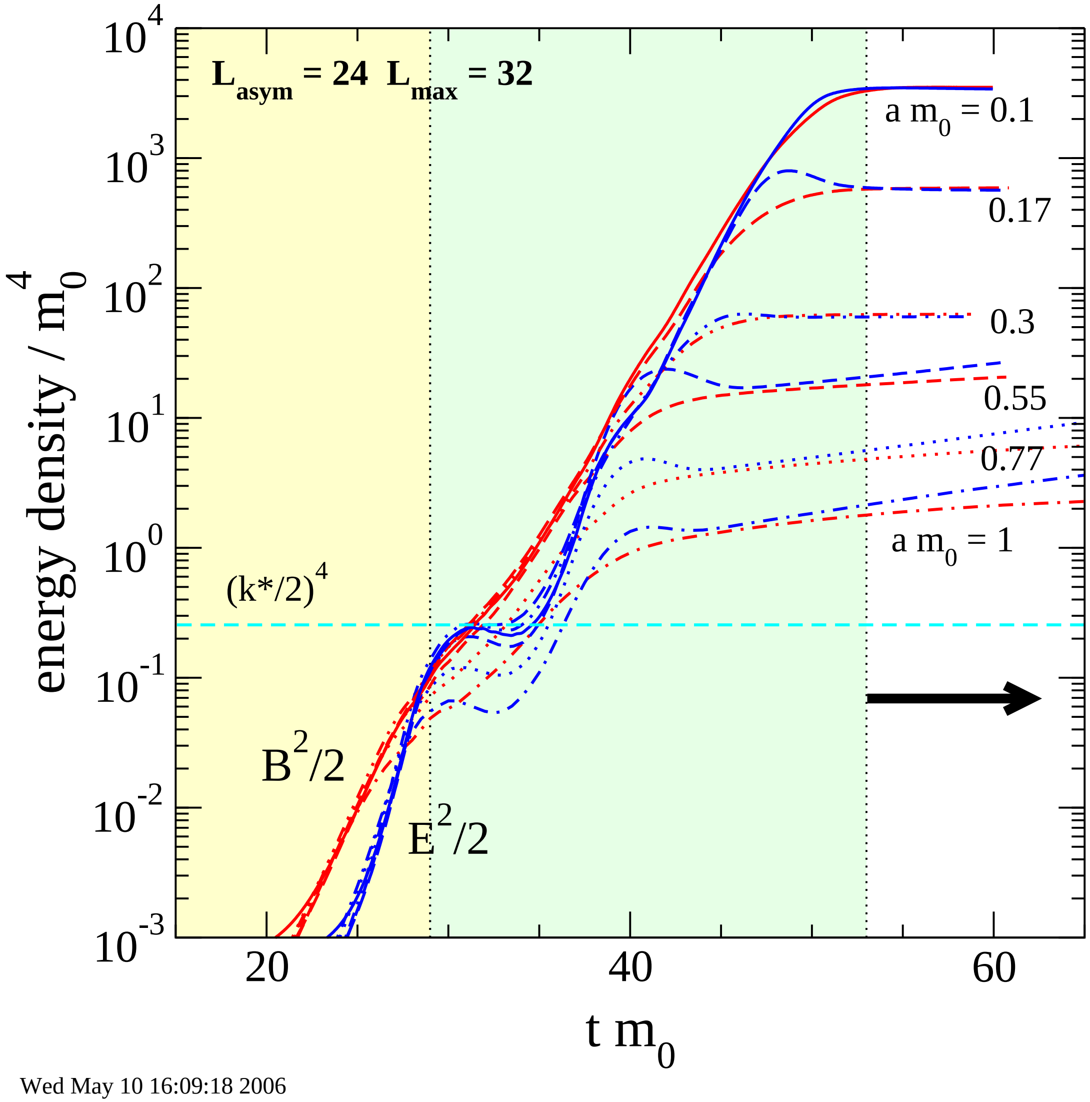
[AR. Romatschke & Strickland. JHEP 09 (2005) 041



3D+3V - Strong anisotropy

Strong anisotropy: no saturation before lattice saturation

[Bödeker & Rummukainen, JHEP 07 (2007) 022



Increasing anisotropy with time: Bjorken expansion

Notation: proper time $\tau = \sqrt{t^2 - z^2}$ and space-time rapidity $\eta = \mathrm{atanh}\frac{z}{t}$

$x^\mu \to x^\alpha = (\tau, x^i, \eta)$ with $g_{\alpha\beta} = (1, -1, -1, -\tau^2)$

momentum rapidity $y = \mathrm{atanh}\frac{p^0}{p^z}$:

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solved by $f_0(\mathbf{p}, \mathbf{x}, t) = f_0(\mathbf{p}_\perp, p_\eta(x))$ because $(p^\alpha \partial_\alpha) p_\eta(x)|_{fixed\ p^\mu} = 0$

$$p^\tau \partial_\tau p_\eta(x)\Big|_{y,\mathbf{p}_\perp} = -p_\perp^2 \sinh(y-\eta)\cosh(y-\eta) = -p^\eta \partial_\eta p_\eta(x)\Big|_{y,\mathbf{p}_\perp}$$

Boost-invariant free-streaming background

Will use:

$$f_0(\mathbf{p}, x) = f_{\rm iso}\left(\sqrt{p_\perp^2 + p_\eta^2/\tau_{\rm iso}^2}\right) = f_{iso}\left(\sqrt{p_\perp^2 + (p'^z\tau/\tau_{iso})^2}\right)$$

space-time dependent anisotropy parameter $\xi(\tau) = (\tau/\tau_{iso})^2 - 1$

increasingly oblate momentum space anisotropy at $\tau > \tau_{iso}$

(but prolate anisotropy for $\tau < \tau_{iso}$)

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Will start at finite τ_0 (mostly $\gg \tau_{iso}$)

as motivated by CGC initial conditions at $\tau_0 \sim Q_s^{-1}$

strong initial anisotropy which gets even stronger, $\xi \sim \tau^2$

(bottom-up scenario: $\xi \sim \tau^{\# < 2/3}$

Bödeker: $\tau^{1/2}$; Arnold & Moore: $\tau^{1/4}$)

Hard-Expanding-Loop formalism

Since $p^\beta \partial_\beta \left[\partial^\alpha_{(p)} f_0(\mathbf{p}_\perp, p_\eta)\right]\Big|_{p^\mu=const.} = 0$ (with index α upstairs!) can solve

$$p \cdot D\, \delta f_a(\mathbf{p}, \mathbf{x}, t)\Big|_{p^\mu=const.} = g p^\beta F^a_{\beta\alpha} \partial^\alpha_{(p)} f_0(\mathbf{p}, \mathbf{x}, t),$$

by introducing auxiliary fields

$$\delta f^a(x; p) = -g W^a_\alpha(\tau, x^i, \eta; \phi, y) \partial^\alpha_{(p)} f_0(p_\perp, p_\eta)$$

that obey

$$\boxed{v \cdot D\, W_\alpha(\tau, x^i, \eta; \phi, y)\Big|_{\phi,y} = v^\beta F_{\alpha\beta},}$$

where $v^\alpha \equiv \frac{p^\alpha}{|\mathbf{p}_\perp|} = (\cosh(y-\eta), \cos\phi, \sin\phi, \frac{\sinh(y-\eta)}{\tau})$.

[P. Romatschke & AR, PRL 97 (2006)]

Discretized HEL

For $f_0(\mathbf{p}, x) = f_{\text{iso}}\left(\sqrt{p_\perp^2 + p_\eta^2/\tau_{\text{iso}}^2}\right)$

$$j^\alpha(\tau, x^i, \eta) = -\frac{m_D^2(\tau = \tau_{iso})}{2} \int_0^{2\pi} \frac{d\phi}{2\pi} \int_{-\infty}^{\infty} dy\, v^\alpha \left(1 + \frac{\tau^2}{\tau_{iso}^2} \sinh^2(y-\eta)\right)^{-} \times \underbrace{\left\{\cos\phi\, W_1 + \sin\phi\, W_2 - \frac{\tau}{\tau_{iso}^2} \sinh(y-\eta)\, W_\eta\right\}}_{\mathcal{W}(\tau, x^i, \eta; \phi, y)}$$

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instead of discoballs [$\mathcal{W}(t,\mathbf{x};\phi_n,\theta_m)$ with equally spaced ϕ_n, $\cos\theta_m$]

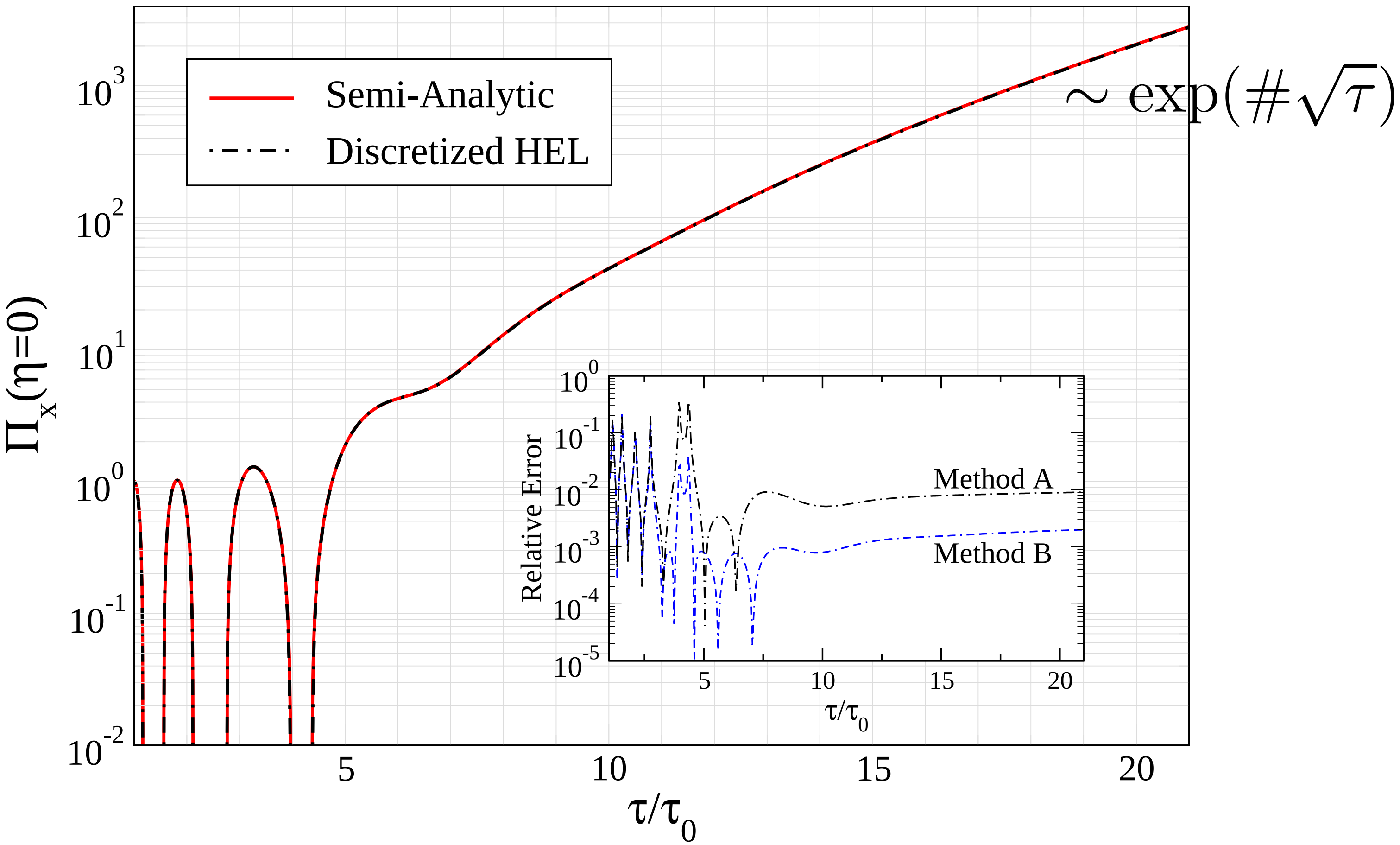
now *disco cylinders*: $\mathcal{W}(\tau,x^i,\eta;\phi,y)$ with equally spaced ϕ_n, y_m

finite rapidity interval for $y-\eta$ because of exponential suppression

$\longrightarrow$ numerical simulation on space-time & $\phi, y-\eta$ grid

Discretized HEL - Abelian checks

1D+3V Abelian: can solve e.o.m. for $\mathcal{W}$ to give 1D integro-differential equation ("semi-analytic")



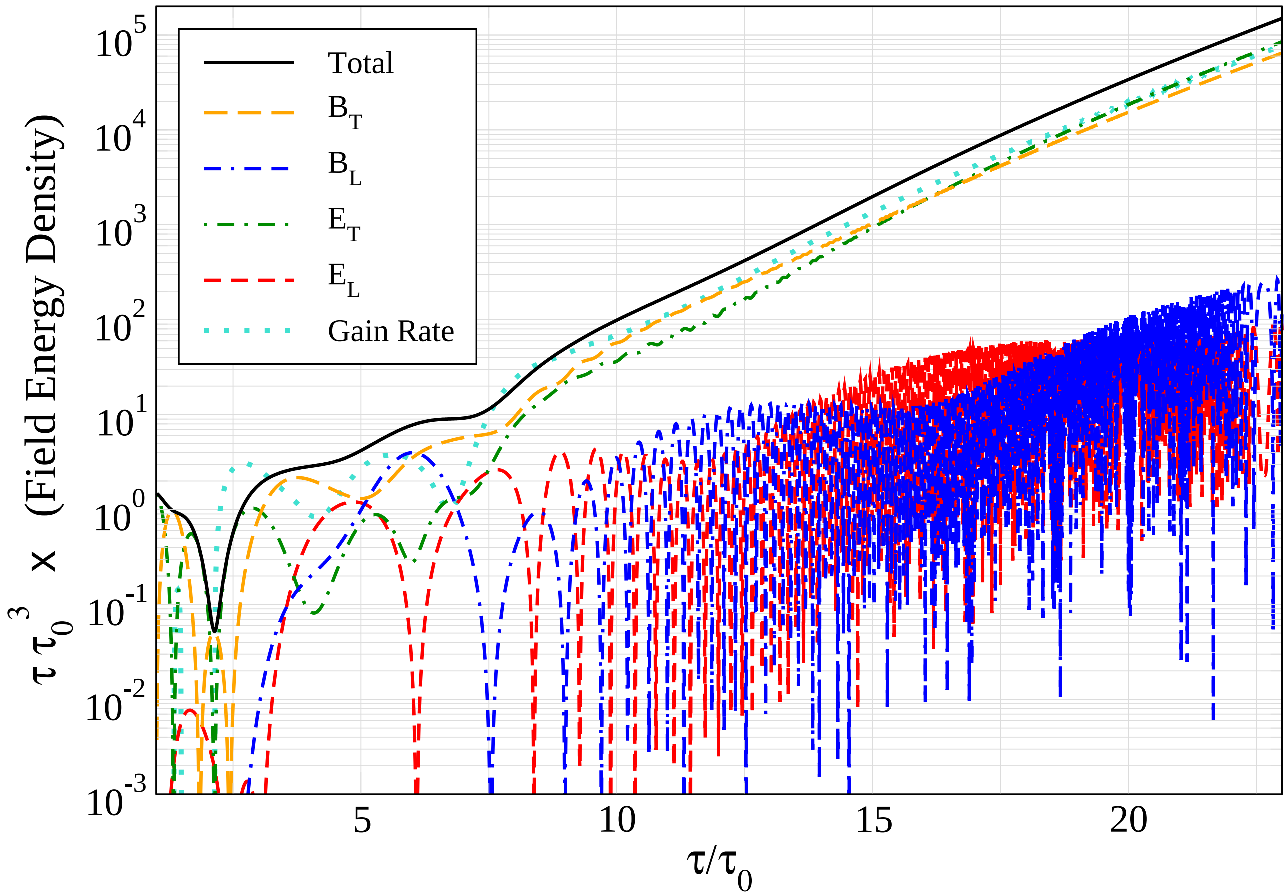
$\tau_{\rm iso} = 0.1$, $\tau_0 = 1.0$, $m_D = 10$, $a = 0.0025$, $\epsilon = 0.001$, $N_\eta = 250$, $N_\phi = 8$,

and (Method A) $N_y = 1000$, (Method B: $x = \tanh(y - \eta)$) $N_x = 1000 \ldots$ $\boxed{N_{\mathcal{W}} = 8000}$

Non-Abelian Discretized HEL

Non-Abelian 1D+3V: Reference (upper bound) on full 3D+3V

single mode, with convenient (unrealistically large) hard-loop mass parameter:

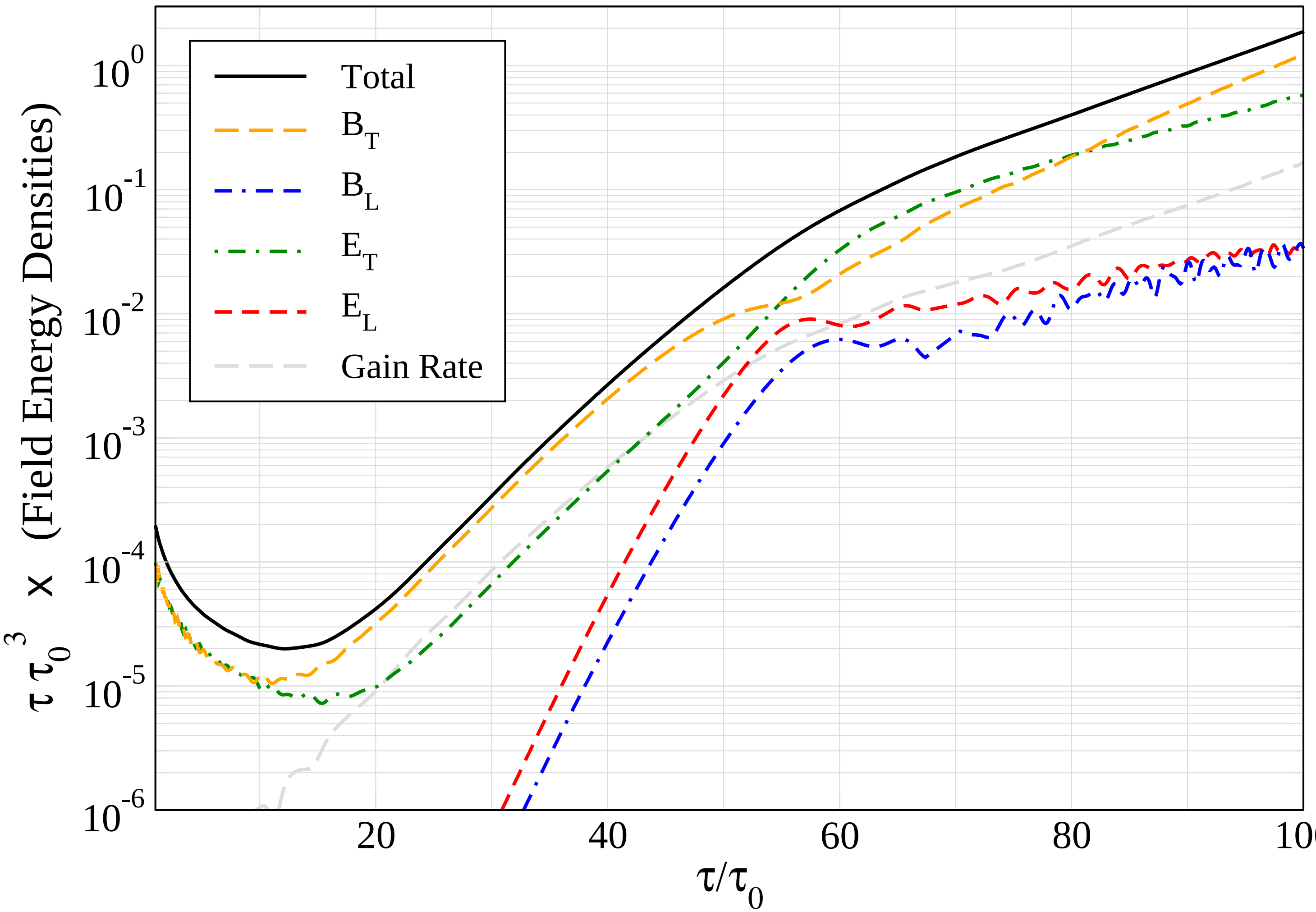


AR, M. Strickland, M. Attems: arXiv:0802.1714

Non-Abelian Discretized HEL

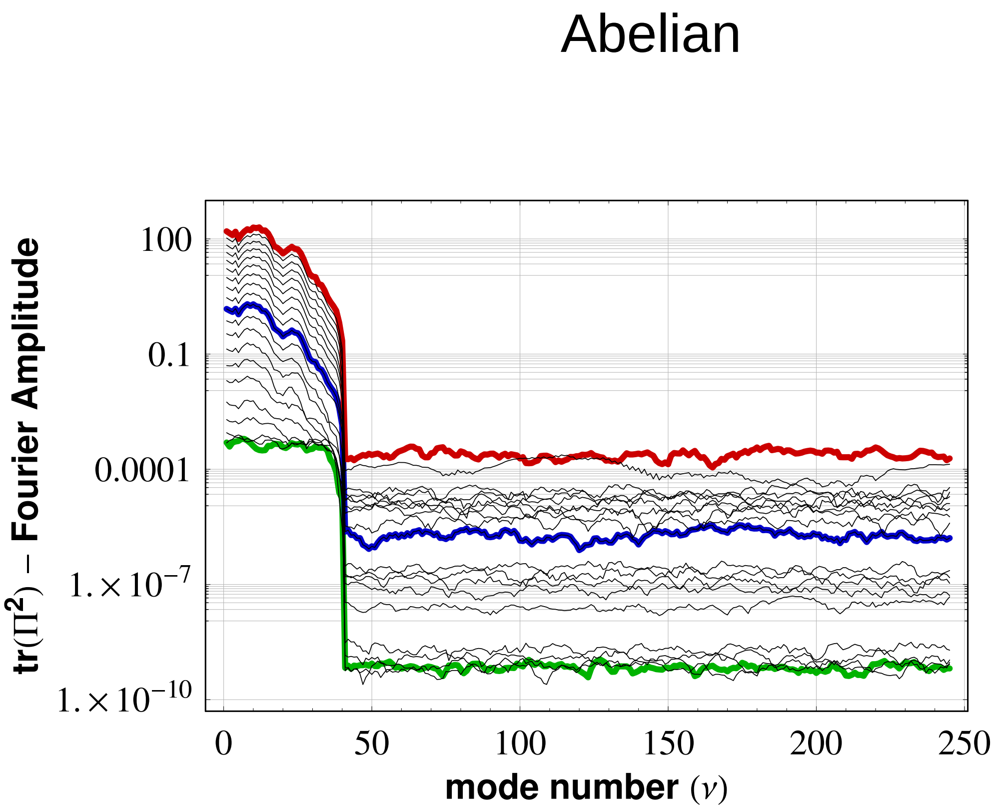
Hard gluon number density and initial fluctuation spectrum from CGC:

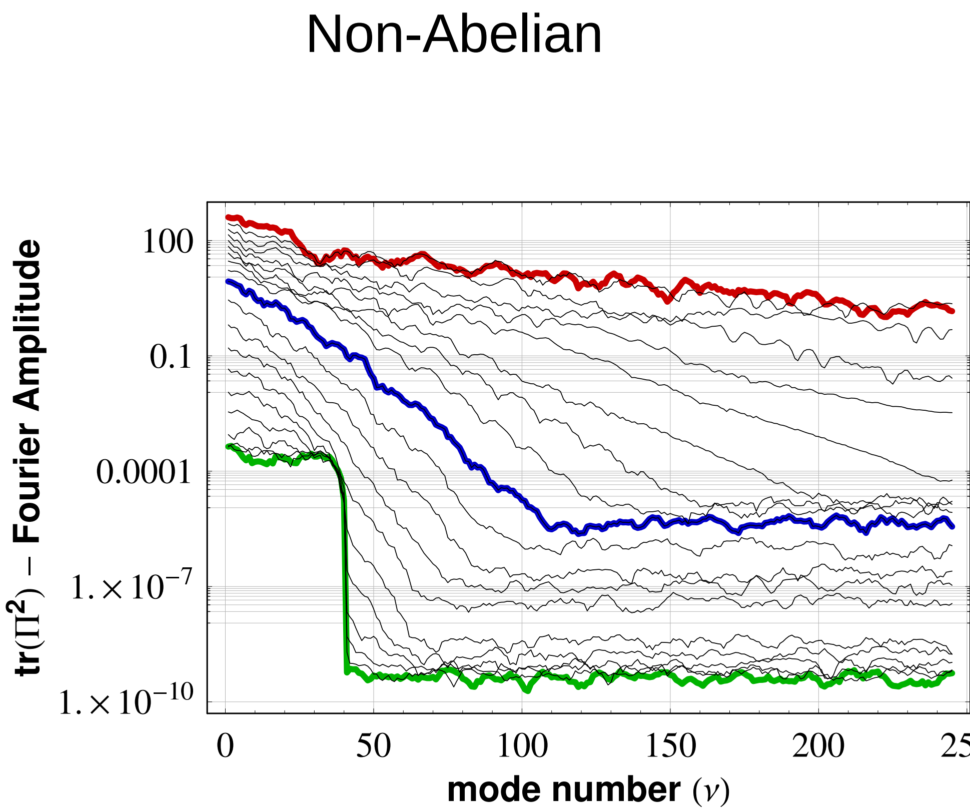
- gluon liberation factor $c = 2\ln 2 \approx 1.386$ (recent numerical value by Lappi $\simeq 1.1$)
- initial transverse el./magn. fields à la Fukushima/Gelis/McLerran (with UV cutoff)



$= \begin{cases} 20 \text{ fm/c (RHIC)} \\ 7 \text{ fm/c (LHC)} \end{cases}$

Non-Abelian Discretized HEL – Cascade





Nonabelian dynamics $\longrightarrow$ quasithermal spectrum of soft modes

Conclusions and Outlook

Conclusions:

- Uncomfortably long delay of onset of PI for role in early thermalization, if starting from small rapidity fluctuations
(much more leeway at LHC!)

Upcoming:

- Full 3D+3V (or at least 2D+3V acc.to Arnold & Leang PRD76 (2007))
- Generic large initial fields

To do:

- Impact on bottom-up thermalization
- Generalization of HEL to non-free streaming?

Backup slides

Matching to CGC parameters

Parameters from saturation scenario $\tau_0 \simeq Q_s^{-1}$:

$$n(\tau_0) = c\frac{(N_c^2-1)Q_s^3}{4\pi^2 N_c \alpha_s(Q_s\tau_0)}$$

with gluon liberation factor $c = \begin{cases} 0.5 & \text{Krasnitz et al. (numerical)} \\ 2\ln 2 & \text{Kovchegov (analytical estimate)} \end{cases}$

Lappi 2008: $c \simeq 1.1$

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pure glue $\quad \longrightarrow \quad$ $$\mathcal{N} = \frac{1}{\alpha_s} \frac{c}{8N_c (0.47)^3 \zeta(3)} \frac{\tau_0}{\tau_{iso}} \frac{1}{Q_s \tau_0}$$

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$f_{\rm iso} = \mathcal{N} f_{\rm thermal}$ with (transverse) temperature $T = 0.47 Q_s$ [Krasnitz et al.]

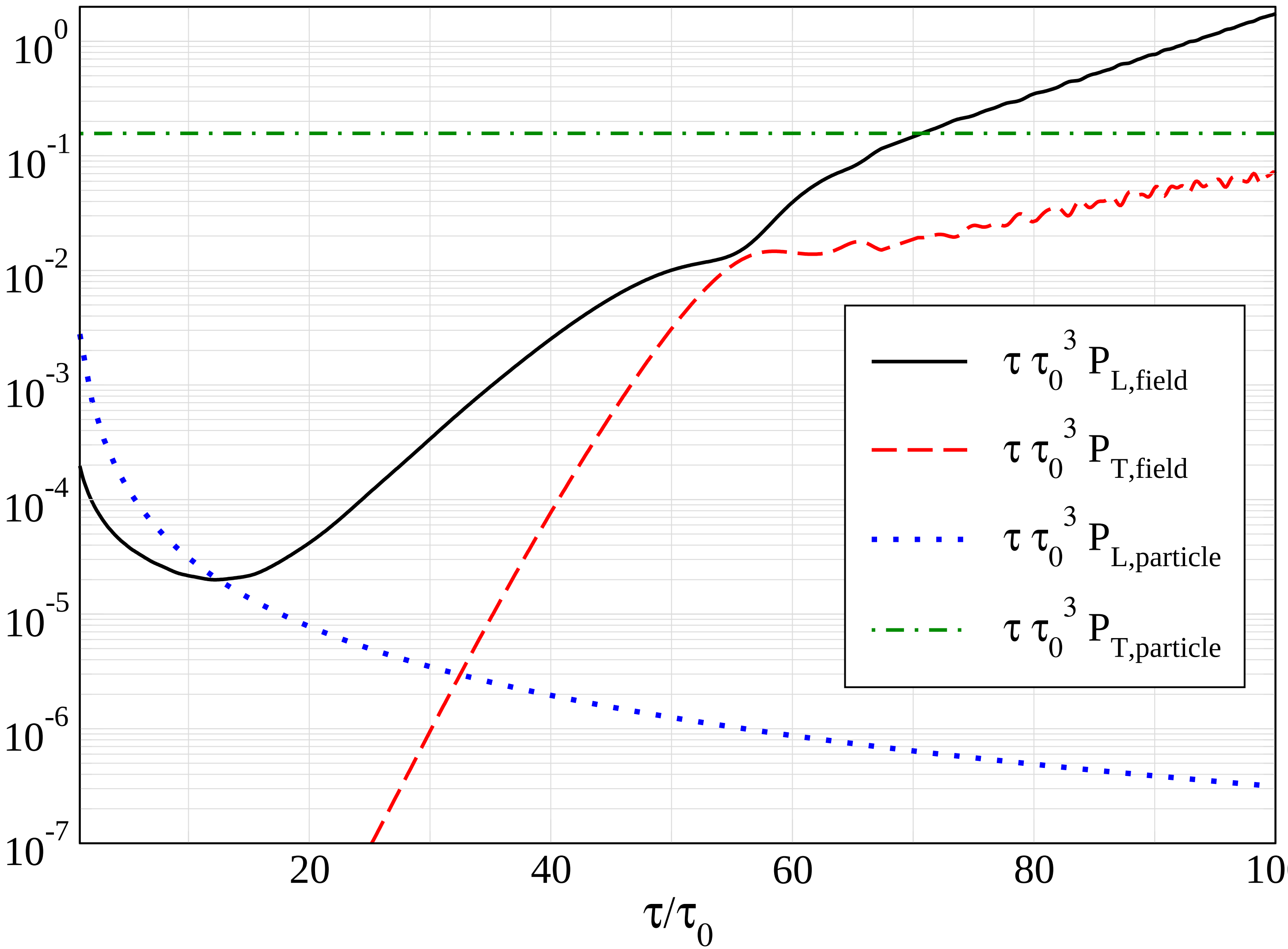
pure glue $\longrightarrow$ $\mathcal{N} = \frac{1}{\alpha_s}\frac{c}{8N_c(0.47)^3\zeta(3)}\frac{\tau_0}{\tau_{iso}}\frac{1}{Q_s\tau_0}$

$$\longrightarrow \quad \frac{\mu}{Q_s} = \frac{1}{8} m_D^2 \pi \tau_{\rm iso} Q_s^{-1} = \frac{\pi^2}{48\cdot 0.47\cdot\zeta(3)}\, c \approx \begin{cases} 0.182 & (c = 0.5) \\ \underline{0.505} & \underline{(c = 2\ln 2)} \end{cases}$$

$Q_s \simeq 1$ GeV (RHIC) . . . 3 GeV (LHC) ?

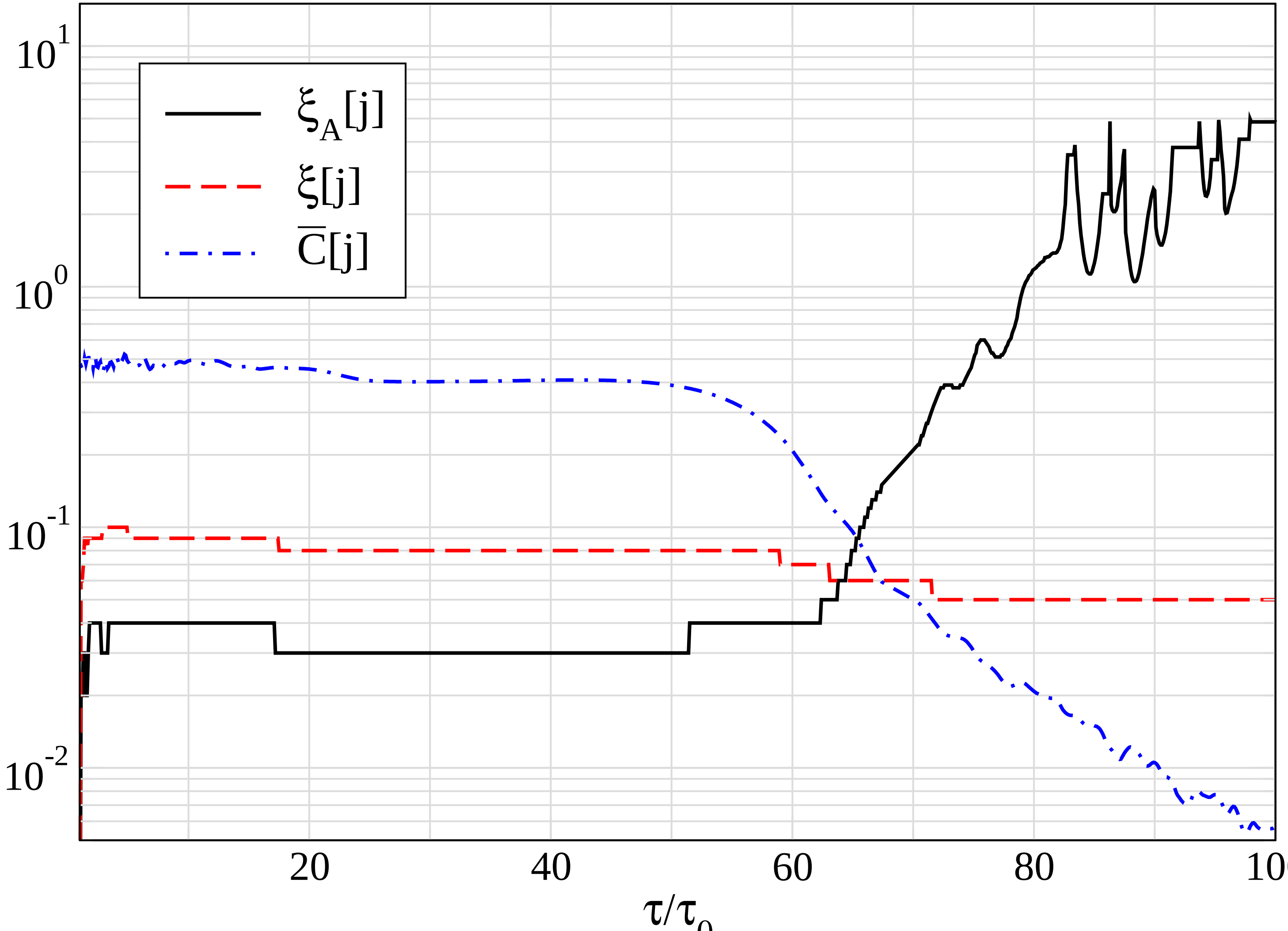
Non-Abelian Discretized HEL

Transverse/longitudinal pressures in hard particles/chromofields:



= { 20 fm/c (RHIC)
7 fm/c (LHC)

Non-Abelian Discretized HEL – Abelianization



$= \begin{cases} 20 \text{ fm/c (RHIC)} \\ 7 \text{ fm/c (LHC)} \end{cases}$