

Static potentials for finite temperature quarkonia

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in collaborations with O. Jahn, M. Laine, P. Romatschke, M. Tassler

- Non-perturbative aspects of potential models
- Critical assessment of static potentials used so far
- New real-time potential, cf. [Mikko Laine](#)

Potential models

T=0: solve static Schrödinger eqn. with confining V (from the lattice)

$$\left(-\frac{\nabla_{\mathbf{r}}^2}{M} + V(r) \right) \psi = (E - 2M)\psi$$

very successful spectroscopy ($\sim 1\%$), search for hybrids, ...

can be derived from QFT in effective theory \rightarrow **pNRQCD**
expansion parameter $(E - 2M)/M$;

$V =$ perturbative matching coefficient

finite T: argue for a 'finite T potential', proceed as above

Matsui, Satz

$$V(r) \approx -\frac{g^2 C_F \exp(-m_D r)}{4\pi r}$$

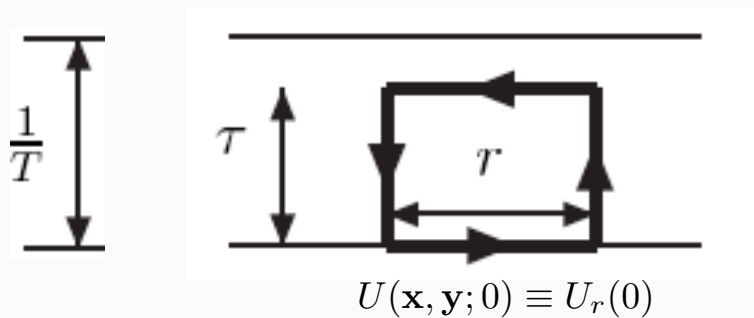
Problems:

- (matching coeff. **the same** as non-pert. static potential ?)
- **which non-pert. potential to take?**

The static potential at $T=0$: Wilson loop

- Euclidean correlator of gauge invariant meson operator
- integrate out quarks in the limit $M \rightarrow \infty$

$$\langle \bar{\psi}(\mathbf{x}, \tau) U(\mathbf{x}, \mathbf{y}; \tau) \psi(\mathbf{y}, \tau) \bar{\psi}(\mathbf{y}, 0) U^\dagger(\mathbf{x}, \mathbf{y}; 0) \psi(\mathbf{x}, 0) \rangle \longrightarrow e^{-2M\tau} W_E(|\mathbf{x} - \mathbf{y}|, \tau)$$



$$r = |\mathbf{x} - \mathbf{y}|$$

- spectral decomposition, $T \rightarrow 0$ $W_E(r, \tau \rightarrow \infty) \longrightarrow c_{01}^2 [U(\mathbf{x}, \mathbf{y}; 0)] e^{-V(r)\tau}$
- lattice generalization to finite T not clear, $N_\tau = \frac{1}{aT}$ on lattices short

Static 'potentials' at finite T, free energies: the Polyakov loop

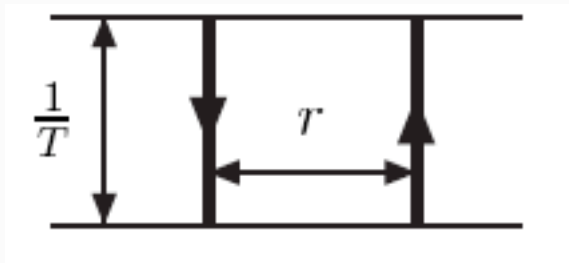
McLerran, Svetitsky PRD 81

- Static quarks propagate through periodic boundary
- finite T: sum over Boltzmann weighted excited states

➔ free energy of a static quark in a plasma: $\langle \text{Tr } L_{\mathbf{x}} \rangle \sim e^{-F_q/T}$

order parameter for confinement: $\langle L \rangle \begin{cases} = 0 \Leftrightarrow \text{confined phase,} & T < T_c \\ > 0 \Leftrightarrow \text{deconfined phase,} & T > T_c \end{cases}$

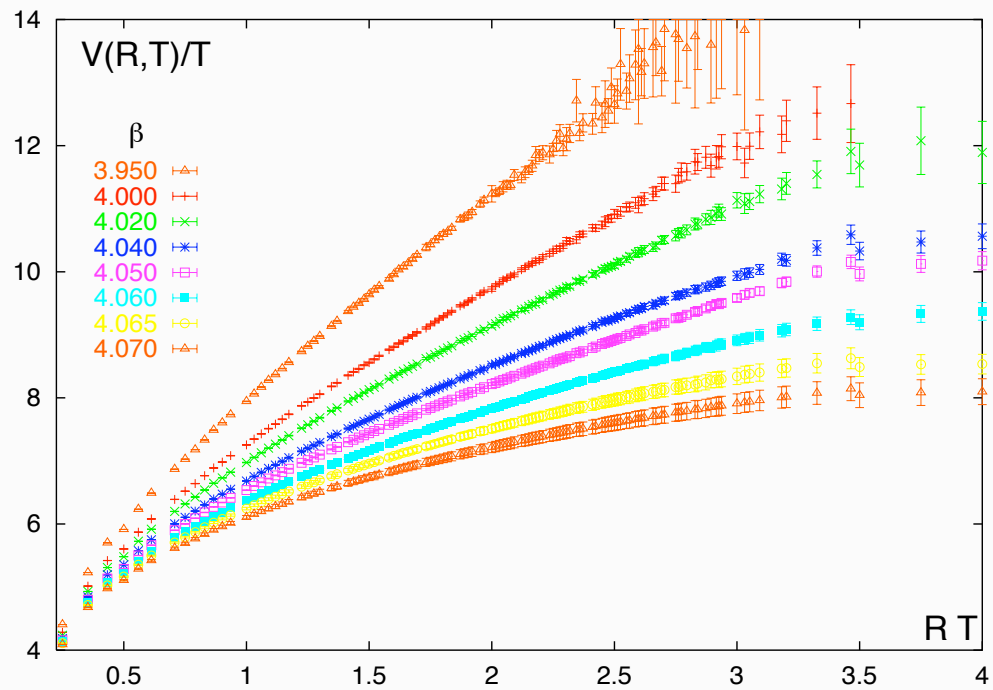
- free energy of static quark anti-quark pair:



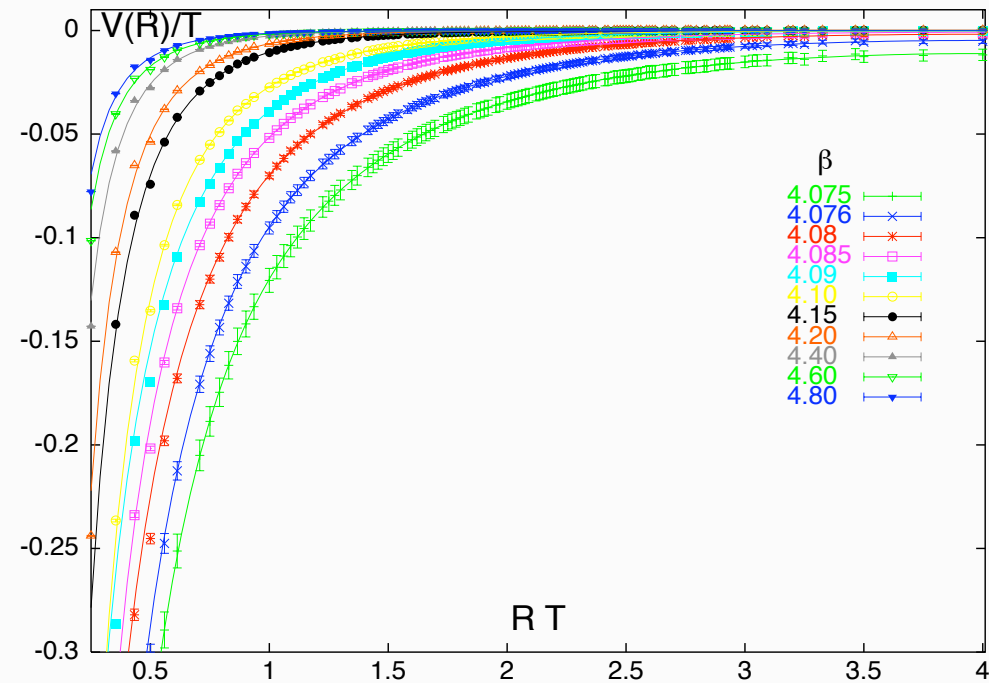
$$e^{-F_{\bar{q}q}(r,T)/T} = \frac{1}{N^2} \langle \text{Tr } L^\dagger(\mathbf{x}) \text{Tr } L(\mathbf{y}) \rangle, \quad r = |\mathbf{x} - \mathbf{y}|$$

The static quark free energy in the quenched limit

Kaczmarek et al., PRD 00



$T < T_c$



$T > T_c$

eff. string:

$$\frac{\sigma(T)}{\sigma(0)} = a \sqrt{1 - b \frac{T^2}{T_c^2}}$$

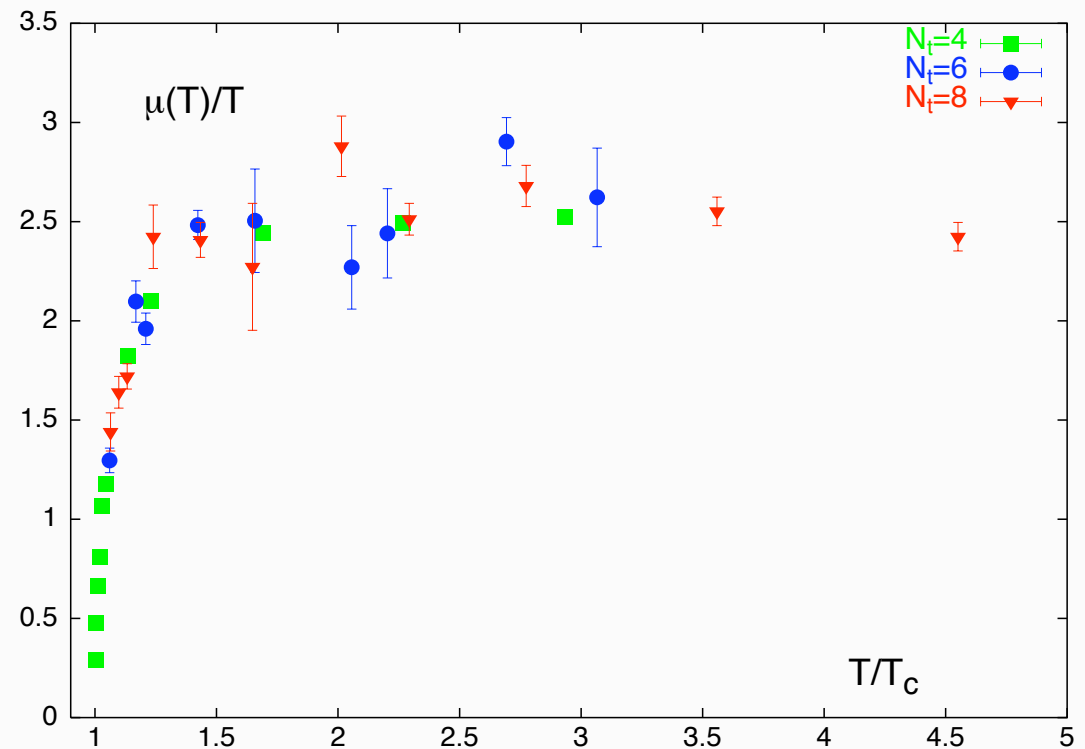
$$\frac{F_{q\bar{q}}(r, T)}{T} = -\frac{c(T)}{(rT)^d} e^{-\mu(T)r}$$

Screening of static quark free energy

...in pure gauge theory

Kaczmarek et al., PRD 00

$$\frac{F_{q\bar{q}}(r, T)}{T} = -\frac{c(T)}{(rT)^d} e^{-\mu(T)r}$$



numerically:

$$d \approx 3/2, \mu \approx M_{0^{++}}$$

lightest gauge inv. screening mass

LO perturbation theory:

$$d = 2, \mu = 2m_D^0$$

exchange of two A_0



$F_{\bar{q}q}$ does **not** correspond to the Debye-screened static potential!

Decomposition in different colour channels

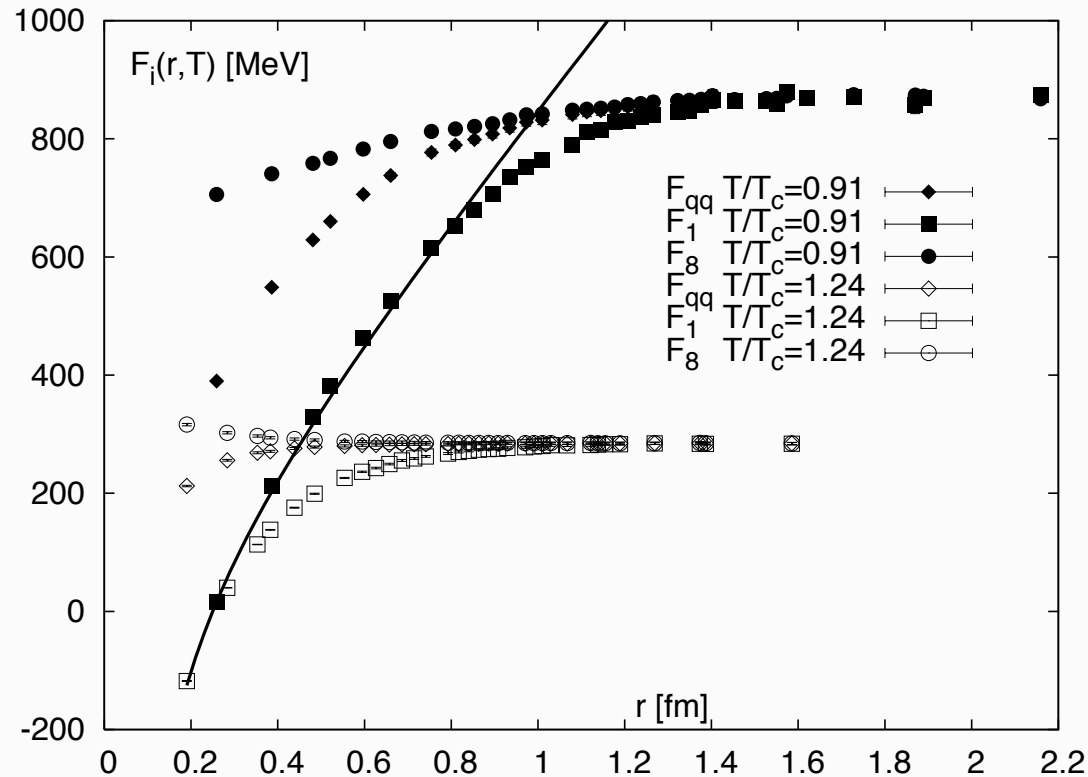
McLerran, Svetitsky PRD 81

$$\begin{aligned}e^{-F_{\bar{q}q}(r,T)/T} &= \frac{1}{N^2} \langle \text{Tr} L^\dagger(\mathbf{x}) \text{Tr} L(\mathbf{y}) \rangle = \frac{1}{N^2} e^{-F_1(r,T)/T} + \frac{N^2 - 1}{N^2} e^{-F_8(r,T)/T} \\e^{-F_1(r,T)/T} &= \frac{1}{N} \langle \text{Tr} L^\dagger(\mathbf{x}) L(\mathbf{y}) \rangle, \\e^{-F_8(r,T)/T} &= \frac{1}{N^2 - 1} \langle \text{Tr} L^\dagger(\mathbf{x}) \text{Tr} L(\mathbf{y}) \rangle - \frac{1}{N(N^2 - 1)} \langle \text{Tr} L^\dagger(\mathbf{x}) L(\mathbf{y}) \rangle.\end{aligned}$$

- correlators in 'singlet' and 'octet' channels **gauge dependent**, non-pert. meaning?

- $$F_1(r, T) \sim \frac{e^{-m_D(T)r}}{4\pi r}$$

Nadkarni PRD 86



- **'Potentiology'**: solve Schrödinger eq. in various potentials, check if solution allows to reconstruct lattice correlators

$$F_i, \quad U_i = F_i + TS_i \dots$$

cf. Mocsy, Petreczky PRD 08

- different **r-dependence** results in different **binding behaviour** in Schrödinger eq.
- F_1, U_1 **gauge artefact or physics?**

Singlet and octet channels from gauge invariant correlators

start from meson operators:

Jahn, O.P., PRD 05

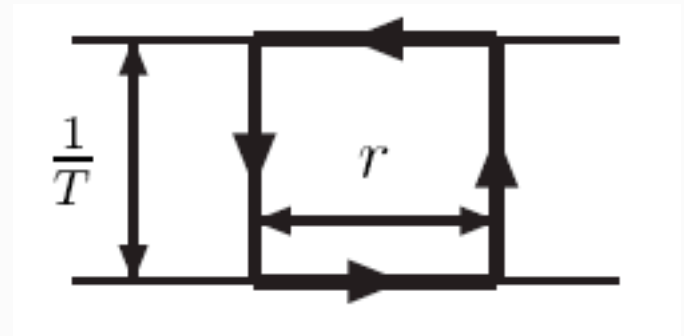
$$O(\mathbf{x}, \mathbf{y}) = \bar{\psi}(\mathbf{x})U(\mathbf{x}, \mathbf{y})\psi(\mathbf{y}), \quad O^a(\mathbf{x}, \mathbf{y}) = \bar{\psi}(\mathbf{x})U(\mathbf{x}, \mathbf{x}_0)T^a U(\mathbf{x}_0, \mathbf{y})\psi(\mathbf{y}),$$

integrate out static quarks:

$$\begin{aligned} \langle O(\mathbf{x}, \mathbf{y}; 0)O^\dagger(\mathbf{x}, \mathbf{y}; N_t) \rangle &\propto \langle \text{Tr} L^\dagger(\mathbf{x})U(\mathbf{x}, \mathbf{y}; 0)L(\mathbf{y})U^\dagger(\mathbf{x}, \mathbf{y}; N_t) \rangle \\ \langle O^a(\mathbf{x}, \mathbf{y}; 0)O^{a\dagger}(\mathbf{x}, \mathbf{y}; N_t) \rangle &\propto \frac{1}{N^2 - 1} \langle \text{Tr} L^\dagger(\mathbf{x}) \text{Tr} L(\mathbf{y}) \rangle \\ &\quad - \frac{1}{N(N^2 - 1)} \langle \text{Tr} L^\dagger(\mathbf{x})U(\mathbf{x}, \mathbf{y}; 0)L(\mathbf{y})U^\dagger(\mathbf{x}, \mathbf{y}; N_t) \rangle \end{aligned}$$

identical to gauge fixed correlators in axial gauge $U(\mathbf{x}, \mathbf{y}) = g^{-1}(\mathbf{x})g(\mathbf{y})$

singlet channel: periodic Wilson loop



Spectral analysis of Polyakov loop correlators

Jahn, O.P., PRD 05

$\hat{T}_0 = e^{-a\hat{H}_0}$ with Kogut-Susskind Hamiltonian in temporal gauge

$$e^{-F_{\bar{q}q}/T} = \frac{1}{ZN^4} \hat{\text{Tr}} [\hat{T}_0^{N_t} \hat{P}^{\mathbf{F} \otimes \bar{\mathbf{F}}}] = \frac{1}{ZN^2} \sum_n \langle n_{\alpha\beta} | n_{\beta\alpha} \rangle e^{-E_n/T}$$

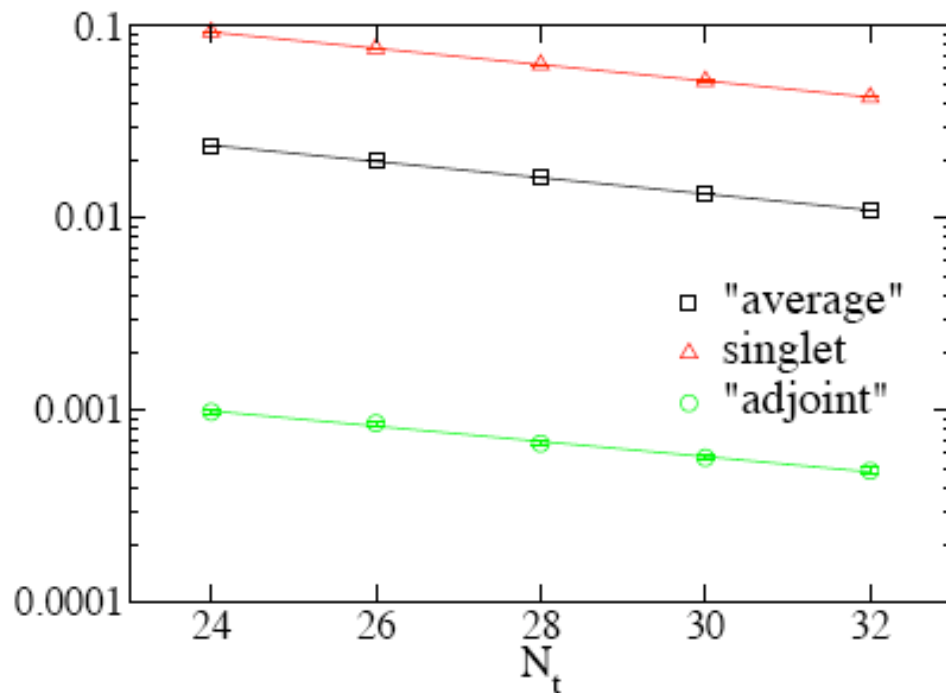
$$e^{-F_1/T} = \frac{1}{ZN^2} \sum_n \langle n_{\delta\gamma} | \hat{U}_{\gamma\delta}(\mathbf{x}, \mathbf{y}) \hat{U}_{\alpha\beta}^\dagger(\mathbf{x}, \mathbf{y}) | n_{\beta\alpha} \rangle e^{-E_n/T}$$

$$e^{-F_8/T} = \frac{1}{ZN^2} \sum_n \langle n_{\delta\gamma} | \hat{U}_{\gamma\delta}^a(\mathbf{x}, \mathbf{y}) \hat{U}_{\alpha\beta}^{\dagger a}(\mathbf{x}, \mathbf{y}) | n_{\beta\alpha} \rangle e^{-E_n/T}$$

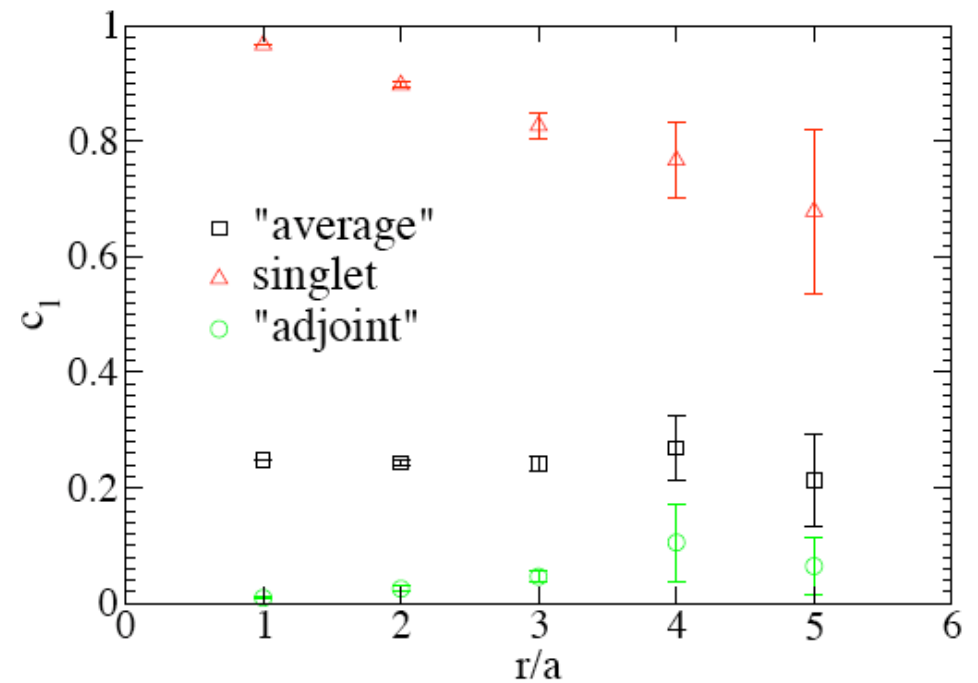
- energies: usual (T=0) colour singlet potential + excit. in **all three** channels
- **non-vanishing matrix elements** in singlet and octet channel
- matrix elements **path/gauge dependent** but **contribute**

Numerical demonstration in 3d SU(2), T=0 limit

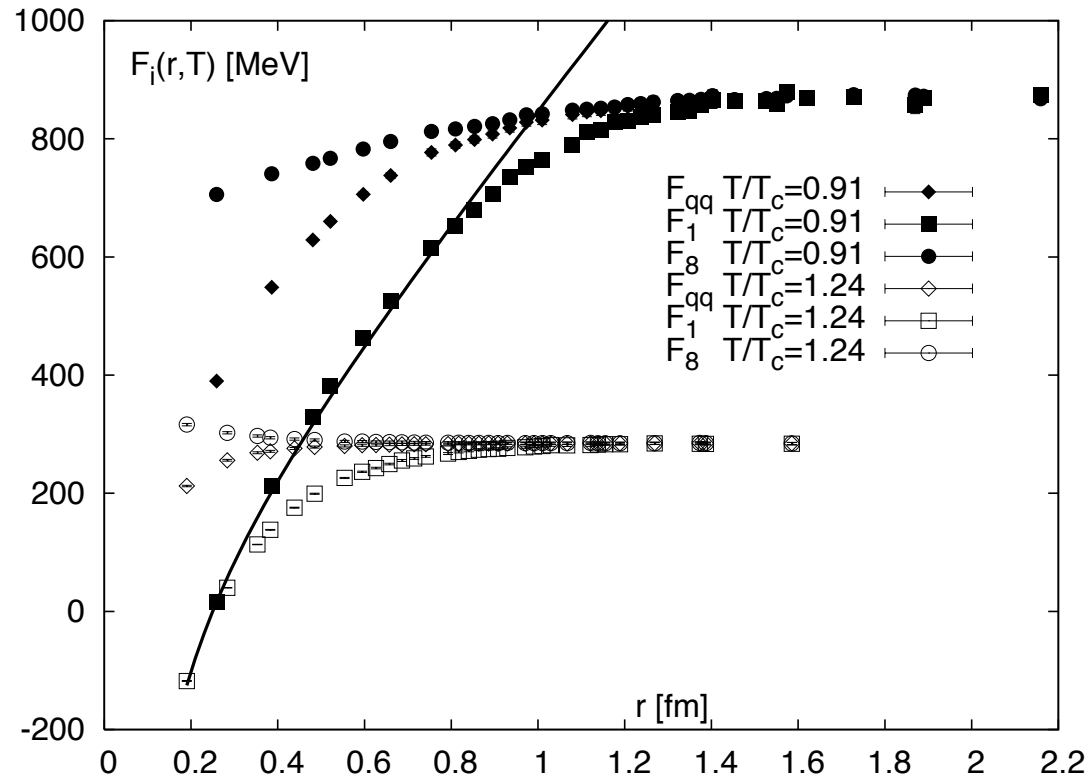
allows to isolate ground state energy and matrix element for each channel



correlators, $r/a=1$



matrix elements



➔ difference between $F_1, F_{\bar{q}q}$ is exclusively in gauge fixing function!

➔ binding energies from F_1, U_1 artefacts ?!

➔ similarly, careful with smearing techniques!

A real time static potential for finite T quarkonia

Laine, O.P., Romatschke, Tassler JHEP 07
Brambilla, Ghiglieri, Petreczky, Vairo PRD 08

- generalise effective theory (pNRQCD) approach to finite T

quarkonium correlator, infinite M limit:

$$C_{>}(t, \mathbf{r}) \propto W_E(it, \mathbf{r}) \quad \text{Wilson loop in Minkowski-time}$$

- finite T correlator related to quarkonium spectral function

$$\tilde{C}_{>}(Q) \equiv \int dt e^{iq^0 x} C_{>}(t, \mathbf{0}) \quad \text{point splitting technical, not physical:}$$

$$\tilde{C}_{>}(Q) \equiv \int dt \int d^3x e^{iQx} \langle J^\mu(x) J_\mu(0) \rangle_T, \quad J^\mu(x) = e_q \bar{\psi}(x) \gamma^\mu \psi(x)$$

$$\rho(Q) = \frac{1}{2} \left(1 - e^{-\frac{q^0}{T}} \right) \tilde{C}_{>}(Q)$$

evolution equation for the correlator:

$$\left\{ i\partial_t - \left[2M + V_{>}(t, r) - \frac{\nabla_{\mathbf{r}}^2}{M} + O\left(\frac{1}{M^2}\right) \right] \right\} C_{>}(t, \mathbf{r}) = 0$$

potential = coefficient scaling as $O(M^0)$ in t-derivative of correlator
= matching coefficient in EFT

required scale hierarchy: $g^2 M < T < gM$

absorb heavy mass by rescaling, leading behaviour:



$$i\partial_t W_E(it, \mathbf{r}) = V_{>}(t, r) W_E(it, \mathbf{r})$$

Non-perturbative effects?

- Wilson loop in Minkowski time not calculable on the lattice
 - scales in perturbative expansion:
vertices $g^2 T$, cut-off Λ , 3d confinement scale $m \sim g^2 T$
 - HTL resummation accounts for effect of hard modes on soft modes,
perturbative corrections $\sim g^2 T / \Lambda$
 - **non-perturbative** corrections from infrared modes $\sim g^2 T / m$
- ➔ **test for non-pert. infrared corrections via classical lattice simulations !**

cf. sphaleron rate in e.-w. theory, plasma instabilities...

Classical limit in perturbation theory

$$V_{>}(\infty, r) = -\frac{g^2 C_F}{4\pi} \left[m_D + \frac{\exp(-m_D r)}{r} \right] - \frac{ig^2 T C_F}{4\pi} \phi(m_D r)$$

re-instate factors of \hbar in perturbative calculation: $g^2 \rightarrow g^2 \hbar$, $\frac{1}{T} \rightarrow \frac{\hbar}{T}$

$$\lim_{\hbar \rightarrow 0} V_{>}(\infty, r) = -\frac{ig^2 T C_F}{4\pi} \phi(m_D r)$$



only the imaginary part survives in the classical limit !



test for non-perturbative effects in this sector

Imaginary part from classical lattice simulations

Laine, O.P., Tassler, JHEP 07

- Hamiltonian approach: t continuous, discretise on 3d lattice
- temporal gauge; conjugate fields $\rightarrow U_i(\mathbf{x}, t), \dot{U}_i(\mathbf{x}, t) = iE_i(\mathbf{x}, t)U_i(\mathbf{x}, t)$
- Gauss constraint on non-abelian charges
- quantum treatment of UV modes via HTL effective theory [Bödeker et al., PRD 00](#)

$$Z = \int \mathcal{D}U_i \mathcal{D}E_i \delta(G) e^{-\beta H}, \quad \beta = \frac{2N}{g^2 T a}, \quad H = \frac{1}{N} \sum_x \left[\sum_{i < j} \text{Re Tr} (1 - U_{ij}) + \frac{1}{2} \text{Tr} (E_i^2) \right]$$

real time evolution:

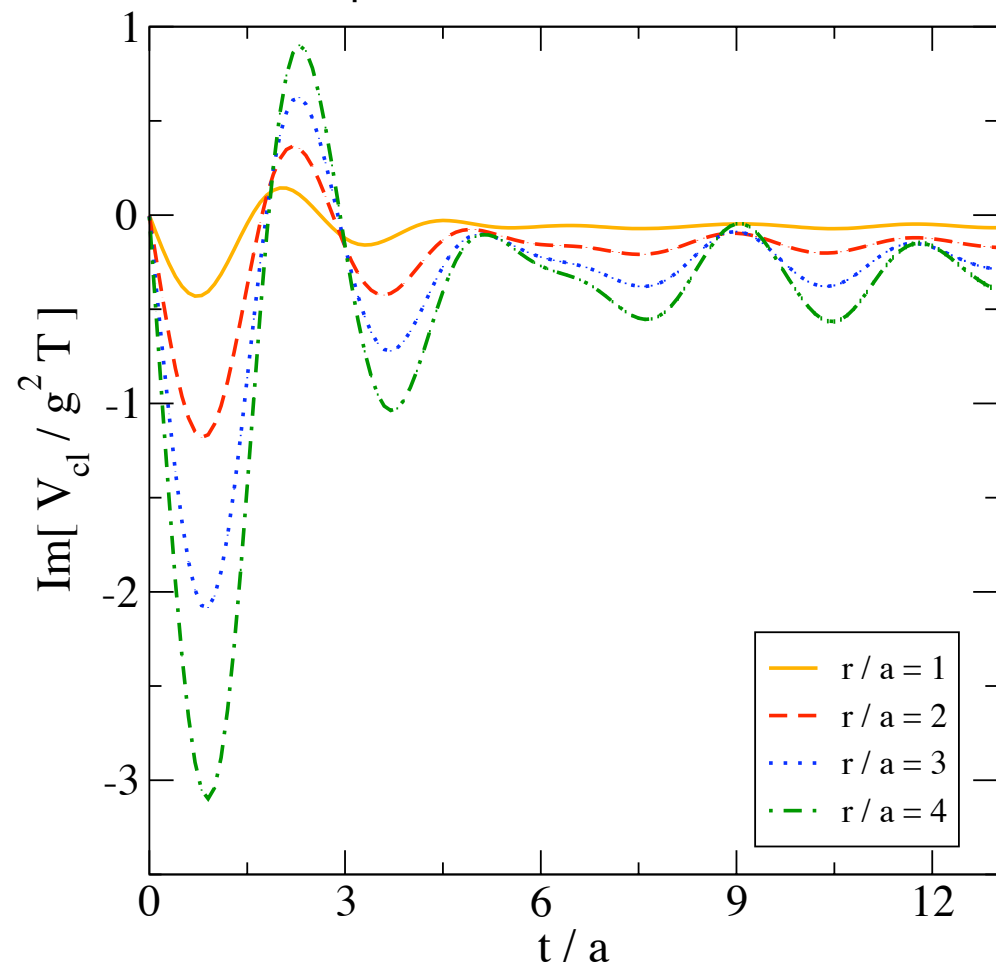
$$\dot{U}_i(x) = iE_i(x)U_i(x), \quad \dot{E}_i^a(x) = -2 \text{Im Tr} [T^a \sum_{|j| \neq i} U_{ij}(x)]$$



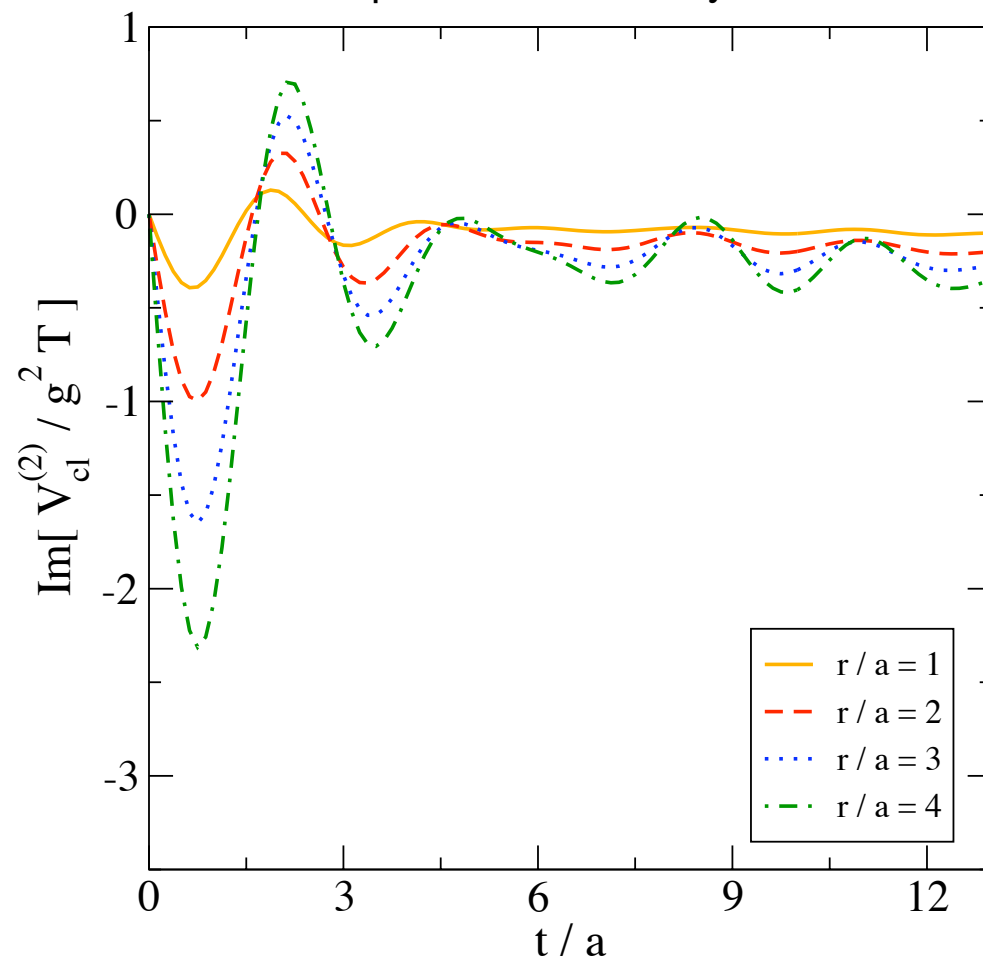
$$C_{\text{cl}}(t, r) = \frac{1}{N} \text{Tr} \langle U_r^\dagger(t) U_r(0) \rangle$$

$$V_{\text{cl}}(t, r) = \frac{i\partial_t C_{\text{cl}}(t, r)}{C_{\text{cl}}(t, r)}$$

$\beta = 16, N = 12$, simulation

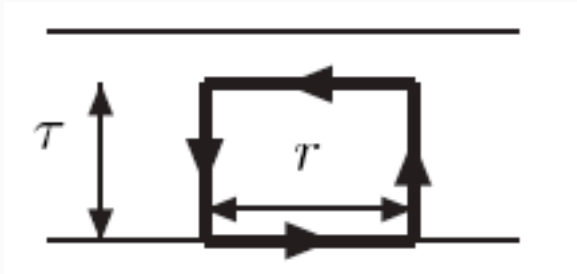


$\beta = 16, N = 12$, analytic



non-perturbative strengthening of damping!

What is needed for the whole object, including real part?



$$W_E(\tau = it, r)$$

within potential approach, do not need full time dependence,

‘only’ $\lim_{t \rightarrow \infty} W_E(\tau = it, r)$

Which euclidean operator corresponds to this ???

Conclusions

- old potential models field-theoretically unclear:
gauge dependence? relation to spectral function?
- reformulation of potential approach in terms of effective QFT
 - ➔ new real-time static potential
- solution possible in HTL-resummed perturbation theory
 - ➔ Debye screening and Landau damping ($\text{Im } V$)
- $\text{Im } V$ calculable non-perturbatively in classical lattice simulations;
comparison: HTL pert. theory captures qualitative physics
- Still needed: non-pert. operator for $\text{Re } V$