Exotic phases of Finite Temperature $SU(N)$ gauge theories

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Outline

Motivation:

- To understand how confinement comes about in an $SU(N)$ gauge theory with fundamental representation fermions.
- To understand the effects of fermion mass on the phase diagram of $SU(N)$ gauge theories for fermions in various representations.

Phase diagrams of three extensions to Yang-Mills theory:

- a heavy quark in the adjoint representation : a simple deformed Yang-Mills theory formulated on the lattice
- multiply wound adjoint quarks : center-stabilized Yang-Mills theory (compare with QCD(Adj))
- fermions with nonzero mass from the one-loop effective potential
  - Fermion representations: Fundamental (F), Antisymmetric (A), Symmetric (S), and Adjoint (Adj)
  - Boundary conditions: periodic (PBC), antiperiodic (ABC)
  - $N_c = 2$ through 9.
  - various N_f

Yang-Mills theory + heavy adjoint fermion

In supersymmetric Yang-Mills theory, which adds massless adjoint fermions with PBC to YM theory, perturbative confinement was shown in an instanton background (Davies et. al. 1999). This led us to test the effect of adding an adjoint particle, with a variable mass, to YM theory (Myers and Ogilvie 2008).

On the lattice:

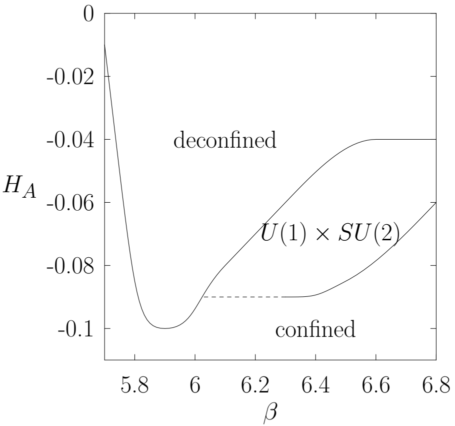
$$S = S_W + \sum_{\vec{x}} H_A \, Tr_A P(\vec{x})$$

- ABC $\implies H_A > 0$
- PBC $\implies H_A < 0$

Two important results for $H_A < 0$:

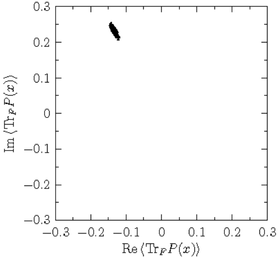
1. Confinement is restored at high temperature.
2. Exotic phases are found.

   $SU(3)$:
   skewed phase: $\mathrm{Proj}_{Z_3} \langle Tr_F P \rangle < 0$

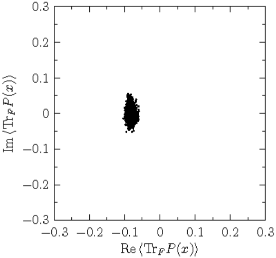


Phase diagram in $SU(3)$ for lattice simulations of YM theory extended with a $Tr_A P$ term.

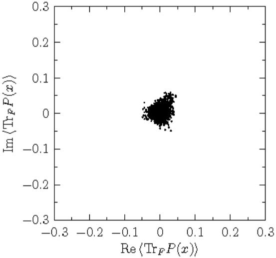
$SU(3)$ histograms of the order parameter $\langle \mathrm{Tr}_F P \rangle$, $\beta = 6.5$



$H_A = -0.04$, deconfined, $\mathrm{Proj}_{Z_3} \langle \mathrm{Tr}_F P \rangle > 0$



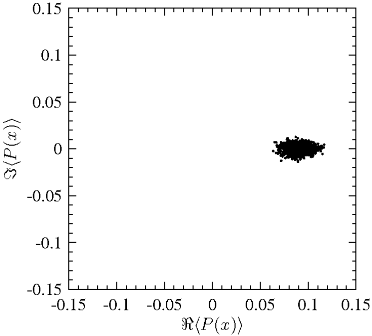
$H_A = -0.06$, skewed, $\mathrm{Proj}_{Z_3} \langle \mathrm{Tr}_F P \rangle < 0$



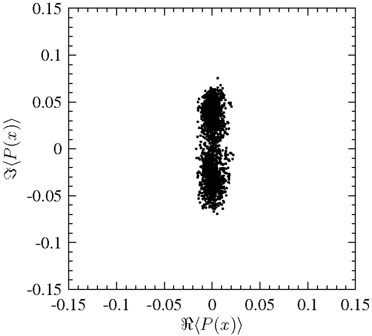
$H_A = -0.10$, confined, $\langle \mathrm{Tr}_F P \rangle = 0$

- In $SU(3)$ simulations the confined and deconfined phases were found in a perturbatively accessible regime.
- In addition, an $SU(2) \times U(1)$ phase which is "skewed" with respect to the deconfined phase is also found.

$SU(4)$ histograms of the order parameter $\langle \mathrm{Tr}_F P \rangle$, $\beta = 11$



$H_A = -0.10$, deconfined, $\langle \mathrm{Tr}_F P \rangle \neq 0$



$H_A = -0.12$, partially confined, $\langle \mathrm{Tr}_F P \rangle = 0$, $\langle \mathrm{Tr}_F P^2 \rangle \neq 0$

- In $SU(4)$ simulations, the deconfined phase, and a partially-confined $Z(2)$-invariant phase were found in the perturbatively accessible regime, however the confined phase was not.

Center-stabilized Yang-Mills theory

To keep the confined phase accessible for $N > 3$ additional terms were required in the extension to Yang-Mills (Ogilvie et al 2007, Unsal and Yaffe 2008):

$$V_{ext}(P) \equiv \frac{1}{\beta} \sum_{n=1}^{\lfloor N/2 \rfloor} a_n \mathrm{Tr}_F\left(P^n\right) \mathrm{Tr}_F(P^{\dagger n}) = \frac{1}{\beta} \sum_{n=1}^{\lfloor N/2 \rfloor} a_n \sum_{i,j=1}^{N} \cos\left[n\left(v_i - v_j\right)\right]$$

where $\lfloor N/2 \rfloor$ is the integer part of $N/2$.
Including the boson contribution from pure Yang-Mills theory

$$V_{CYM}(P) = -\frac{2}{\pi^2 \beta^4} \sum_{n=1}^{\infty} \frac{1}{n^4} \left[\mathrm{Tr}_A\left(P^n\right)\right] + \frac{1}{\beta} \sum_{n=1}^{\lfloor N/2 \rfloor} a_n \mathrm{Tr}_F(P^n) \mathrm{Tr}_F(P^{\dagger n})$$

- We minimize V_{CYM} with respect to the Polaykov loop eigenvalues v_i to determine the phase diagram for a range of values of the a_n.

One-loop effective potential with massive fermions in rep R

The one-loop effective potential for N_f Majorana fermions ($N_{f,Dirac} = \frac{1}{2}N_f$) of mass m in a background Polyakov loop P is (Meisinger and Ogilvie 2002):

$$\begin{aligned} V_{1-loop}(P,m) &\equiv -\frac{1}{\beta V_3}\ln Z(P,m) \\ &= \frac{1}{\beta V_3}\left[-N_f \ln\det\left(-D_R^2(P)+m^2\right) + \ln\det\left(-D_{adj}^2(P)\right)\right] \\ &= \frac{m^2 N_f}{\pi^2\beta^2}\sum_{n=1}^{\infty}\frac{(\pm 1)^n}{n^2}\mathrm{Re}\left[\mathrm{Tr}_R\left(P^n\right)\right]K_2\left(n\beta m\right) - \frac{2}{\pi^2\beta^4}\sum_{n=1}^{\infty}\frac{1}{n^4}\mathrm{Tr}_A\left(P^n\right) \end{aligned}$$

where we have $(+1)^n$ for periodic boundary conditions (PBC) and $(-1)^n$ for antiperiodic boundary conditions (ABC) applied to fermions.

To obtain the preferred phases for a range of $m\beta$ we numerically minimize V_{1-loop} with respect to the eigenvalues of the Polyakov loop $P = diag\{e^{iv_1}, e^{iv_2}, ..., e^{iv_N}\}$.

Chiral Condensate:

$$\langle\bar{\psi}\psi\rangle_{1-loop}(m) = -\lim_{V_4\to\infty}\frac{1}{V_4 N_f}\frac{\partial}{\partial m}\ln Z(m) = \frac{1}{N_f}\frac{\partial}{\partial m}V_{\mathrm{eff}}(P,m)$$

Possible phases of perturbative QCD for ABC and PBC

- ABC
  - deconfined phase

$$\mathbf{v} = \{0, 0, 0\}$$

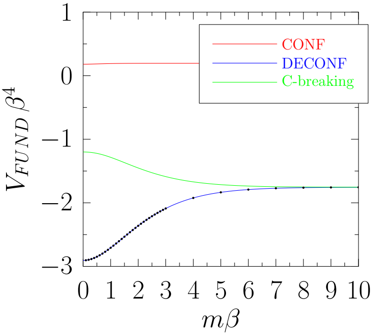
- PBC
  - confined phase
  - deconfined phase
  - $\mathscr{C}$-breaking phase (P is not invariant under $P \to P^*$.)

$$\mathbf{v} = \{\frac{2\pi}{3}, \frac{2\pi}{3}, \frac{2\pi}{3}\}, \{\frac{4\pi}{3}, \frac{4\pi}{3}, \frac{4\pi}{3}\}$$

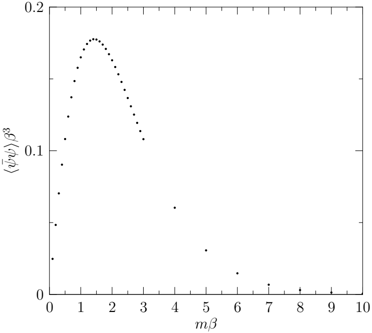
Note: In QCD(F) with PBC on fermions, $\mathscr{C}$-symmetry is only broken for N odd. For N even, $Tr_F P$ is magnetized along the negative real axis ($\mathbf{v} = \{\pi, \pi, ...\}$).

V_{1-loop} and $\langle\bar{\psi}\psi\rangle$ in perturbative QCD

- We calculate V_{1-loop} for fermions in the fundamental representation to which ABC are applied.



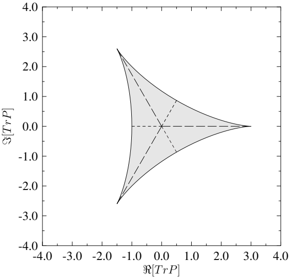
$V_{F,-}$, $Nc = 3$, $Nf = 1$



$\langle\bar{\psi}\psi\rangle_{F,1-loop}$ for $N_c = 3$

- Only the deconfined phase is accessible in the perturbative limit.
- The fermion contribution to V_{1-loop} vanishes as $m\beta \to \infty$.
- The inflection point in V_{1-loop} at $m\beta \approx 1.4$ implies a large one-loop contribution to $\langle\bar{\psi}\psi\rangle$.

Phases of adjoint QCD: $N_c = 3$, $N_c = 4$, PBC, $N_f > 1$



$N_c = 3$

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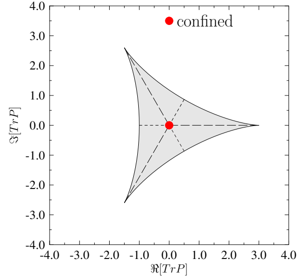
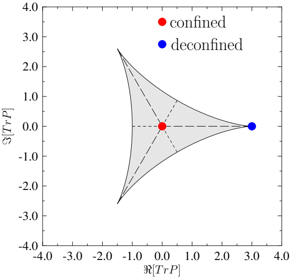


$N_c = 3$

confined: $\mathbf{v} = \{0, \frac{2\pi}{3}, \frac{4\pi}{3}\}$

Phases of adjoint QCD: $N_c = 3$, $N_c = 4$, PBC, $N_f > 1$



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deconfined: $\mathbf{v} = \{0, 0, 0\}$

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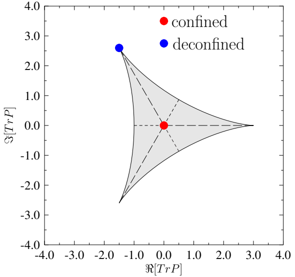

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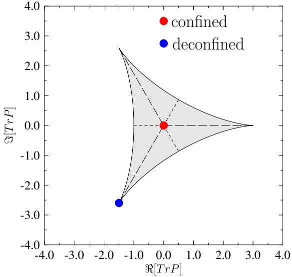

$N_c = 3$

confined: $\mathbf{v} = \{0, \frac{2\pi}{3}, \frac{4\pi}{3}\}$

deconfined: $\mathbf{v} = \{\frac{4\pi}{3}, \frac{4\pi}{3}, \frac{4\pi}{3}\}$

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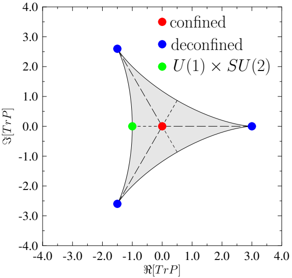

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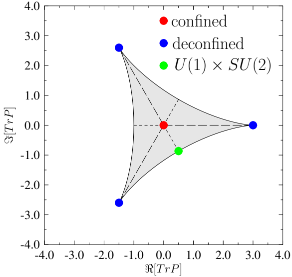
confined: $\mathbf{v} = \{0, \frac{2\pi}{3}, \frac{4\pi}{3}\}$

deconfined: $\mathbf{v} = \{0, 0, 0\}$

$U(1) \times SU(2)$: $\mathbf{v} = \{0, \pi, \pi\}$

Phases of adjoint QCD: $N_c = 3$, $N_c = 4$, PBC, $N_f > 1$



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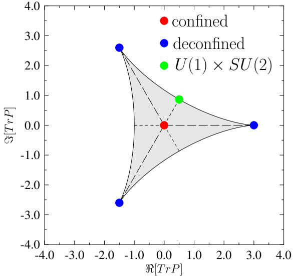

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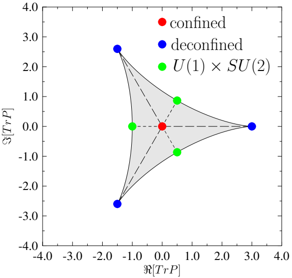

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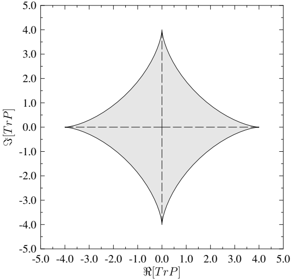


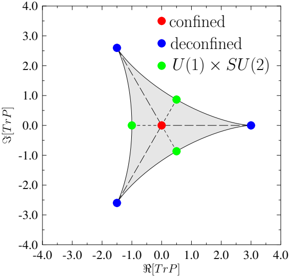
$N_c = 4$

confined: $\mathbf{v} = \{0, \frac{2\pi}{3}, \frac{4\pi}{3}\}$

deconfined: $\mathbf{v} = \{0, 0, 0\}$

$U(1) \times SU(2)$: $\mathbf{v} = \{0, \pi, \pi\}$

Phases of adjoint QCD: $N_c = 3$, $N_c = 4$, PBC, $N_f > 1$



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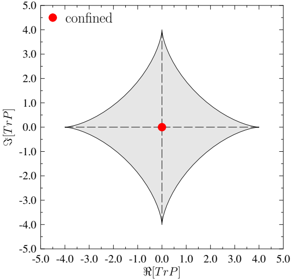

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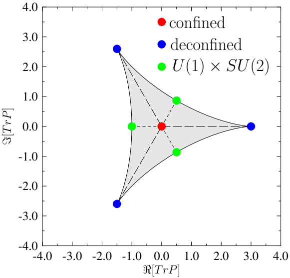
deconfined: $\mathbf{v} = \{0, 0, 0\}$

$U(1) \times SU(2)$: $\mathbf{v} = \{0, \pi, \pi\}$

confined: $\mathbf{v} = \{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\}$

Phases of adjoint QCD: $N_c = 3$, $N_c = 4$, PBC, $N_f > 1$



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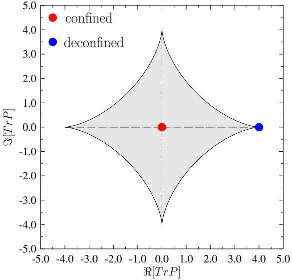

$N_c = 4$

confined: $\mathbf{v} = \{0, \frac{2\pi}{3}, \frac{4\pi}{3}\}$

deconfined: $\mathbf{v} = \{0, 0, 0\}$

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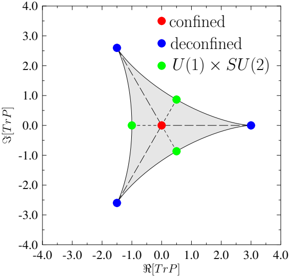
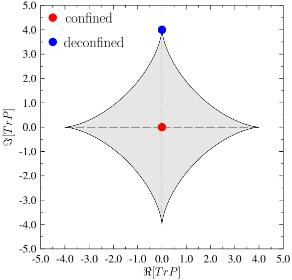
Phases of adjoint QCD: $N_c = 3$, $N_c = 4$, PBC, $N_f > 1$



$N_c = 3$



$N_c = 4$

$N_c = 3$:

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deconfined: $\mathbf{v} = \{0, 0, 0\}$

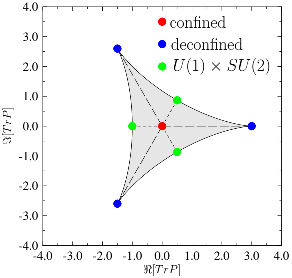
$U(1) \times SU(2)$: $\mathbf{v} = \{0, \pi, \pi\}$

$N_c = 4$:

confined: $\mathbf{v} = \{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\}$

deconfined: $\mathbf{v} = \{\frac{\pi}{2}, \frac{\pi}{2}, \frac{\pi}{2}, \frac{\pi}{2}\}$

Phases of adjoint QCD: $N_c = 3$, $N_c = 4$, PBC, $N_f > 1$



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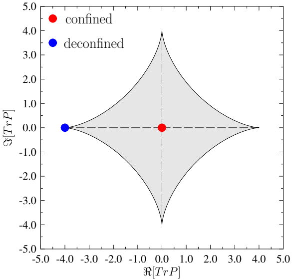

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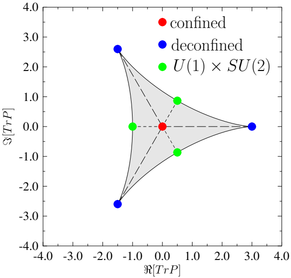
Phases of adjoint QCD: $N_c = 3$, $N_c = 4$, PBC, $N_f > 1$



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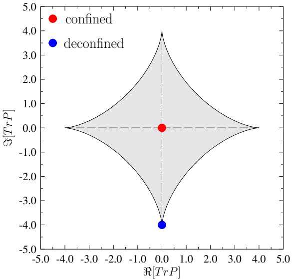

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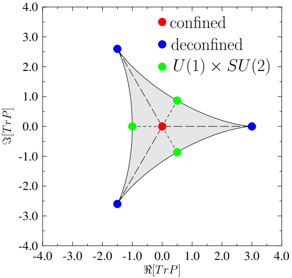
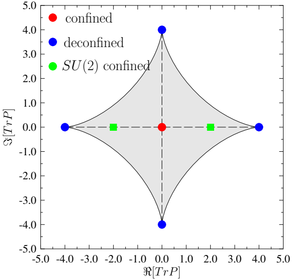
Phases of adjoint QCD: $N_c = 3$, $N_c = 4$, PBC, $N_f > 1$



$N_c = 3$



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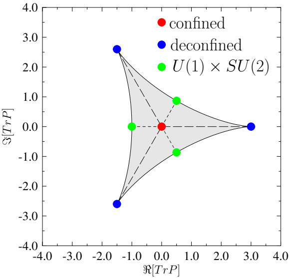
$SU(2)$ conf: $\mathbf{v} = \{0, 0, \pi, \pi\}$

Phases of adjoint QCD: $N_c = 3$, $N_c = 4$, PBC, $N_f > 1$



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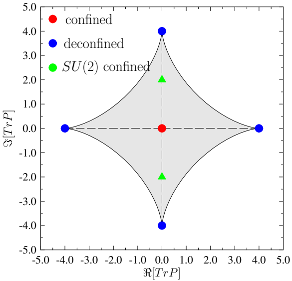

$N_c = 4$

confined: $\mathbf{v} = \{0, \frac{2\pi}{3}, \frac{4\pi}{3}\}$

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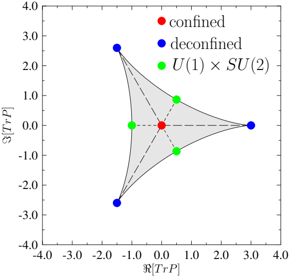

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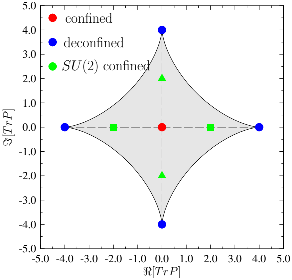


$N_c = 4$

confined: $\mathbf{v} = \{0, \frac{2\pi}{3}, \frac{4\pi}{3}\}$

deconfined: $\mathbf{v} = \{0, 0, 0\}$

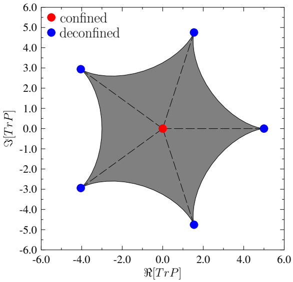
$U(1) \times SU(2)$: $\mathbf{v} = \{0, \pi, \pi\}$

confined: $\mathbf{v} = \{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\}$

deconfined: $\mathbf{v} = \{0, 0, 0, 0\}$

$SU(2)$ conf: $\mathbf{v} = \{0, 0, \pi, \pi\}$

Phases of adjoint QCD: $N = 5$, $N = 6$, PBC, $N_f > 1$

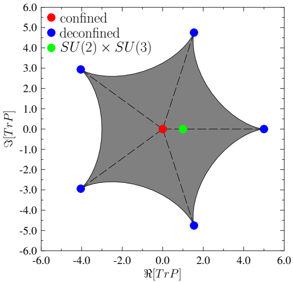


$N = 5$

confined: $\mathbf{v} = \{0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}\}$

deconfined: $\mathbf{v} = \{0, 0, 0, 0, 0\}$

Phases of adjoint QCD: $N=5$, $N=6$, PBC, $N_f > 1$



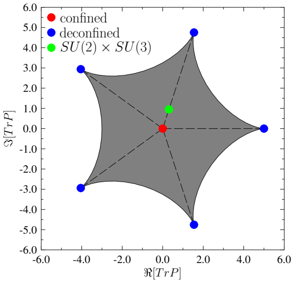
$N=5$

confined: $\mathbf{v} = \{0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}\}$

deconfined: $\mathbf{v} = \{0,0,0,0,0\}$

$SU(2) \times SU(3)$: $\mathbf{v} = \{0,0,0,\pi,\pi\}$

Phases of adjoint QCD: $N = 5$, $N = 6$, PBC, $N_f > 1$



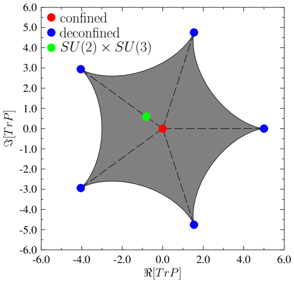
$N = 5$

confined: $\mathbf{v} = \{0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}\}$

deconfined: $\mathbf{v} = \{0, 0, 0, 0, 0\}$

$SU(2) \times SU(3)$: $\mathbf{v} = \{\frac{2\pi}{5}, \frac{2\pi}{5}, \frac{2\pi}{5}, \frac{7\pi}{5}, \frac{7\pi}{5}\}$

Phases of adjoint QCD: $N=5$, $N=6$, PBC, $N_f>1$



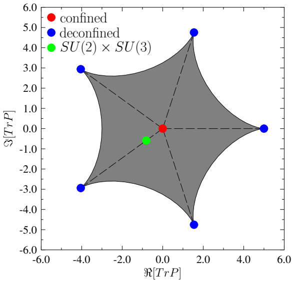
$N=5$

confined: $\mathbf{v}=\{0,\frac{2\pi}{5},\frac{4\pi}{5},\frac{6\pi}{5},\frac{8\pi}{5}\}$

deconfined: $\mathbf{v}=\{0,0,0,0,0\}$

$SU(2)\times SU(3)$: $\mathbf{v}=\{\frac{4\pi}{5},\frac{4\pi}{5},\frac{4\pi}{5},\frac{9\pi}{5},\frac{9\pi}{5}\}$

Phases of adjoint QCD: $N = 5$, $N = 6$, PBC, $N_f > 1$



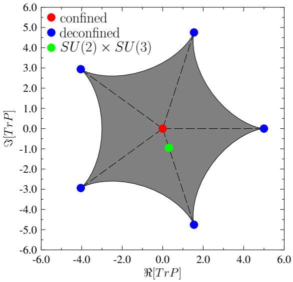
$N = 5$

confined: $\mathbf{v} = \{0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}\}$

deconfined: $\mathbf{v} = \{0, 0, 0, 0, 0\}$

$SU(2) \times SU(3)$: $\mathbf{v} = \{\frac{6\pi}{5}, \frac{6\pi}{5}, \frac{6\pi}{5}, \frac{\pi}{5}, \frac{\pi}{5}\}$

Phases of adjoint QCD: $N=5$, $N=6$, PBC, $N_f>1$



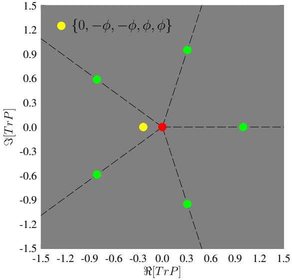
$N=5$

confined: $\mathbf{v}=\{0,\frac{2\pi}{5},\frac{4\pi}{5},\frac{6\pi}{5},\frac{8\pi}{5}\}$

deconfined: $\mathbf{v}=\{0,0,0,0,0\}$

$SU(2)\times SU(3)$: $\mathbf{v}=\{\frac{8\pi}{5},\frac{8\pi}{5},\frac{8\pi}{5},\frac{3\pi}{5},\frac{3\pi}{5}\}$

Phases of adjoint QCD: $N=5$, $N=6$, PBC, $N_f>1$



$N=5$

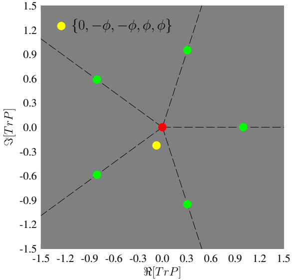
confined: $\mathbf{v}=\{0,\frac{2\pi}{5},\frac{4\pi}{5},\frac{6\pi}{5},\frac{8\pi}{5}\}$

deconfined: $\mathbf{v}=\{0,0,0,0,0\}$

$SU(2)\times SU(3)$: $\mathbf{v}=\{0,0,0,\pi,\pi\}$

$SU(2)\times SU(2)\times U(1)$:
$\mathbf{v}=\{0,-\phi,-\phi,\phi,\phi\}$

Phases of adjoint QCD: $N=5$, $N=6$, PBC, $N_f>1$



$N=5$

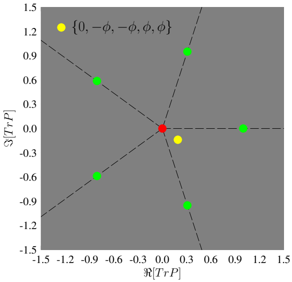
confined: $\mathbf{v}=\{0,\frac{2\pi}{5},\frac{4\pi}{5},\frac{6\pi}{5},\frac{8\pi}{5}\}$

deconfined: $\mathbf{v}=\{0,0,0,0,0\}$

$SU(2)\times SU(3)$: $\mathbf{v}=\{0,0,0,\pi,\pi\}$

$SU(2)\times SU(2)\times U(1)$:
$\mathbf{v}=\{\frac{2\pi}{5},\frac{2\pi}{5}-\phi,\frac{2\pi}{5}-\phi,\frac{2\pi}{5}+\phi,\frac{2\pi}{5}+\phi\}$

Phases of adjoint QCD: $N = 5$, $N = 6$, PBC, $N_f > 1$



$N = 5$

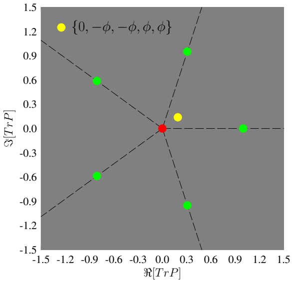
confined: $\mathbf{v} = \{0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}\}$

deconfined: $\mathbf{v} = \{0, 0, 0, 0, 0\}$

$SU(2) \times SU(3)$: $\mathbf{v} = \{0, 0, 0, \pi, \pi\}$

$SU(2) \times SU(2) \times U(1)$:
$\mathbf{v} = \{\frac{4\pi}{5}, \frac{4\pi}{5} - \phi, \frac{4\pi}{5} - \phi, \frac{4\pi}{5} + \phi, \frac{4\pi}{5} + \phi\}$

Phases of adjoint QCD: $N=5$, $N=6$, PBC, $N_f > 1$



$N=5$

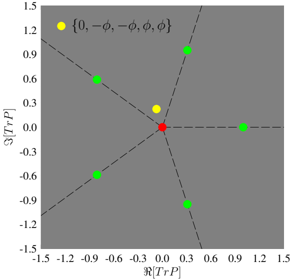
confined: $\mathbf{v} = \{0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}\}$

deconfined: $\mathbf{v} = \{0,0,0,0,0\}$

$SU(2) \times SU(3)$: $\mathbf{v} = \{0,0,0,\pi,\pi\}$

$SU(2) \times SU(2) \times U(1)$:
$\mathbf{v} = \{\frac{6\pi}{5}, \frac{6\pi}{5} - \phi, \frac{6\pi}{5} - \phi, \frac{6\pi}{5} + \phi, \frac{6\pi}{5} + \phi\}$

Phases of adjoint QCD: $N=5$, $N=6$, PBC, $N_f>1$



$N=5$

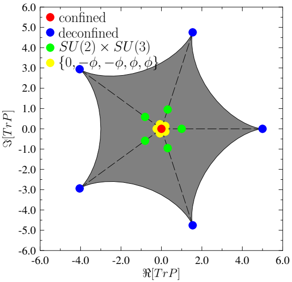
confined: $\mathbf{v}=\{0,\frac{2\pi}{5},\frac{4\pi}{5},\frac{6\pi}{5},\frac{8\pi}{5}\}$

deconfined: $\mathbf{v}=\{0,0,0,0,0\}$

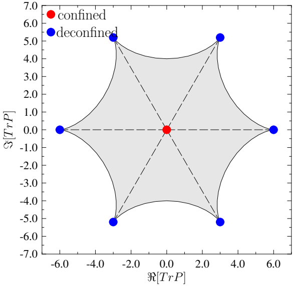
$SU(2)\times SU(3)$: $\mathbf{v}=\{0,0,0,\pi,\pi\}$

$SU(2)\times SU(2)\times U(1)$:
$\mathbf{v}=\{\frac{8\pi}{5},\frac{8\pi}{5}-\phi,\frac{8\pi}{5}-\phi,\frac{8\pi}{5}+\phi,\frac{8\pi}{5}+\phi\}$

Phases of adjoint QCD: $N=5$, $N=6$, PBC, $N_f>1$



$N=5$



$N=6$

confined: $\mathbf{v}=\{0,\frac{2\pi}{5},\frac{4\pi}{5},\frac{6\pi}{5},\frac{8\pi}{5}\}$

deconfined: $\mathbf{v}=\{0,0,0,0,0\}$

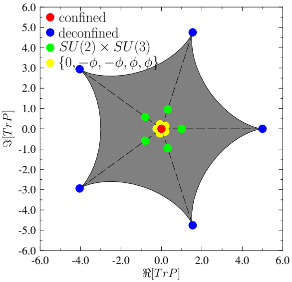
$SU(2)\times SU(3)$: $\mathbf{v}=\{0,0,0,\pi,\pi\}$

$SU(2)\times SU(2)\times U(1)$:
$\mathbf{v}=\{0,-\phi,-\phi,\phi,\phi\}$

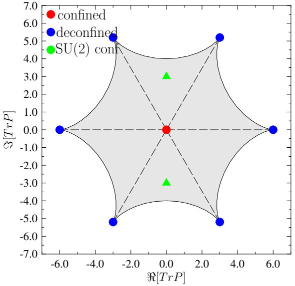
confined: $\mathbf{v}=\{\frac{\pi}{6},\frac{3\pi}{6},\frac{5\pi}{6},\frac{7\pi}{6},\frac{9\pi}{6},\frac{11\pi}{6}\}$

deconfined: $\mathbf{v}=\{0,0,0,0,0,0\}$

Phases of adjoint QCD: $N=5$, $N=6$, PBC, $N_f>1$



$N=5$



$N=6$

confined: $\mathbf{v}=\{0,\frac{2\pi}{5},\frac{4\pi}{5},\frac{6\pi}{5},\frac{8\pi}{5}\}$

deconfined: $\mathbf{v}=\{0,0,0,0,0\}$

$SU(2)\times SU(3)$: $\mathbf{v}=\{0,0,0,\pi,\pi\}$

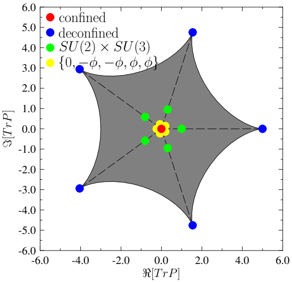
$SU(2)\times SU(2)\times U(1)$:
$\mathbf{v}=\{0,-\phi,-\phi,\phi,\phi\}$

confined: $\mathbf{v}=\{\frac{\pi}{6},\frac{3\pi}{6},\frac{5\pi}{6},\frac{7\pi}{6},\frac{9\pi}{6},\frac{11\pi}{6}\}$

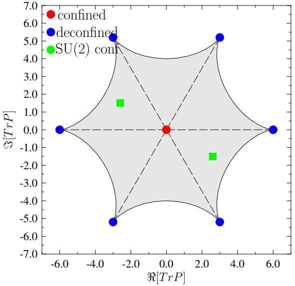
deconfined: $\mathbf{v}=\{0,0,0,0,0,0\}$

$SU(2)$ conf: $\mathbf{v}=\{\frac{\pi}{2},\frac{3\pi}{2},\frac{\pi}{2},\frac{3\pi}{2},\frac{\pi}{2},\frac{3\pi}{2}\}$

Phases of adjoint QCD: $N=5$, $N=6$, PBC, $N_f>1$



$N=5$



$N=6$

confined: $\mathbf{v}=\{0,\frac{2\pi}{5},\frac{4\pi}{5},\frac{6\pi}{5},\frac{8\pi}{5}\}$

deconfined: $\mathbf{v}=\{0,0,0,0,0\}$

$SU(2)\times SU(3)$: $\mathbf{v}=\{0,0,0,\pi,\pi\}$

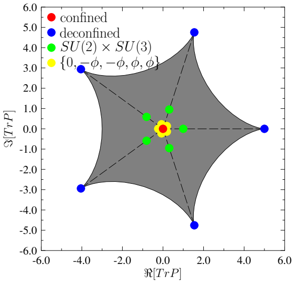
$SU(2)\times SU(2)\times U(1)$:
$\mathbf{v}=\{0,-\phi,-\phi,\phi,\phi\}$

confined: $\mathbf{v}=\{\frac{\pi}{6},\frac{3\pi}{6},\frac{5\pi}{6},\frac{7\pi}{6},\frac{9\pi}{6},\frac{11\pi}{6}\}$

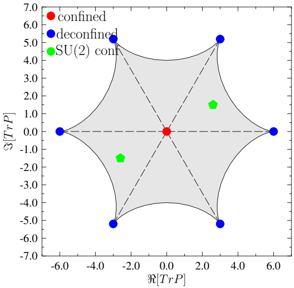
deconfined: $\mathbf{v}=\{0,0,0,0,0,0\}$

$SU(2)$ conf:
$\mathbf{v}=\{\frac{5\pi}{6},\frac{11\pi}{6},\frac{5\pi}{6},\frac{11\pi}{6},\frac{5\pi}{6},\frac{11\pi}{6}\}$

Phases of adjoint QCD: $N=5$, $N=6$, PBC, $N_f>1$



$N=5$



$N=6$

confined: $\mathbf{v}=\{0,\frac{2\pi}{5},\frac{4\pi}{5},\frac{6\pi}{5},\frac{8\pi}{5}\}$

deconfined: $\mathbf{v}=\{0,0,0,0,0\}$

$SU(2)\times SU(3)$: $\mathbf{v}=\{0,0,0,\pi,\pi\}$

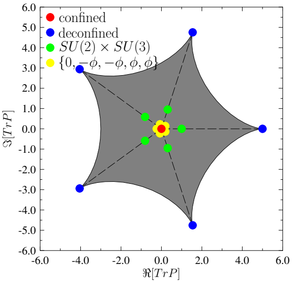
$SU(2)\times SU(2)\times U(1)$:
$\mathbf{v}=\{0,-\phi,-\phi,\phi,\phi\}$

confined: $\mathbf{v}=\{\frac{\pi}{6},\frac{3\pi}{6},\frac{5\pi}{6},\frac{7\pi}{6},\frac{9\pi}{6},\frac{11\pi}{6}\}$

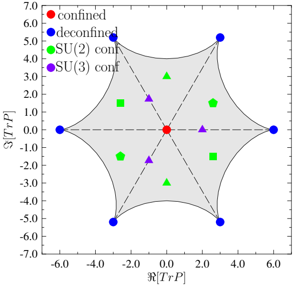
deconfined: $\mathbf{v}=\{0,0,0,0,0,0\}$

$SU(2)$ conf: $\mathbf{v}=\{\frac{7\pi}{6},\frac{\pi}{6},\frac{7\pi}{6},\frac{\pi}{6},\frac{7\pi}{6},\frac{\pi}{6}\}$

Phases of adjoint QCD: $N = 5$, $N = 6$, PBC, $N_f > 1$



$N = 5$



$N = 6$

confined: $\mathbf{v} = \{0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}\}$

deconfined: $\mathbf{v} = \{0, 0, 0, 0, 0\}$

$SU(2) \times SU(3)$: $\mathbf{v} = \{0, 0, 0, \pi, \pi\}$

$SU(2) \times SU(2) \times U(1)$:
$\mathbf{v} = \{0, -\phi, -\phi, \phi, \phi\}$

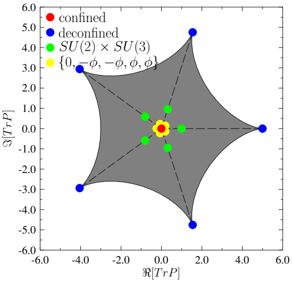
confined: $\mathbf{v} = \{\frac{\pi}{6}, \frac{3\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{9\pi}{6}, \frac{11\pi}{6}\}$

deconfined: $\mathbf{v} = \{0, 0, 0, 0, 0, 0\}$

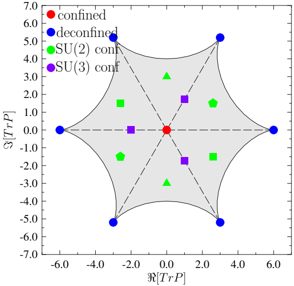
$SU(2)$ conf: $\mathbf{v} = \{\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}\}$

$SU(3)$ conf: $\mathbf{v} = \{0, \frac{2\pi}{3}, \frac{4\pi}{3}, 0, \frac{2\pi}{3}, \frac{4\pi}{3}\}$

Phases of adjoint QCD: $N=5$, $N=6$, PBC, $N_f>1$



$N=5$



$N=6$

confined: $\mathbf{v}=\{0,\frac{2\pi}{5},\frac{4\pi}{5},\frac{6\pi}{5},\frac{8\pi}{5}\}$

deconfined: $\mathbf{v}=\{0,0,0,0,0\}$

$SU(2)\times SU(3)$: $\mathbf{v}=\{0,0,0,\pi,\pi\}$

$SU(2)\times SU(2)\times U(1)$:
$\mathbf{v}=\{0,-\phi,-\phi,\phi,\phi\}$

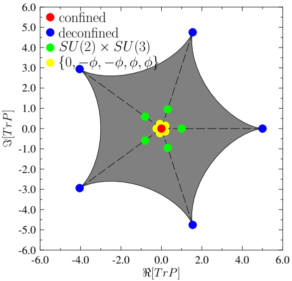
confined: $\mathbf{v}=\{\frac{\pi}{6},\frac{3\pi}{6},\frac{5\pi}{6},\frac{7\pi}{6},\frac{9\pi}{6},\frac{11\pi}{6}\}$

deconfined: $\mathbf{v}=\{0,0,0,0,0,0\}$

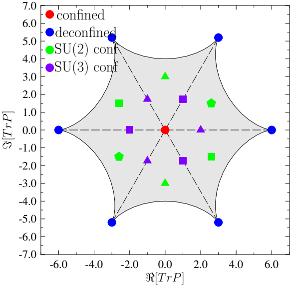
$SU(2)$ conf: $\mathbf{v}=\{\frac{\pi}{2},\frac{3\pi}{2},\frac{\pi}{2},\frac{3\pi}{2},\frac{\pi}{2},\frac{3\pi}{2}\}$

$SU(3)$ conf: $\mathbf{v}=\{\frac{\pi}{3},\pi,\frac{5\pi}{3},\frac{\pi}{3},\pi,\frac{5\pi}{3}\}$

Phases of adjoint QCD: $N=5$, $N=6$, PBC, $N_f > 1$



$N=5$



$N=6$

confined: $\mathbf{v} = \{0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}\}$

deconfined: $\mathbf{v} = \{0,0,0,0,0\}$

$SU(2) \times SU(3)$: $\mathbf{v} = \{0,0,0,\pi,\pi\}$

$SU(2) \times SU(2) \times U(1)$:
$\mathbf{v} = \{0, -\phi, -\phi, \phi, \phi\}$

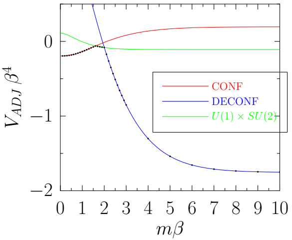
confined: $\mathbf{v} = \{\frac{\pi}{6}, \frac{3\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{9\pi}{6}, \frac{11\pi}{6}\}$

deconfined: $\mathbf{v} = \{0,0,0,0,0,0\}$

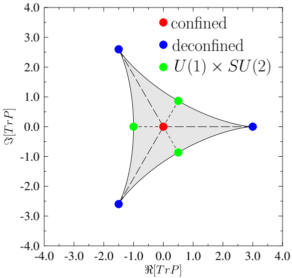
$SU(2)$ conf: $\mathbf{v} = \{\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}\}$

$SU(3)$ conf: $\mathbf{v} = \{0, \frac{2\pi}{3}, \frac{4\pi}{3}, 0, \frac{2\pi}{3}, \frac{4\pi}{3}\}$

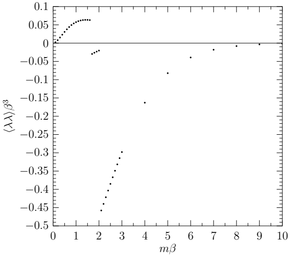
$SU(3)$ Adjoint QCD (PBC) $N_f = 2$



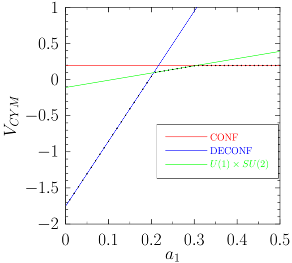
V_{ADJ}, $Nc = 3$, $Nf = 2$



V_{ADJ}, $Nc = 3$ phases



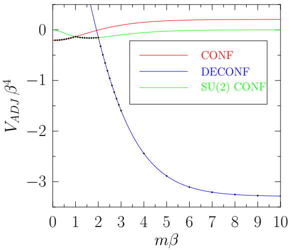
$\langle\lambda\lambda\rangle_{ADJ}$



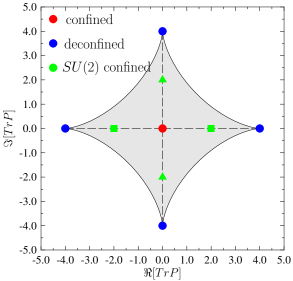
V_{CYM} vs. a_1, $Nc = 3$

- The data points (black dots) were found by minimizing V_{eff} with respect to the Polyakov loop eigenvalues v_i.
- The confined phase is accessible perturbatively for $m\beta \leq 1.6$.
- There is a dramatic jump in $\langle\bar{\psi}\psi\rangle$ corresponding to the deconfinement transition
- The model has the same phases as QCD(Adj)

$SU(4)$ Adjoint QCD (PBC) $N_f = 2$

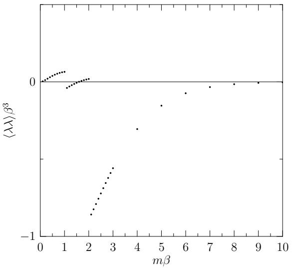


V_{ADJ}, $Nc = 4$, $Nf = 2$

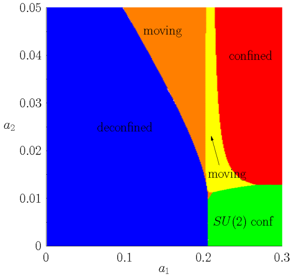


V_{ADJ}, $Nc = 4$ phases

- The confined phase is accessible perturbatively for $m\beta \leq 1.0$.



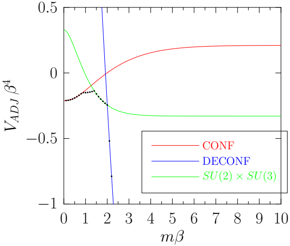
$\langle\lambda\lambda\rangle_{ADJ}$



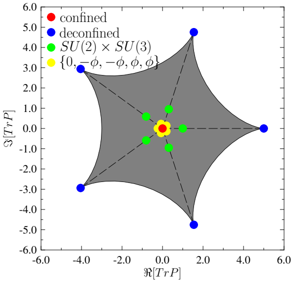
phase diagram of V_{CYM} in the $a_1 - a_2$ plane, $Nc = 4$

- The model has the same phases as QCD(Adj), and more, but the additional phases can be circumnavigated.

$SU(5)$ Adjoint QCD (PBC) $N_f = 2$

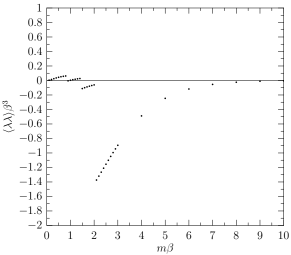


V_{ADJ}, $Nc = 5$, $Nf = 2$

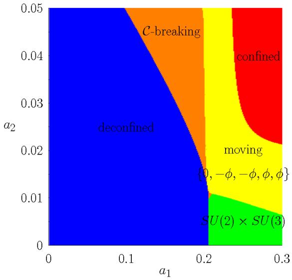


V_{ADJ}, $Nc = 5$ phases

- The confined phase is accessible perturbatively for $m\beta \leq 0.8$.
- A moving phase is found between the confined and $SU(2)\times SU(3)$ phases.



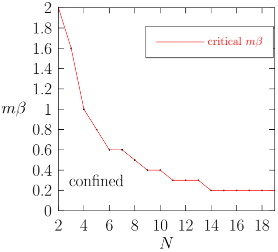
$\langle\lambda\lambda\rangle_{ADJ}$



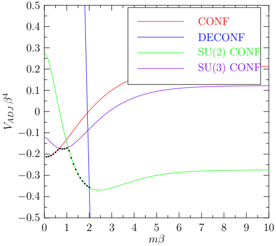
phase diagram of V_{CYM} in the $a_1 - a_2$ plane, $Nc = 5$

- The model includes the phases of QCD(Adj).
- The (non-$\mathscr{C}$-breaking) moving phase of the model is the same as that of QCD(Adj).

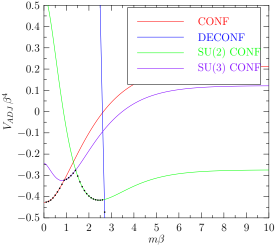
Accessibility of the confined phase as $N \to \infty$, or as N_f is increased



Range of $m\beta$ for which the confined phase is accessible in QCD(Adj) with $N_f = 2$



$V_{ADJ(+)}$ for $N_c = 6$, $N_f = 2$



$V_{ADJ(+)}$ for $N_c = 6$, $N_f = 3$

- As $N \to \infty$ the maximum $m\beta$ for which the confined phase is accessible, $(m\beta)_{crit}$, decreases.
- However, as N_f increases, $(m\beta)_{crit}$ increases (we must have $N_f \leq 5$ Majorana flavours to preserve asymptotic freedom).

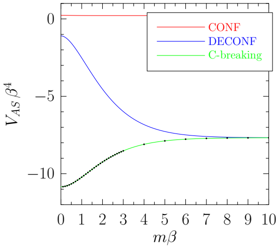
Application 1: Orientifold Planar Equivalence

The story:

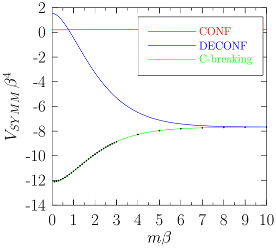
- Armoni, Shifman, and Veneziano (2003 - 2004) prove non-perturbatively the equivalence of the bosonic sectors of QCD(Adj) with N_f Majorana fermions and QCD(AS/S) with N_f Dirac fermions, in the planar limit.
- Unsal and Yaffe (2006) show that on $S^1 \times \mathbb{R}^3$ $\mathscr{C}$-symmetry is broken in QCD(AS/S) when PBC are applied to fermions.
- DeGrand and Hoffman (2007), Lucini et al (2007) showed using lattice simulations that the $\mathscr{C}$-breaking is lifted as S^1 is decompactified
- Lucini et al. (2008) non-perturbatively prove orientifold equivalence in the quenched approximation (in the absence of $\mathscr{C}$-breaking) using lattice simulations and calculate the quark condensate in QCD(AS/S/Adj)

We compare (to 1-loop order) the phase diagrams of $QCD(AS), QCD(S)$ with $N_f = 1$ Dirac flavour to $QCD(Adj)$ with $N_f = 1$ Majorana flavour, for massive fermions with PBC.

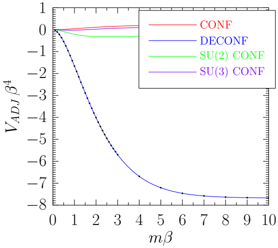
$SU(6)$ QCD(A) (left), (S) (middle), and (Adj) (right) for PBC on fermions



$V_{A,+}$, $Nc = 6$, $Nf = 1$ Dirac flavour



$V_{S,+}$, $Nc = 6$, $Nf = 1$ Dirac flavour



$V_{Adj,+}$, $Nc = 6$, $Nf = 1$ Majorana flavour

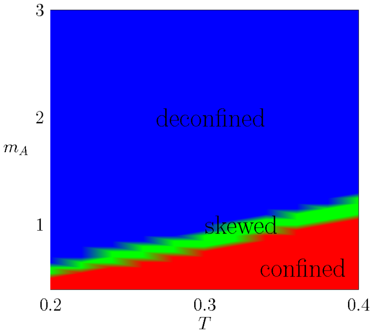
- The $\mathscr{C}$-breaking phase is favoured in the case where PBC are applied to fermions in the A and S representations (When ABC are used the deconfined phase is favoured).
- When $N_c = 6$ the $\mathscr{C}$-breaking phase has the Polyakov loop eigenvalues corresponding to

$$\mathbf{v} = \pm\{\frac{2\pi}{6}, \frac{2\pi}{6}, \frac{2\pi}{6}, \frac{2\pi}{6}, \frac{2\pi}{6}, \frac{2\pi}{6}\} \text{ or } \pm\{\frac{4\pi}{6}, \frac{4\pi}{6}, \frac{4\pi}{6}, \frac{4\pi}{6}, \frac{4\pi}{6}, \frac{4\pi}{6}\}$$

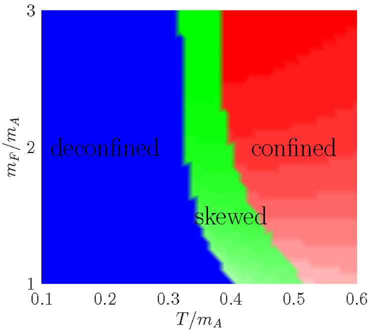
- P is clearly not invariant under $P \to P^*$.

Application 2: QCD with fund. and adj. rep fermions

- Using the effective potential for V_{ADJ} to perturbatively obtain the confined phase, fundamental fermions with ABC can be added in to determine the effect they have on confinement.
- The potential is $V_{DQCD} = V_{ADJ(+)} + V^F_{F(-)}$.



The phase diagram of $V_{ADJ(+)}$ at high temperature as it varies with m_A for $N_A = 4$



The phase diagram of V_{DQCD} at high temperature as it varies with m_F/m_A ($N_F = 3$, $N_A = 4$, $m_A = 1$)

- When the m_F is decreased from infinity the deconfined phase is favoured. In the confined region the Polyakov loop eigenvalues are given by $v = \{-\phi, 0, \phi\}$. When $m_F = \infty$, $\phi = 2\pi/3$. As $m_F \to 0$, ϕ decreases to some limit.

Conclusions

- All extensions to Yang-Mills theory result in an exotic phase structure.
- All extensions lead to perturbative access of the confined phase for $N = 3$.
- The center-stabilized theory and adjoint QCD with $N_f \geq 2$ Majorana flavours leads to perturbative access to the confined phase for all N.
  - As N increases, $(m\beta)_{crit}$, below which the confined phase is accessible, decreases.
  - However, as N_f is increased within the limits allowed by asymptotic freedom, the confined phase becomes accessible for a larger $(m\beta)_{crit}$.
- In $QCD(AS/S)$ with PBC for fermions the $\mathscr{C}$-breaking phase is favoured for all $m\beta$.
- For all representations there is a clear one-loop contribution to $\langle\bar{\psi}\psi\rangle$ for small $m\beta$.
- The center-stabilized theory contains all the phases of adjoint QCD and these can be traversed in the a_n parameter space in the same order as they appear when increasing $m\beta$ in adjoint QCD, avoiding extraneous phases.
- The perturbative access of the confined phase in adjoint QCD allows the addition of fundamental fermions, within the limits allowed by asymptotic freedom, so we can study their effect on confinement analytically.

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DANK U!