Quark Dynamics, Thermal QCD and The Gravity Dual
or
Five Easy Pieces

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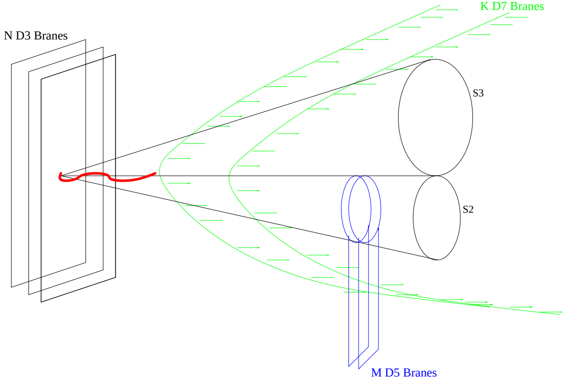
Content

Introduction

- We want to study strongly coupled QGP using string theory methods.
- Perturbative methods fail, so lets be BOLD! String Theory!!
- AdS/CFT teaches us about Conformal Field Theory. What about QCD?
- QCD is a gauge theory. There exists a generalized Gauge/Gravity duality.
- Search for gravity dual of QCD with running gauge coupling: motivation from Klebanov-Strassler Model(**hep-th/0007191**).
- Introduce flavor using D7 branes (Peter Ouyang **hep-th/0311084**), introduce temperature with a black hole(Pando Zayas et al **hep-th/0605170,0707.2737**).
- Compute gauge theory observables.

- Consider 6 dimensional conifold with base $T^{11} = S^2 \times S^3$ with 4 dimensinal flat space; D3 , D5 and D7 branes in following set up



The Gauge Theory

SU(N+M) × SU(N) Co lo r Symmetry

K Flavors

The Gauge Theory

$SU(N+M) \times SU(N)$ Color Symmetry
K Flavors

$$\frac{d}{d \log \Lambda}\left[\frac{4\pi}{g_1^2} + \frac{4\pi}{g_2^2}\right] = -\frac{3K}{4}$$

$$\frac{d}{d \log \Lambda}\left[\frac{4\pi}{g_1^2} - \frac{4\pi}{g_2^2}\right] = 3M\left(1 + \frac{3 g_s K}{4\pi} \log(\Lambda^2)\right)$$

The Gauge Theory

$SU(N+M) \times SU(N)$ Color Symmetry
K Flavors

Seiberg* ↓ *Duality

$SU(N-(M-K)) \times SU(N)$ Color Symmetry
K Flavors

$$\frac{d}{d\log\Lambda}\left[\frac{4\pi}{g_1^2}+\frac{4\pi}{g_2^2}\right] = -\frac{3K}{4}$$

$$\frac{d}{d\log\Lambda}\left[\frac{4\pi}{g_1^2}-\frac{4\pi}{g_2^2}\right] = 3M\left(1+\frac{3 g_s K}{4\pi}\log(\Lambda^2)\right)$$

The Gauge Theory

SU(N+M) × SU(N) Color Symmetry
K Flavors

↓ ***Seiberg Duality***

SU(N−(M−K)) × SU(N) Color Symmetry
K Flavors

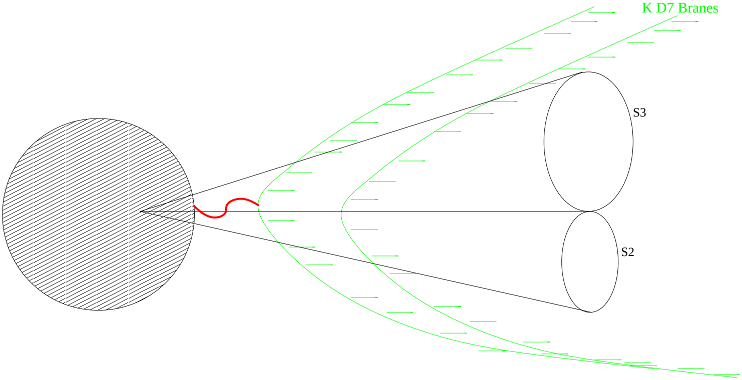
↓ ***Cascade of Seiberg Dualities***

SU(N) Color Symmetry
K Flavors

$$\frac{d}{d\log\Lambda}\left[\frac{4\pi}{g_1^2}+\frac{4\pi}{g_2^2}\right] = -\frac{3K}{4}$$

$$\frac{d}{d\log\Lambda}\left[\frac{4\pi}{g_1^2}-\frac{4\pi}{g_2^2}\right] = 3M\left(1+\frac{3g_sK}{4\pi}\log(\Lambda^2)\right)$$

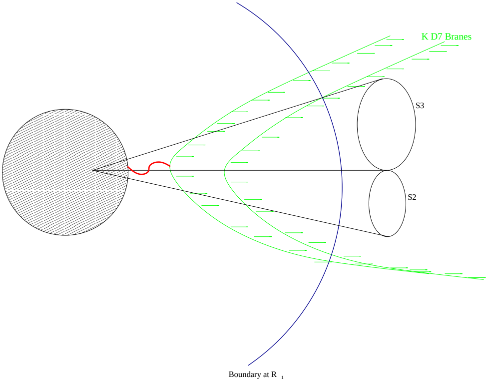
- The geometry that contains the Hilbert space of the above gauge theory for finite temperature is



The Gravity Dual

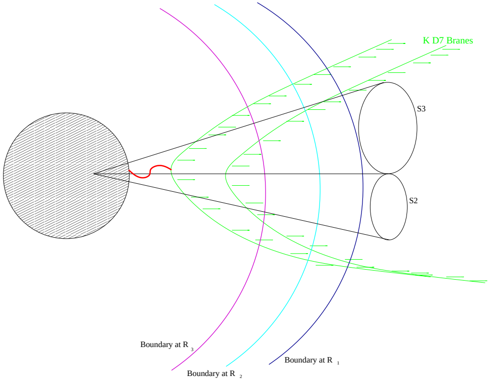
- Introduce radial cutoff $R_1 = \Lambda_1$ in the gravity action, then

$$\mathcal{Z}_{\mathrm{QCD}}[\phi_0(\Lambda_1)] \equiv \exp(S_{\mathrm{total}}[\phi_0(\Lambda_1)]) = \exp \int_{r_h}^{R_1} dr \int d^4x \mathcal{L}_{\mathrm{gravity}}[\phi_0]$$



- Introduce radial cutoff $R_i = \Lambda_i$ in the gravity action, then

$$\mathcal{Z}_{\rm QCD}[\phi_0(\Lambda_i)] \;\equiv\; \exp(S_{\rm total}[\phi_0(\Lambda_i)]) = \exp \int_{r_h}^{R_i} dr \int d^4x \, \mathcal{L}_{\rm gravity}[\phi_0]$$



Quark Mass and Drag

- We study gravity in with the five dimensional effective action coming from the ten dimensional geometry.
- The energy of the static string is the thermal mass of the quark in the dual gauge theory.

$$m(T) = T_0 L^2 (r_0 - r_h) = T_0 L^2 (r_0 - \pi T) \qquad (3)$$

Quark Mass and Drag

- To obtain the product of quark's *effective* mass M and drag coefficient μ, consider strings moving with constant velocity

$$x(t,r) = \bar{x}(r) + vt \tag{4}$$

- The rate at which a quark dumps energy and momentum into the thermal medium is precisely the rate at which the string loses energy and momentum to the black hole. Thus we have $\mu\, Mv/\sqrt{1-v^2} = \Pi_1^x(r = r_h)$ and

$$\mu = T_0 L^2 C/M\sqrt{1-v^2}$$
$$\frac{T^2\pi^2}{\sqrt{\left(1+\frac{3g_s\alpha M^2}{2\pi N}\log(\frac{T\pi}{(1-v^2)^{\frac{1}{4}}})(1+\frac{3g_s\beta K}{2\pi}(\log(\frac{T\pi}{(1-v^2)^{\frac{1}{4}}})+\frac{1}{2})\right)}} \tag{5}$$

Wake

- Energy momentum tensor is obtained from supergravity action

$$\langle T^{pq}\rangle \;=\; \frac{1}{\kappa}\frac{\delta S_{\text{total}}}{\delta l_{pq}}\bigg|_{\kappa l_{pq}=0} \tag{6}$$

- Writing the metric perturbation in Fourier space

$$l^{[i]}_{\mu\nu}(x^\alpha) = \sum_{k=0}^{\infty}\int \frac{d^4q}{(2\pi)^4}\left[e^{-i(q_\nu x^\nu)}s^{(k)[i]}_{\mu\nu}(q^\beta)\frac{1}{r^k}\right] \tag{7}$$

- Obtain the stress tensor

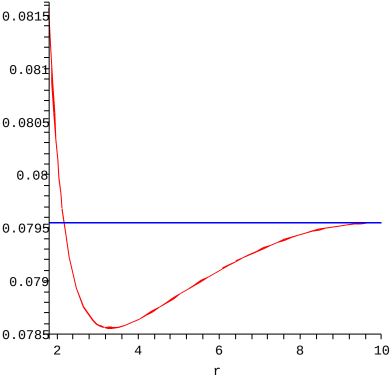
$$T^{mm}_{\text{background+string}} = \int \frac{d^4q}{(2\pi)^4} \Bigg\{ (H^{mn} + H^{nm}) s^{(4)[0]}_{nn} + (\widetilde{H}^{mn} + \widetilde{H}^{nm}) s^{(4)[1]}_{nn} + \left[\sum_{j=0}^{\infty} \frac{\widetilde{b}_{n(j)}}{R_1^j (1-\zeta)^j} \right] \widetilde{J}^n \delta_{nm} + \left[\sum_{j=0}^{\infty} \frac{\widetilde{d}_{m(j)}}{R_1^j (1-\zeta)^j} \right] \widetilde{Q}^m \delta_{nm} \Bigg\} \theta(\zeta - \zeta_0) \qquad (8)$$

- η obtained from Kubo formula:

$$\eta = \lim_{\omega\to 0}\frac{1}{2\omega}\int dt d^3x\, e^{i\omega t}\langle[T_{23}(x),T_{23}(0)]\rangle$$
$$= -\lim_{\omega\to 0}\frac{\mathrm{Im}\, G^R(\omega,0)}{\omega} \qquad (9)$$

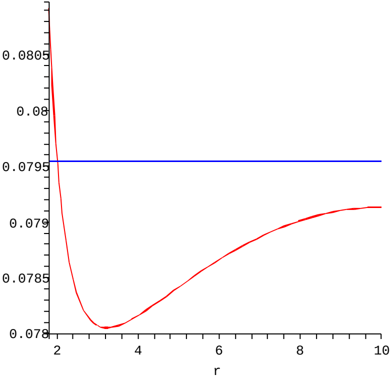
  with $G^R(\omega,0)$ obtained from the gravity dual.
- Entropy density obtained from Wald's formula.

- η/s as a function of renormalizations scale $r = \Lambda$,for theories *without* Riemann Square term is (*assuming entropy doesn't run*)



- Shape is consistent with Csernai,Kapusta and McLerran (**nucl-th/0604032**).

- η/s as a function of renormalizations scale $r = \Lambda$,for theories *with* Riemann Square term (Kats and Petrov **0712.0743**) is (*assuming entropy doesn't run*)



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Conclusion

- We have obtained gravity dual of a gauge theory which has logarithmic running.
- We obtain $1/\sqrt{A\log(T) + B(\log(T))^2}$ correction to AdS/CFT computation of drag.
- We observe $\mathcal{O}(1/R_1), \mathcal{O}(\log(R_1))$ correction to wake , R_1 being the boundary.
- η/s maybe violated.
- Coming up: *Five Not So Easy Pieces* (M.Mia, K.Dasgupta, C.Gale and S.Jeon)