The classical field created in early stages of high energy nucleus-nucleus collisions

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In collaboration with Jean-Paul Blaizot, arXiv:0806.1422 [hep-ph]

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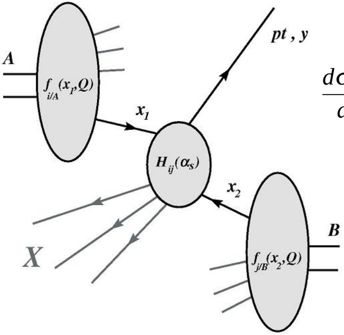
QCD-factorization

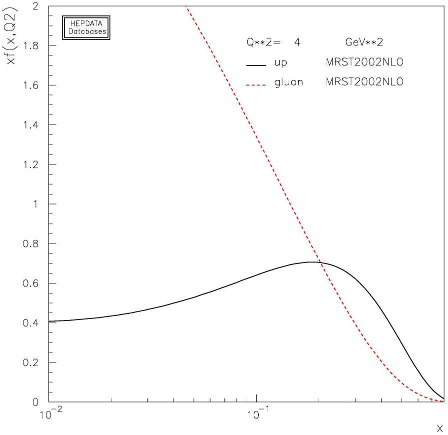
Perturbative QCD describes successfully hard processes : Deep Inelastic Scattering (DIS), proton-proton, etc.



$$\frac{d\sigma^{AB\to qX}}{d^2p_t dy} \equiv \sum_{i,j} f_{i/A}(x_1,Q) \otimes f_{j/B}(x_1,Q) \otimes H_{ij}(\alpha_s)$$

$Q \sim p_t \gg \Lambda_{QCD}$ is the hard scale which justifies the application of perturbative QCD. The Parton Distributions $f_i(x,Q)$ are non-perturbative quantities, but their evolution with respect to Q (DGLAP 1972-1977) is perturbative:

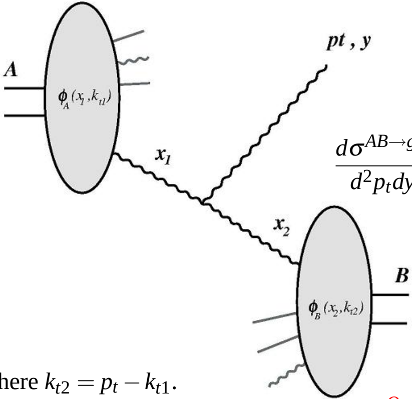
$$Q\frac{df_i(x,Q)}{dQ} = P_{ij} \otimes f_j(x,Q)$$



- At high energy (small x), the gluon parton distribution increases quickly.
- Gluons dominate the dynamics!
- Saturation of gluon distribution is required to preserve unitarity

k_t-factorization

At small $x = k^+/P^+$, the gluon distribution is made of soft gluons ($x \ll 1$).



$$\frac{d\sigma^{AB\to gX}}{d^2p_t dy} \equiv \frac{1}{p_t^2} \int d^2k_{t1}\ \phi_A(x_1, k_{t1})\ \phi_B(x_2, k_{t2})$$

where $k_{t2} = p_t - k_{t1}$.

$$xg(x, Q) = \int^Q d^2k_t\ \phi(x, k_t)$$

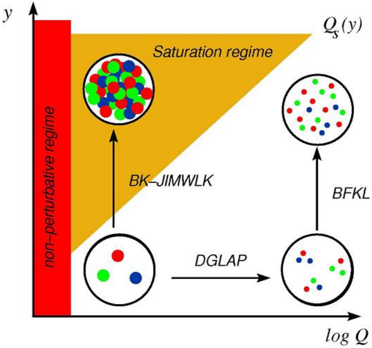
It obeys the BFKL equation (1977-1978)

$$\frac{d\phi(x, k_t)}{d\log\frac{1}{x}} = \chi \otimes \phi(x, k_t)$$

QCD at high parton density

The Color Glass Condensate is nuclear matter made of saturated gluons at high energy . L. McLerran and R. Venugopalan (1995) Yuri. V. Kovchegov and I.Balitsky(1996-1999) J.Jalilian-Marian, E. Iancu, L. McLerran, H. Weigert, A. Leonidov and A. Kovner (1997-2002)

The saturation scale $Q_s(y = \log \frac{1}{x}) >> \Lambda_{QCD}$ is the relevant scale in the problem.



QCD at high parton density

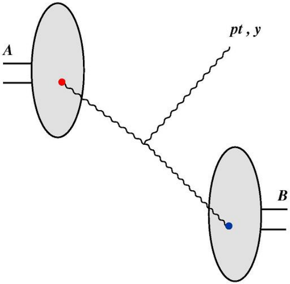
Gluon production at high energy reads:

$$\frac{dN^{AB}}{dy d^2\boldsymbol{p}_\perp} = \sum_\lambda |\mathcal{M}_\lambda|^2.$$

where the amplitude is related to the classical gauge field by the reduction formula:

$$\mathcal{M}_\lambda = \lim_{p^2\to 0} p^2 A_\mu(p)\varepsilon^\mu_\lambda \ ,$$

A_μ is determined by solving the Yang-Mills equations: $[D_\mu, F^{\mu\nu}] = J^\nu[\rho]$.



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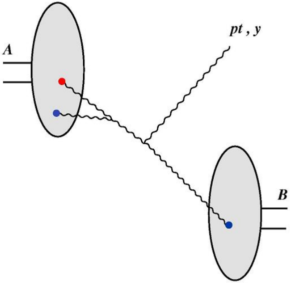

When the color sources are dense, i.e., $\rho \sim 1/g$, gluon recombination starts to be important.

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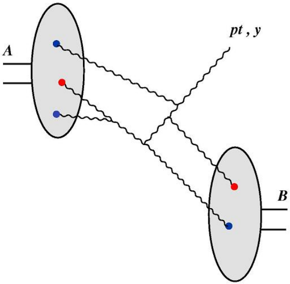

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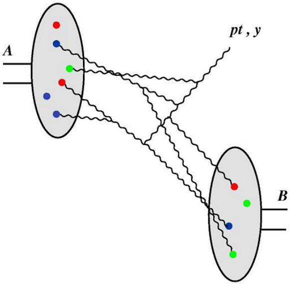

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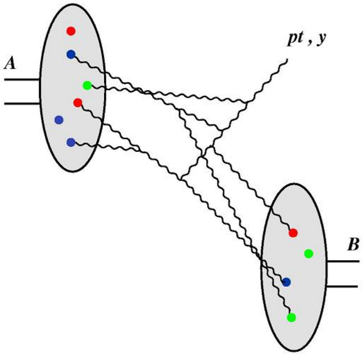
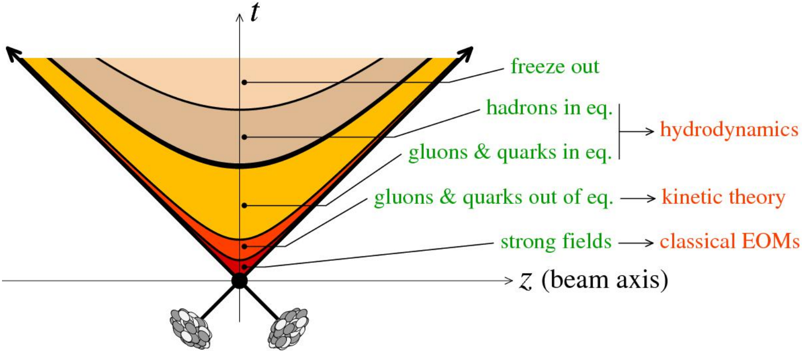


After the collision, $t > 0$, gluons present in the nuclear wave function at high density are freed, and eventually will lead to the formation of the QGP.

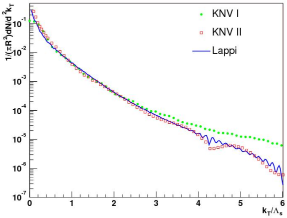
Motivations

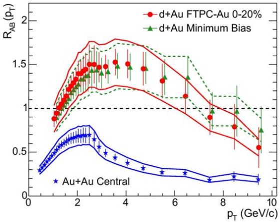


From F. Gelis and R. Venugopalan (2006)

Motivations

- Does k_t-factorization hold in AA collisions?
- Can we disentangle contributions from initial state interactions and final state interactions?
- An analytic approach of the problem is needed to answer these questions.





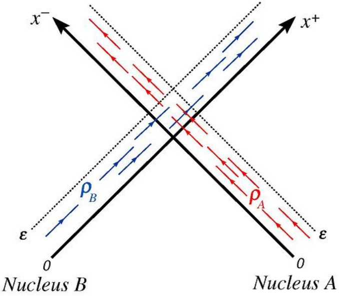
A. Krasnitz, R. Venugopalan (1999-2001), R. T.Lappi(2004)

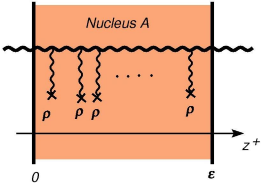
Nuclei fields before the collision

The high energy nuclei A and B are moving respectively in the $-z$ and the $+z$ direction. They are described by the charge densities ρ_A and ρ_B. Because of Lorentz contraction, they are confined on the light cone, i.e. where $x^- = t - z$ and $x^+ = t + z$ are the light cone variables.

The relevant variables in high density QCD are Wilson lines, U and V, associated to each nucleus:

$$U(x^+, \mathbf{x}) \equiv \mathscr{P}_+ \exp\left[ig \int_{-\infty}^{x^+} dz^+ \frac{1}{\partial_\perp^2} \rho_A . T\right] \; , \; V(x^-, \mathbf{x}) \equiv \mathscr{P}_- \exp\left[ig \int_{-\infty}^{x^-} dz^- \frac{1}{\partial_\perp^2} \rho_B . T\right] \; ,$$

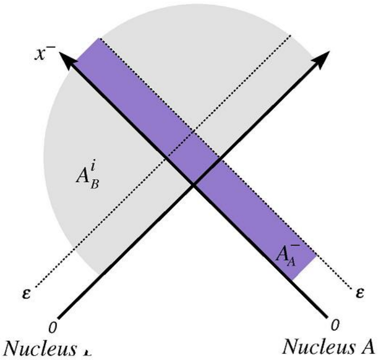




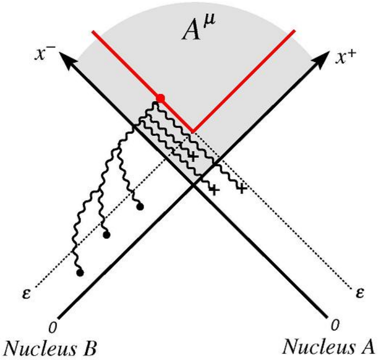
Nuclei fields before the collision

We chose the light-cone gauge $A^+ = 0$.
The fields before the collision are

$$A_A^- = \frac{1}{\partial_\perp^2}\rho_A(x^+,\boldsymbol{x}), \quad A_B^i \cdot T = -\frac{1}{ig}V^\dagger(x^-,\boldsymbol{x})\partial^i V(x^-,\boldsymbol{x})$$



The gauge field immediately after the collision

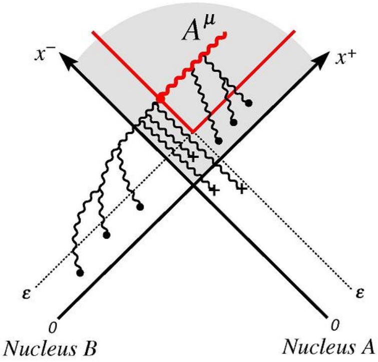


The Yang-Mills equations can be solved exactly near the light-cone:

$$A^i \equiv \alpha_0^i = (U-1)A_B^i \ .$$

A. Kovner, L.D. McLerran and H. Weigert (1995)

How to deal with final state interactions?



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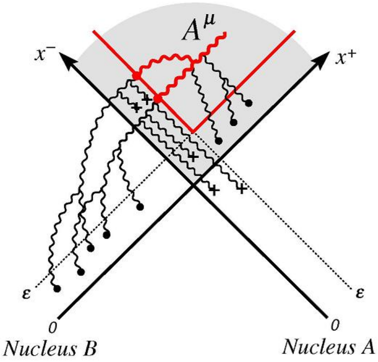


Two sorts of final state interactions :

- *non-physical* f.s.i :
  The produced gluon can interact with *non-physical* gluons of the pure gauge field of nucleus B.
- *physical* f.s.i :
  Merging of two produced gluons.

The gauge field after the collision

To get rid of non-physical f.s.i we perform a gauge rotation which affects only the fields at $x^- > 0$

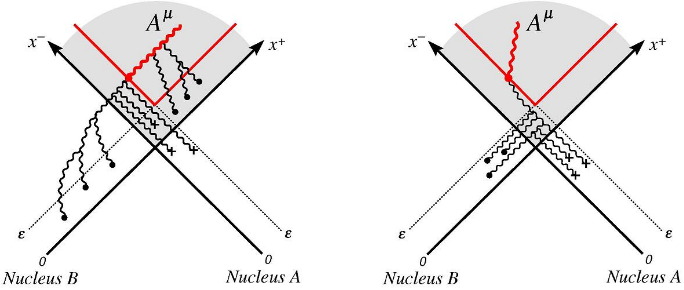
$$A^i_{_B} \cdot T \rightarrow V(A^i_B \cdot T)V^\dagger + \frac{1}{ig}(\partial^i V)V^\dagger = 0$$

$$A^+_{_B} \cdot T \rightarrow \frac{1}{ig}(\partial^+ V)V^\dagger = \frac{1}{\partial^2_\perp}\rho_B \cdot T$$

Corresponding to the gauge choice $A^+ = \frac{1}{\partial^2_\perp}\rho_B = \Phi_{_B}$ or $\partial^- A^+ = 0$ The pure gauge transverse field of nucleus B is removed. With this gauge choice the initial configuration of nuclei fields is linear in the sources (similar to covariant gauge)

$$A^+_{(0)} = \frac{1}{\partial^2_\perp}\rho_B, \quad A^-_{(0)} = \frac{1}{\partial^2_\perp}\rho_A, \quad A^i_{(0)} = 0$$

The gauge field after the collision



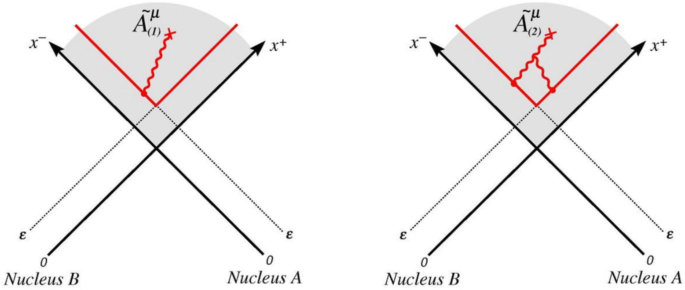
$$A^+ = 0 \Rightarrow A^+ = \Phi_B$$

These two gauges are compatible with the choice $\partial^- A^+ = 0$

Final state interactions expansion

Our strategy will then be to construct the solution through an iterative procedure.

$$\tilde{A}^{\mu} = \tilde{A}^{\mu}_{(0)} + \tilde{A}^{\mu}_{(1)} + \tilde{A}^{\mu}_{(2)} + \ldots$$



Note that near the light-cone $A^i = A^i_{(1)}$ and $A^i_{(n)}(x^- = \varepsilon) = 0$ for $n > 1$. That means that some time is needed for the gluon merging to occur.

Gluon production in the absence of final state interactions

The linearized dynamical equation for the transverse field leads to (for $x^- > 0$ and $x^+ > 0$)

$$\Box A^i_{(1)} = -\partial^i(\partial_\mu A^\mu_{(1)})$$

The gauge field in the absence of final state interactions

$$\Box A^i_{(1)} = 2\delta(x^+)\delta(x^-)\alpha^i_{_0}(\boldsymbol{x}) \\ -\theta(x^+)\theta(x^-)\partial^i\left[\beta_{_0}(\boldsymbol{x}) - \partial^j\alpha^j_{_0}(\boldsymbol{x})\right] ,$$

where $\alpha_{_0}$ and $\beta_{_0}$ are the initial conditions for the field on the light-cone.

$$\alpha^i_{_0} = V(U-1)A^i_{_B} \text{ and } \beta_{_0} = V\partial^i(U-1)A^i_{_B}$$

Gluon production in the absence of final state interaction

We get a compact form for the gluon production cross-section in nucleus-nucleus collision at high energy

$$2E\frac{dN}{d^3q} = \frac{1}{\boldsymbol{q}^2}\langle|\boldsymbol{q}\times\alpha_0(\boldsymbol{q})|^2 + |\beta_0(\boldsymbol{q})|^2\rangle \ ,$$

Already at this order (no final state interactions) k_t-factorization breaks down!
The same result is obtained in the Fock-Schwinger gauge $x^+A^- + x^-A^+ = 0$.

J.-P. Blaizot, Y. M.-T.(2008)

$\langle\ldots\rangle$ stands for the statistical average over the random sources ρ_A and ρ_B.

F. Gelis, R. Venugopalan, R.T. Lappi (2008)

Application: proton-nucleus collisions

Our first order (absence of f.s.i) should reproduce the proton-nucleus case. For A=nucleus and B=proton, the gluon production amplitude is

$$\mathscr{M}^i = -2i \int \frac{d^2\boldsymbol{k}}{(2\pi)^2} \left\{ k^i - \frac{q^i}{\boldsymbol{q}^2}\boldsymbol{k}^2 \right\} U(\boldsymbol{q}-\boldsymbol{k})\Phi_{_B}(\boldsymbol{k})$$

F. Gelis and Y. M-T (2006)

Note that we reproduce the Lipatov vertex $\left(k^i - \frac{q^i}{\boldsymbol{q}^2}\boldsymbol{k}^2\right)^2 = \frac{\boldsymbol{k}^2(\boldsymbol{q}-\boldsymbol{k})^2}{\boldsymbol{q}^2}$

$$\frac{dN^{pA}}{dyd^2\boldsymbol{q}} \propto \frac{1}{\boldsymbol{q}^2} \int d^2\boldsymbol{k}\varphi_A(\boldsymbol{q}-\boldsymbol{k})\varphi_p(\boldsymbol{k}),$$

where $\varphi_A(\boldsymbol{q}-\boldsymbol{k}) = (\boldsymbol{q}-\boldsymbol{k})^2\langle \mathbf{Tr} U^\dagger(\boldsymbol{q}-\boldsymbol{k})U(\boldsymbol{q}-\boldsymbol{k})\rangle$ is the generalized unintegrated gluon distribution of the nucleus described in the framework of the CGC, and $\varphi_p(\boldsymbol{k}) = \boldsymbol{k}^2\langle \Phi_{_p}(\boldsymbol{k})\Phi_{_p}(\boldsymbol{k})\rangle$

Yuri. V. Kovchegov and A. H. Mueller (1998)

A. Dumitru and L. McLerran and R. Venugopalan 2001

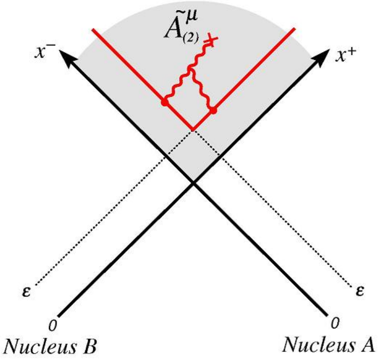
J-P Blaizot, F. Gelis and R. Venugopalan (2004)

Including final state interactions

Are final state interactions important?

Yuri. V. Kovchegov (2000)

To answer this question we must calculate the one gluon merging order i.e. $A^i_{(2)}$



Now the calculation is as involved as the calculation for a usual gluon vertex!
To be compared to I.Balitsky (2004) who proposed a different approached based on an expansion in powers of $[U, V]$. .

Conclusion and Outlook

- We have developed a formalism in L.C.G to calculate the strong gauge field in nucleus-nucleus collisions at high energy.
- We discussed the ambiguity due to the residual gauge freedom, and show that on can get rid of non-physical final state interactions by introducing the L.C.G $A^+ = \Phi_B$ with a proper choice of initial conditions.
- We calculate the gauge field up to the second iteration and provide the gluon production cross-section in the absence of f.s.i. (first iteration). At this order k_t-factorization breaks down in nucleus-nucleus collisions.
- It would be interesting to estimate f.s.i and see their effect on the gluon spectrum, and compare to available numerical calculations.