The glasma initial state and JIMWLK factorization

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Abstract

I will discuss the CGC/glasma framework relating heavy ion collisions to the small-x wavefunction. I describe recent work on a factorization theorem for the leading log corrections to the glasma and the application of methods used in the proof to instabilities.

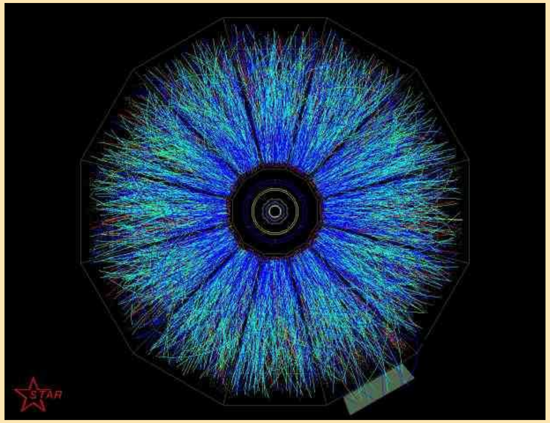
Outline

- The little bang at RHIC and LHC
- Color glass and glasma
- NLO corrections to classical field: JIMWLK factorization

Talk based on

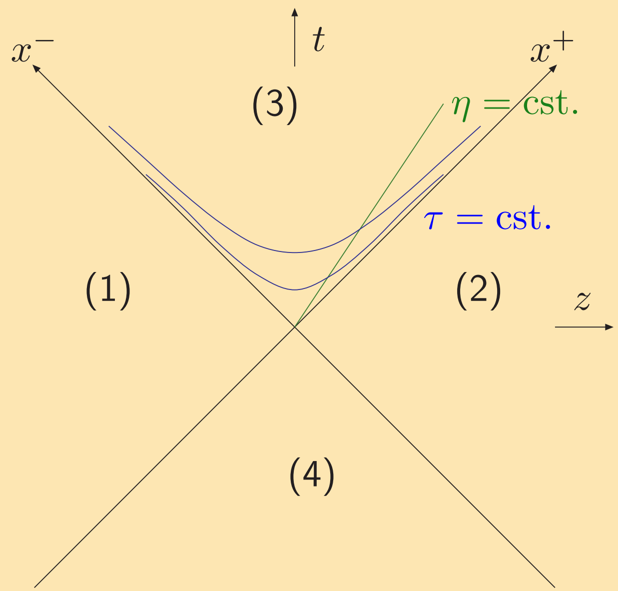
- F. Gelis, T. L. and R. Venugopalan, “High energy factorization in nucleus-nucleus collisions,” arXiv:0804.2630 [hep-ph] (PRD tbp).
- F. Gelis, T. L. and R. Venugopalan, “High energy factorization in nucleus-nucleus collisions II — Multigluon correlations,” arXiv:0807.1306 [hep-ph] (PRD tbp).

Little bang at RHIC



Collide two heavy nuclei at
$\sqrt{s} = 200A\text{GeV}$ (RHIC) or $5500A\text{GeV}$ (LHC).

At early times ($\tau \ll R_A$) expansion is 1-dimensional, to a first approximation boost invariant $\equiv \partial_\eta = 0$.



- Boost invariant *field configurations*
  - ▶ 1 dim. Hubble expansion,
  - ▶ Boost inv. broken by quantum fluctuations
    - ▶ **subject of this talk**
- Coherent initial state
  - ▶ thermalization, isotropization **?**
- Locally isotropic, boost invariant hydrodynamical expansion ?

Coordinate system: Proper time $\tau = \sqrt{2x^-x^+}$, Rapidity $\eta = \frac{1}{2}\ln x^+/x^-$

Glass and Glasma

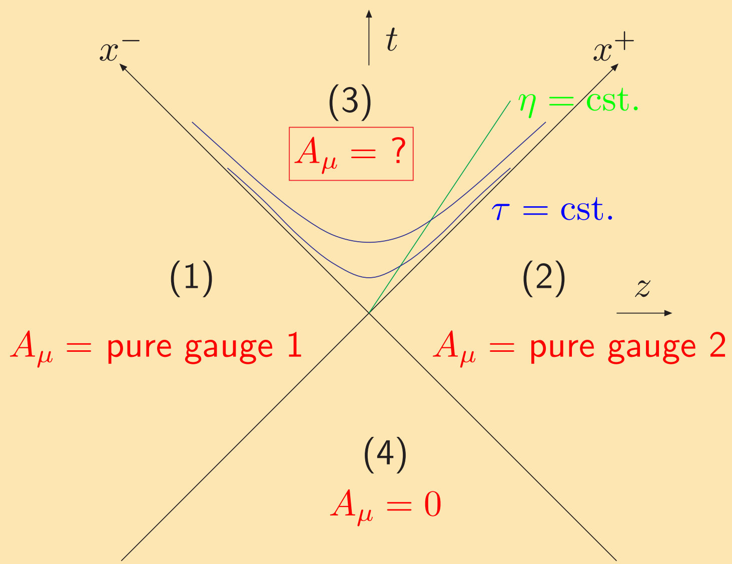
Gluon saturation: At large energies (small x) the hadron/nucleus wavefunction is characterized by saturation scale $Q_s \gg \Lambda_{QCD}$.

At $\mathbf{p}_T \sim Q_s$: strong gluon fields $A_\mu \sim 1/g$ ▶ large occupation numbers $\sim 1/\alpha_s$ ▶ classical field approximation.

CGC: The saturated wavefunction of one hadron/nucleus too many references to mention here

Glasma:[1]

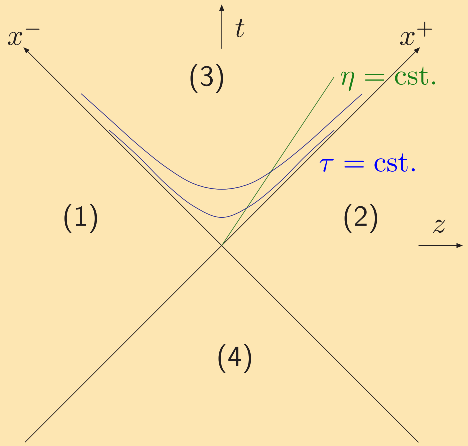
- The coherent, classical field configuration of two colliding sheets of CGC.
- Initial condition for heavy ion collision at $0 < \tau \lesssim 1/Q_s$.



[1] T. Lappi and L. McLerran, *Nucl. Phys.* **A772** (2006) 200 [hep-ph/0602189].

Weizsäcker-Williams color field, MV model

Separation of scales between small x and large x:



classical field **color charge**

$$[D_\mu, F^{\mu\nu}] \quad = \quad J^\nu$$

$$J^\mu = \delta^{\mu+}\rho_{(1)}(\mathbf{x}_T)\delta(x^-) + \delta^{\mu-}\rho_{(2)}(\mathbf{x}_T)\delta(x^+)$$

What is the charge density $\rho(\mathbf{x}_T)$? A static (**glass!**) stochastic variable, distribution

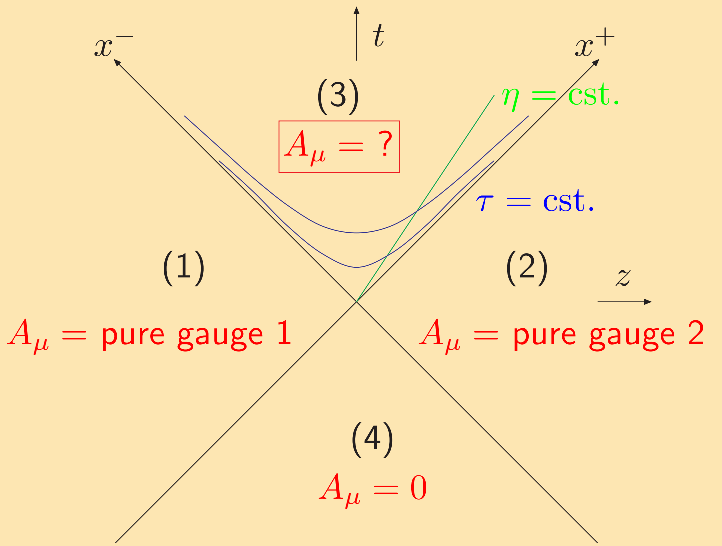
$$\boxed{W_y[\rho(\mathbf{x}_T)]}$$

E.g. MV model [2]: $W[\rho(\mathbf{x}_T)] \sim \exp\left[-\frac{1}{2}\int \mathrm{d}^2\mathbf{x}_T \rho^a(\mathbf{x}_T)\rho^a(\mathbf{x}_T)/g^2\mu^2\right]$

Cannot compute $W_y[\rho(\mathbf{x}_T)]$ from first principles, but can derive evolution equation for $y = \ln 1/x$–dependence: **JIMWLK**. Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner

[2] L. D. McLerran and R. Venugopalan, *Phys. Rev.* **D49** (1994) 2233 [hep-ph/9309289].

From glass to glasma: initial condition

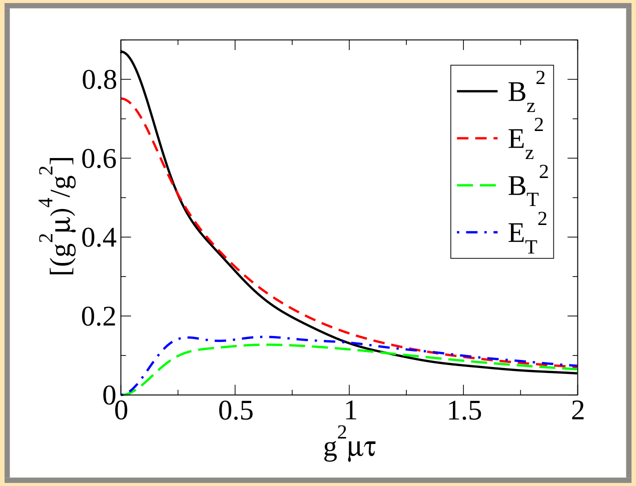


Analytical solution for (1) & (2): pure gauge

▼

Initial condition (Kovner, McLerran, Weigert[3]) for numerical solution (Krasnitz, Venugopalan[4]) in region (3):

▼



See Yacine's talk

For $Q_s\tau \lesssim 1$ negative "longitudinal pressure"

$$T_{zz} = \frac{1}{2}\left(E_T^2 - E_z^2 + B_T^2 - B_z^2\right)$$

▼

no particle interpretation at early times.

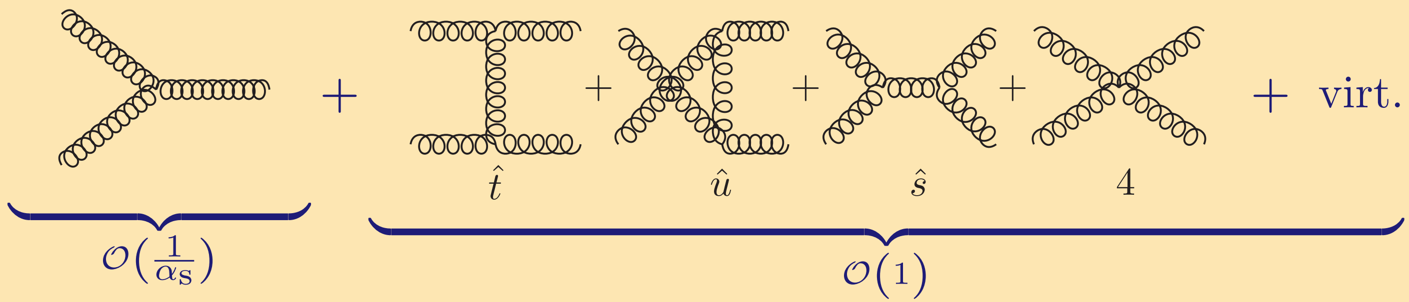
[3] A. Kovner, L. D. McLerran and H. Weigert, *Phys. Rev.* **D52** (1995) 3809 [hep-ph/9505320].
[4] A. Krasnitz and R. Venugopalan, *Nucl. Phys.* **B557** (1999) 237 [hep-ph/9809433].

NLO corrections, factorization: BFKL

Kinematics easily understood in weak field / BFKL limit:

$$\frac{\mathrm{d}N}{\mathrm{d}^2\mathbf{p}_T\,\mathrm{d}y} = \frac{1}{\alpha_\mathrm{s}}\frac{1}{\mathbf{p}_T{}^2}\int\frac{\mathrm{d}^2\mathbf{k}_T}{(2\pi)^2}\phi_y(\mathbf{k}_T)\phi_y(\mathbf{p}_T-\mathbf{k}_T)\left[1+\alpha_\mathrm{s}\int\,\mathrm{d}y'(\cdots)\right]$$

$$\frac{\mathrm{d}N}{\mathrm{d}^2\mathbf{p}_T\,\mathrm{d}y} =$$



Divergence $\Delta y = y - y' \to \infty$ (t–channel) compensated with BFKL evolution of unintegrated pdf's $\phi_y(\mathbf{k}_T)$ ▶ $\mathbf{k}_T$-factorization.

JIMWLK is nonlinear generalization of BFKL: **RGE for** $W_y[\rho(\mathbf{x}_T)]$ (in stead of $\phi_y(\mathbf{k}_T)$)

So far **derived** for DIS, but not **proven** to be universal (for gluon production in AA).

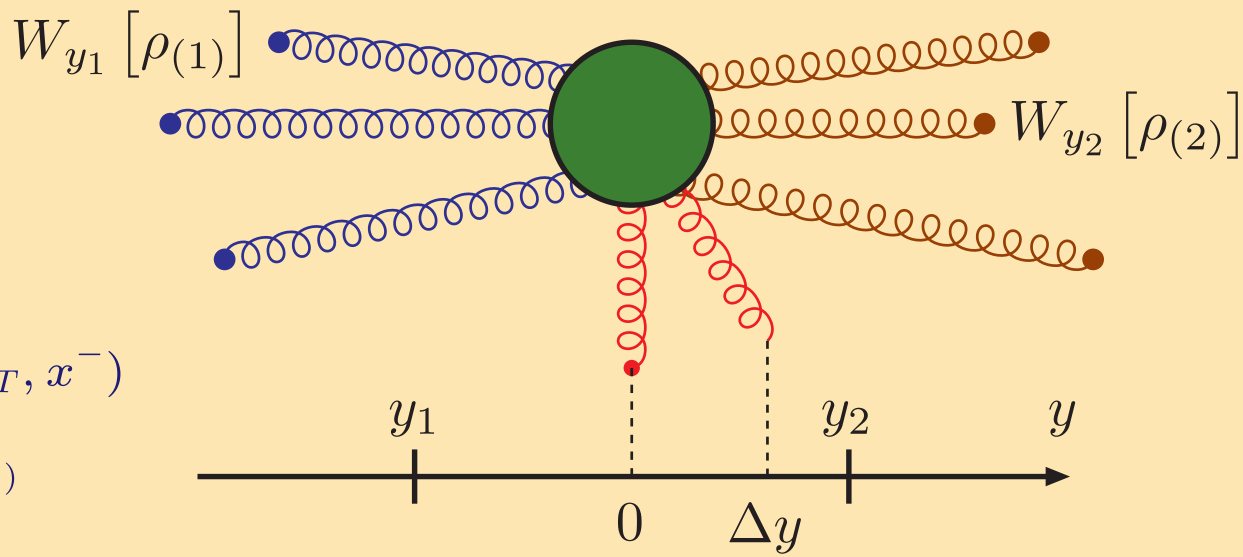
NLO corrections: JIMWLK

Restrict $y_1 < \Delta y < y_2$

▼

Physics indep. of $y_1,\ y_2$

(to appropriate order in α_{s}).



$$\boldsymbol{\nabla}_T{}^2 \mathcal{A}^+(\mathbf{x}_T, x^-) = -g\rho(\mathbf{x}_T, x^-)$$

$$U(\mathbf{x}_T) = \mathrm{P}e^{i\int \mathrm{d}y^- \mathcal{A}^+(\mathbf{x}_T, y^-)}$$

Sources W evolve with JIMWLK Hamiltonian:

$$\mathcal{H} \equiv \frac{1}{2}\int \mathrm{d}^2\mathbf{x}_T\, \mathrm{d}^2\mathbf{y}_T \frac{\delta}{\delta \widetilde{\mathcal{A}}_c^+(\mathbf{y}_T)} \eta^{bc}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp) \frac{\delta}{\delta \widetilde{\mathcal{A}}_b^+(\mathbf{x}_T)}$$

$$\eta^{bc}(\mathbf{x}_T, \mathbf{y}_T) = \frac{1}{\pi}\int \frac{d^2\mathbf{u}_T}{(2\pi)^2} \frac{(\mathbf{x}_T - \mathbf{u}_T)\cdot(\mathbf{y}_T - \mathbf{u}_T)}{(\mathbf{x}_T - \mathbf{u}_T)^2(\mathbf{y}_T - \mathbf{u}_T)^2}$$
$$\times \left[U(\mathbf{x}_T)U^\dagger(\mathbf{y}_T) - U(\mathbf{x}_T)U^\dagger(\mathbf{u}_T) - U(\mathbf{u}_T)U^\dagger(\mathbf{y}_T) + 1\right]_{bc}$$

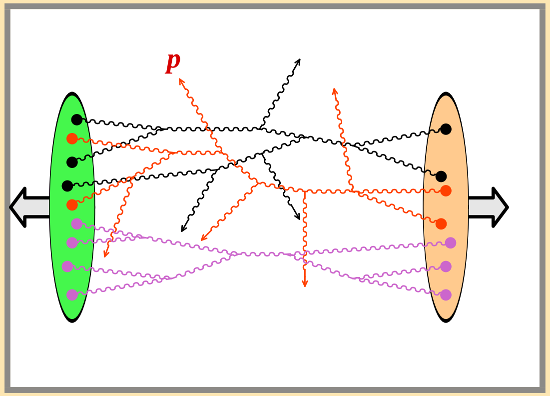
Gluon multiplicity as cut vacuum graphs

Particle production with strong external sources Gelis, Venugopalan [5]:

$$J^\mu \sim 1/g$$

Compute multiplicity

$$\frac{dN}{d^3\vec{\boldsymbol{p}}} \sim \sum_{n=0}^{\infty} \frac{1}{n!} \int \left[d^3\vec{\boldsymbol{p}}_1 \cdots d^3\vec{\boldsymbol{p}}_n \right] \left| \langle \vec{\boldsymbol{p}} \ \vec{\boldsymbol{p}}_1 \cdots \vec{\boldsymbol{p}}_n | 0 \rangle \right|^2$$

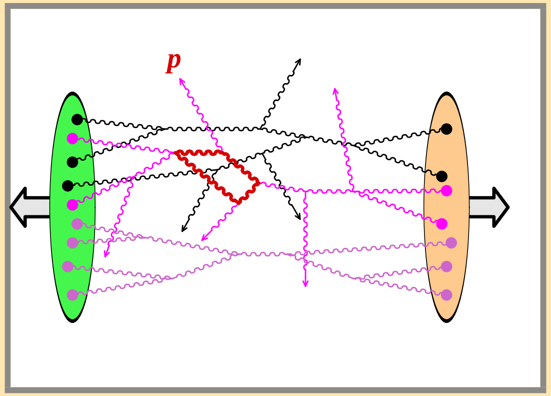


All insertions of source at same order

◀ LO: tree diagrams

NLO: 1 loop ▶

Integrate phase space of additional gluons.



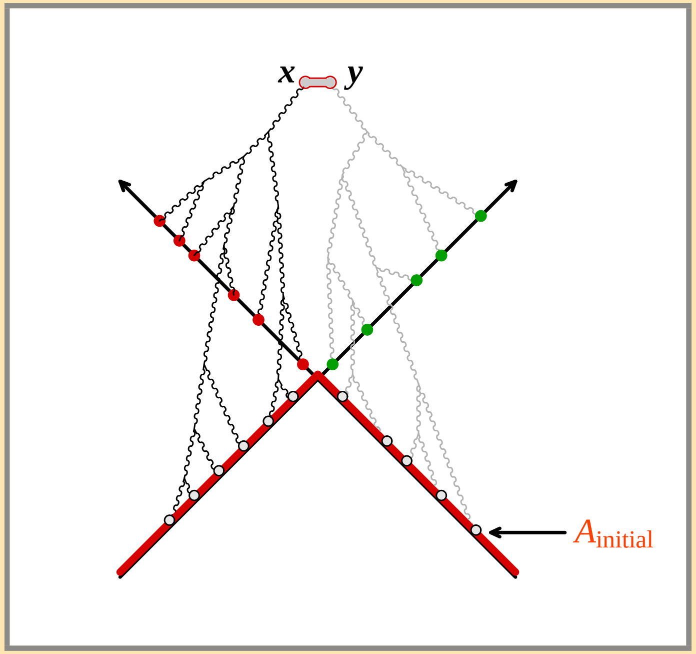
Schwinger-Keldysh formalism, leads to retarded propagators.

[5] F. Gelis and R. Venugopalan, *Nucl. Phys.* **A776** (2006) 135 [hep-ph/0601209].

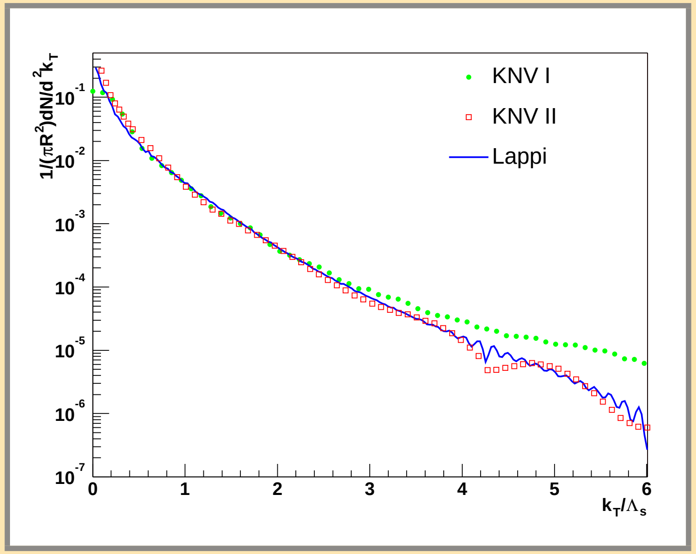
LO is classical field

Leading order multiplicity from **retarded** solution of classical field equations.

$$\left.\frac{dN}{d^3\vec{\boldsymbol{p}}}\right|_{\mathrm{LO}} = \int_{\vec{\boldsymbol{x}}\vec{\boldsymbol{y}}} e^{i\vec{\boldsymbol{p}}\cdot(\vec{\boldsymbol{x}}-\vec{\boldsymbol{y}})} \,(\cdots)\left[\mathcal{A}^{\mu}(t,\vec{\boldsymbol{x}})\mathcal{A}^{\nu}(t,\vec{\boldsymbol{y}})\right]\Bigg|_{t\to\infty}$$



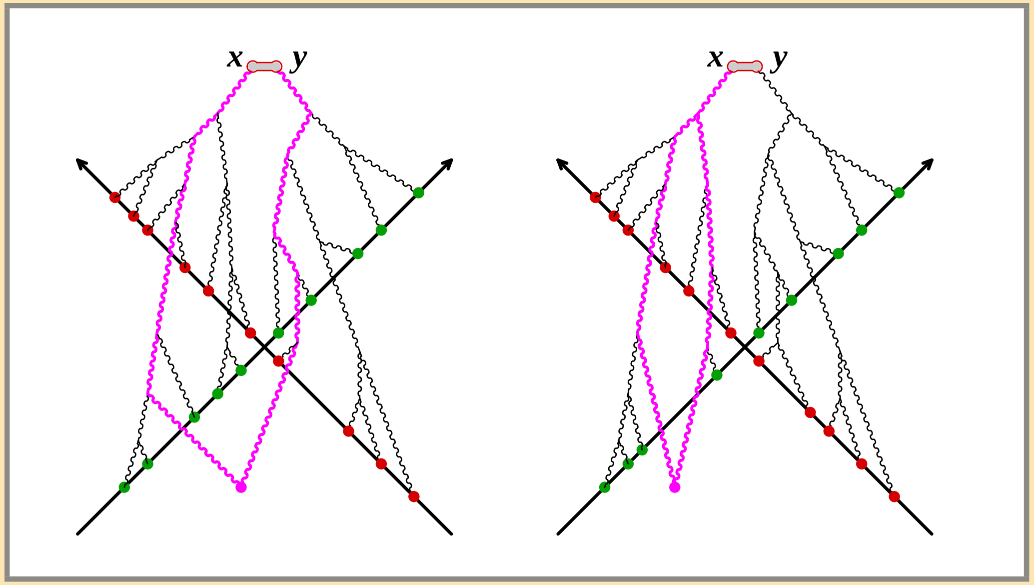
Gluon spectrum from numerical computation



View multiplicity as functional of classical field on initial surface.

NLO is 1 loop

"real", pair production



"virtual", loop correction to field

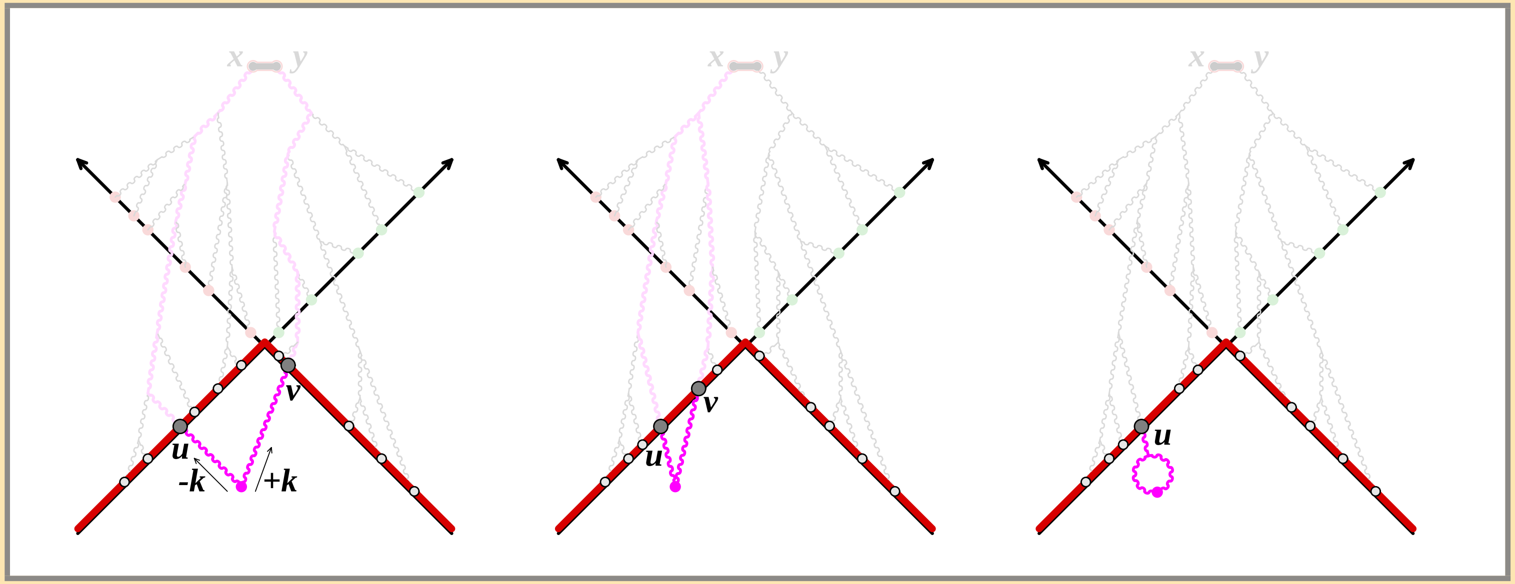
$$\left.\frac{dN}{d^3\vec{\boldsymbol{p}}}\right|_{\mathrm{NLO}} = \int_{\vec{\boldsymbol{x}}\vec{\boldsymbol{y}}} e^{i\vec{\boldsymbol{p}}\cdot(\vec{\boldsymbol{x}}-\vec{\boldsymbol{y}})}\ (\cdots)\left[\mathcal{G}^{\mu\nu}(x,y) + \beta^{\mu}(t,\vec{\boldsymbol{x}})\ \mathcal{A}^{\nu}(t,\vec{\boldsymbol{y}}) + \mathcal{A}^{\mu}(t,\vec{\boldsymbol{x}})\ \beta^{\nu}(t,\vec{\boldsymbol{y}})\right]\Big|_{t\to\infty}$$

- $\mathcal{G}^{\mu\nu}$ is a 2-point function on top of the classical field
- β^{μ} is a small field fluctuation driven by a 1-loop source

Can be expressed in terms of retarded propagators ▶ good for eventual real time numerics

Plasma instability of perturbation ?

NLO: propagators as functional derivatives



$$\left.\frac{dN}{d^3\vec{\boldsymbol{p}}}\right|_{\text{NLO}} = \underbrace{\left[\frac{1}{2} \int\limits_{\vec{\boldsymbol{u}},\vec{\boldsymbol{v}}\in \text{LC}} \mathcal{G}(\mathbf{u}_T, \mathbf{v}_T) \mathbb{T}_{\mathbf{u}_T} \mathbb{T}_{\mathbf{v}_T} + \int\limits_{\vec{\boldsymbol{u}}\in \text{LC}} \beta(\mathbf{u}_T) \mathbb{T}_{\mathbf{u}_T} \right]}_{\text{below LC}} \underbrace{\left.\frac{dN}{d^3\vec{\boldsymbol{p}}}\right|_{\text{LO}}}_{\text{above LC}}$$

$$a^\mu(x) = \int\limits_{\vec{\boldsymbol{u}}\in \text{LC}} a(\vec{\boldsymbol{u}}) \cdot \mathbb{T}_{\mathbf{u}_T} \mathcal{A}^\mu(x) \qquad \mathcal{G}(\vec{\boldsymbol{u}}, \vec{\boldsymbol{v}}) \equiv \int \frac{d^3\vec{\boldsymbol{k}}}{(2\pi)^3 2E_{\boldsymbol{k}}} \, \eta_{-\boldsymbol{k}}(u) \, \eta_{+\boldsymbol{k}}(v)$$

Divergence from $\int \frac{\mathrm{d}k^+}{k^+}$ Functional derivative in $\mathbb{T}$ ▶ $\frac{\delta}{\delta \widetilde{\mathcal{A}}_c^+(\mathbf{y}_T)}$ in JIMWLK

Some aspects of the factorization theorem

- High energy kinematics: fixed $Q^2 \sim Q_s^2$, large $\sqrt{s} \sim e^y$ ▶ weak field limit is BFKL
- Not factorization of pdf's, but color charge distributions
- Power counting: sources $\sim 1/g$
  - Nonperturbative, all orders in classical field
  - NLO in weak coupling/loop expansion (not all orders)
- Work with multiplicities, not cross sections
  - Most natural thing to look at in multiparticle production
  - Retarded propagation
  - Diffractive observables **?**
- Express retarded propagators as functional derivatives wrt. initial condition ▶ relate to functional derivatives in JIMWLK Hamiltonian

Conclusions

- CGC and glasma framework for heavy ion collisions
- Factorize leading $\ln 1/x$ 1-loop corrections into RG evolution.