Four-loop Screened Perturbation Theory

Lars Kyllingstad
in collaboration with Jens O. Andersen

Department of Physics
Norwegian University of Science and Technology

2008 Conference on Strong and Electroweak Matter
Amsterdam, 29 August 2008

Outline

Motivation

Screened Perturbation Theory

Results and summary

Motivation

- Want to study the thermodynamics of massless ϕ^4 theory.

Motivation

- Want to study the thermodynamics of massless ϕ^4 theory.
- Pressure was recently[1] calculated to order g^6 in weak-coupling expansion.

[1] A. Gynther, M. Laine, Y. Schröder, C. Torrero, and A. Vuorinen, JHEP **0704**, 094 (2007)

Motivation

- Want to study the thermodynamics of massless ϕ^4 theory.
- Pressure was recently[1] calculated to order g^6 in weak-coupling expansion.
- Present work: Pressure to four-loop order in screened perturbation theory.

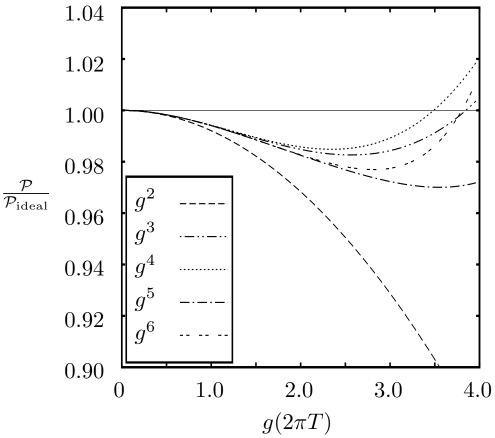
[1] A. Gynther, M. Laine, Y. Schröder, C. Torrero, and A. Vuorinen, JHEP **0704**, 094 (2007)

Motivation

- Want to study the thermodynamics of massless ϕ^4 theory.
- Pressure was recently[1] calculated to order g^6 in weak-coupling expansion.
- Present work: Pressure to four-loop order in screened perturbation theory.
- Motivation: Corresponding problem in QCD.

[1] A. Gynther, M. Laine, Y. Schröder, C. Torrero, and A. Vuorinen, JHEP **0704**, 094 (2007)

Weak-coupling expansion



g^4: P. Arnold and C. Zhai, Phys. Rev. D **50**, 7603 (1994)

g^5: E. Braaten and A. Nieto, Phys. Rev. D **51**, 6990 (1995)

g^6: A. Gynther et al., JHEP **0704**, 094 (2007)

The need for resummation

- The weak-coupling expansion converges poorly.

The need for resummation

- The weak-coupling expansion converges poorly.
- Need a way of reorganising the perturbative series to improve convergence.

The need for resummation

- The weak-coupling expansion converges poorly.
- Need a way of reorganising the perturbative series to improve convergence.
- Many methods are variational:
  - $\mathcal{F}$ depends on variational parameters m_i.

The need for resummation

- The weak-coupling expansion converges poorly.
- Need a way of reorganising the perturbative series to improve convergence.
- Many methods are variational:
  - $\mathcal{F}$ depends on variational parameters m_i.
  - Thermodynamic quantities are found by evaluating $\mathcal{F}$ at the point where $\delta\mathcal{F}/\delta m_i = 0$.

The need for resummation

- The weak-coupling expansion converges poorly.
- Need a way of reorganising the perturbative series to improve convergence.
- Many methods are variational:
  - $\mathcal{F}$ depends on variational parameters m_i.
  - Thermodynamic quantities are found by evaluating $\mathcal{F}$ at the point where $\delta\mathcal{F}/\delta m_i = 0$.
  - Example: 2PI effective action formalism, where the propagator is treated as a variational function.

The need for resummation

- The weak-coupling expansion converges poorly.
- Need a way of reorganising the perturbative series to improve convergence.
- Many methods are variational:
  - $\mathcal{F}$ depends on variational parameters m_i.
  - Thermodynamic quantities are found by evaluating $\mathcal{F}$ at the point where $\delta\mathcal{F}/\delta m_i = 0$.
  - Example: 2PI effective action formalism, where the propagator is treated as a variational function.
- Screened perturbation theory[2] (SPT) is one of these.

[2] F. Karsch, A. Patkós, and P. Petreczky, Phys. Lett. B **401**, 69 (1997)

Basics of SPT (1)

Lagrangian for massless ϕ^4 theory:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{g^2}{24}\phi^4$$

Basics of SPT (1)

Lagrangian for massless ϕ^4 theory:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{g^2}{24}\phi^4$$

Add and subtract mass terms:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}m^2\phi^2 + \frac{1}{2}m_1^2\phi^2 - \frac{g^2}{24}\phi^4$$

Basics of SPT (1)

Lagrangian for massless ϕ^4 theory:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{g^2}{24}\phi^4$$

Add and subtract mass terms:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}m^2\phi^2 + \frac{1}{2}m_1^2\phi^2 - \frac{g^2}{24}\phi^4$$

When $m_1^2 = m^2$, we recover the original theory.

Basics of SPT (2)

Treat one mass term as an interaction,

$$\mathcal{L} = \mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int}},$$

where

$$\begin{aligned} \mathcal{L}_{\text{free}} &= \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2} m^2 \phi^2 \\ \mathcal{L}_{\text{int}} &= \frac{1}{2} m_1^2 \phi^2 - \frac{g^2}{24} \phi^4 \end{aligned}$$

Basics of SPT (2)

Treat one mass term as an interaction,

$$\mathcal{L} = \mathcal{L}_{\rm free} + \mathcal{L}_{\rm int},$$

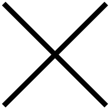
where

$$\begin{aligned} \mathcal{L}_{\rm free} &= \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2} m^2 \phi^2 \\ \mathcal{L}_{\rm int} &= \frac{1}{2} m_1^2 \phi^2 - \frac{g^2}{24}\phi^4 \end{aligned}$$

We now expand around an ideal gas of massive particles.

Feynman rules

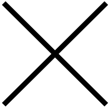
Quartic interaction:



$$= -\frac{g^2}{24}$$

Feynman rules

Quartic interaction:



$$= -\frac{g^2}{24}$$

Mass insertion:

$$= \frac{1}{2}m_1^2$$

Calculating $\mathcal{F}$

The free energy is calculated using a double power expansion:

Calculating $\mathcal{F}$

The free energy is calculated using a double power expansion:

1. Expand in powers of $\alpha = (g/4\pi)^2$ (loop expansion).

Calculating $\mathcal{F}$

The free energy is calculated using a double power expansion:

1. Expand in powers of $\alpha = (g/4\pi)^2$ (loop expansion).
2. Expand each diagram in powers of m/T.

Calculating $\mathcal{F}$

The free energy is calculated using a double power expansion:

1. Expand in powers of $\alpha = (g/4\pi)^2$ (loop expansion).
2. Expand each diagram in powers of m/T.

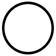
Truncate resulting expressions at g^7.

Loop expansion

The free energy $\mathcal{F}$ is the sum of these diagrams:

Loop expansion

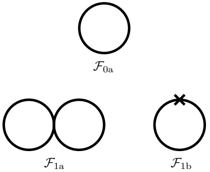
The free energy $\mathcal{F}$ is the sum of these diagrams:



$\mathcal{F}_{0\mathrm{a}}$

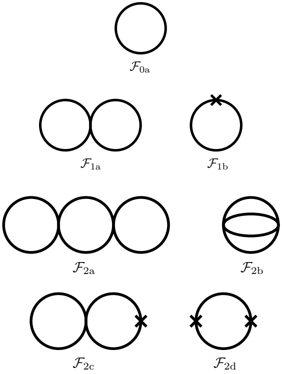
Loop expansion

The free energy $\mathcal{F}$ is the sum of these diagrams:



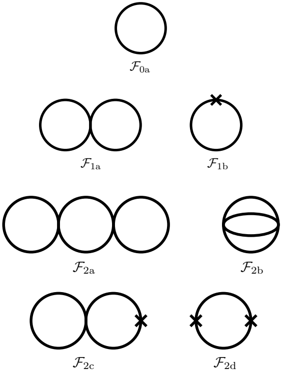
Loop expansion

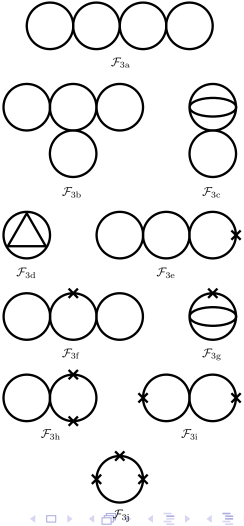
The free energy $\mathcal{F}$ is the sum of these diagrams:



Loop expansion

The free energy $\mathcal{F}$ is the sum of these diagrams:





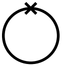
m/T expansion

$$
\begin{aligned}
m &\sim gT \\
p_0 &= 2\pi n T, \qquad n = 0, \pm 1, \pm 2, \ldots
\end{aligned}
$$

m/T expansion

$$
\begin{aligned}
m &\sim gT \\
p_0 &= 2\pi n T, \qquad n = 0, \pm 1, \pm 2, \ldots
\end{aligned}
$$

Example:

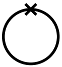


$$= -\frac{1}{2} m_1^2 T \sum_{p_0} \int_p \frac{1}{P^2 + m^2}$$

m/T expansion

$$\begin{aligned} m &\sim gT \\ p_0 &= 2\pi n T, \qquad n = 0, \pm 1, \pm 2, \ldots \end{aligned}$$

Example:

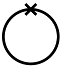


$$\begin{aligned} &= -\frac{1}{2} m_1^2 T \sum_{p_0} \int_p \frac{1}{P^2 + m^2} \\ &= -\frac{1}{2} m_1^2 T \left[\int_p \frac{1}{p^2 + m^2} + \sum_{p_0 \neq 0} \int_p \frac{1}{P^2 + m^2} \right] \end{aligned}$$

m/T expansion

$$\begin{aligned} m &\sim gT \\ p_0 &= 2\pi n T, \qquad n = 0, \pm 1, \pm 2, \ldots \end{aligned}$$

Example:



$$\begin{aligned} &= -\frac{1}{2} m_1^2 T \sum_{p_0} \int_p \frac{1}{P^2 + m^2} \\ &= -\frac{1}{2} m_1^2 T \left[\int_p \frac{1}{p^2 + m^2} + \sum_{p_0 \neq 0} \int_p \frac{1}{P^2 + m^2} \right] \\ &= -\frac{1}{2} m_1^2 T \left[\int_p \frac{1}{p^2 + m^2} + \sum_{p_0 \neq 0} \int_p \frac{1}{P^2} \left(\frac{m^2}{P^2} + \frac{m^4}{P^4} + \cdots \right) \right] \end{aligned}$$

Pressure

The pressure is defined as

$$\mathcal{P} = -\mathcal{F}\big|_{m_1^2=m^2}$$

Pressure

The pressure is defined as

$$\mathcal{P} = -\mathcal{F}\big|_{m_1^2 = m^2}$$

- m is an arbitrary parameter.

Pressure

The pressure is defined as

$$\mathcal{P} = -\mathcal{F}\big|_{m_1^2=m^2}$$

- m is an arbitrary parameter.
- If SPT calculations were carried out to all loop orders, the results would be independent of m.

Pressure

The pressure is defined as

$$\mathcal{P} = -\mathcal{F}\big|_{m_1^2=m^2}$$

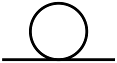
- m is an arbitrary parameter.
- If SPT calculations were carried out to all loop orders, the results would be independent of m.
- Need a prescription for m as a function of g and T.

Pressure

The pressure is defined as

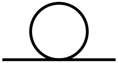
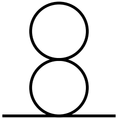
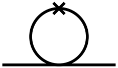
$$\mathcal{P} = -\mathcal{F}\big|_{m_1^2 = m^2}$$

- m is an arbitrary parameter.
- If SPT calculations were carried out to all loop orders, the results would be independent of m.
- Need a prescription for m as a function of g and T.
- To obtain weak-coupling limit:

$m^2 =$  $= \dfrac{g^2 T^2}{24}$

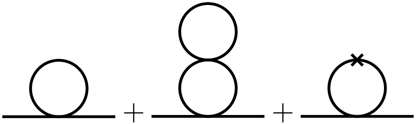
Tadpole mass

We choose m to be the tadpole mass,

$m^2 =$  $+$  $+$  $+ \cdots = g^2 \left. \frac{\partial \mathcal{F}}{\partial (m^2)} \right|_{m_1^2 = m^2}$

Tadpole mass

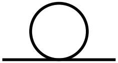
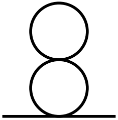
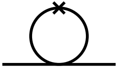
We choose m to be the tadpole mass,

$m^2 =$  $+ \cdots = g^2 \left. \frac{\partial \mathcal{F}}{\partial (m^2)} \right|_{m_1^2 = m^2}$

- Self-consistent gap equation for m.

Tadpole mass

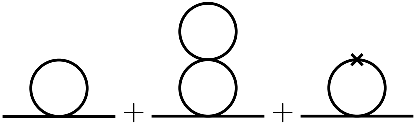
We choose m to be the tadpole mass,

$m^2 =$  $+$  $+$  $+ \cdots = g^2 \left. \frac{\partial \mathcal{F}}{\partial (m^2)} \right|_{m_1^2 = m^2}$

- Self-consistent gap equation for m.
- m is well-defined at all loop orders.

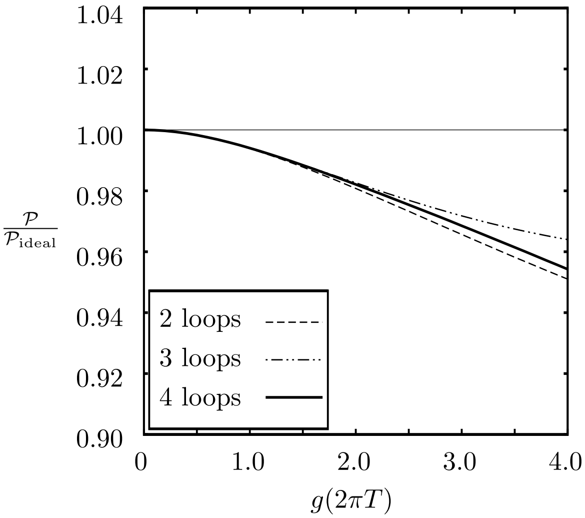
Tadpole mass

We choose m to be the tadpole mass,

$m^2 =$  $+ \cdots = g^2 \left. \frac{\partial \mathcal{F}}{\partial (m^2)} \right|_{m_1^2 = m^2}$

- Self-consistent gap equation for m.
- m is well-defined at all loop orders.
- Selective resummation of diagrams from all loop orders in the original (massless) theory.

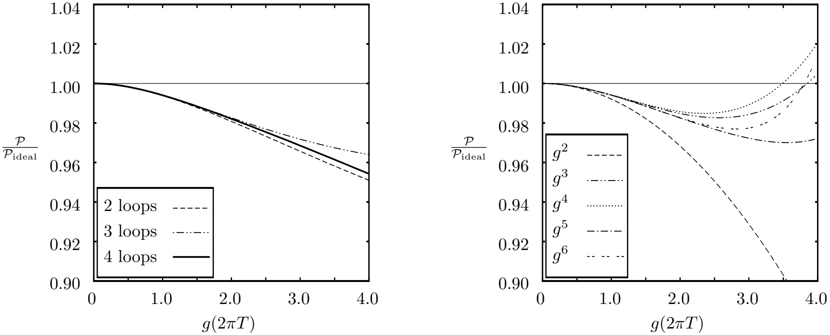
Results


1.04
1.02
1.00
0.98
0.96
0.94
0.92
0.90
$\frac{\mathcal{P}}{\mathcal{P}_{\text{ideal}}}$
0
1.0
2.0
3.0
4.0
$g(2\pi T)$
2 loops
3 loops
4 loops

Comparison

Screened perturbation theory:

Weak-coupling expansion:



Convergence properties

- Two-loop result:
  - Indistinguishable from exact numerical result[3].

[3] J. O. Andersen et al., Phys. Rev. D **63**, 105008 (2001)

Convergence properties

- Two-loop result:
  - Indistinguishable from exact numerical result[3].
  - Very fast convergence (g^4).

[3] J. O. Andersen et al., Phys. Rev. D **63**, 105008 (2001)

Convergence properties

- Two-loop result:
  - Indistinguishable from exact numerical result[3].
  - Very fast convergence (g^4).
- Three-loop result:
  - Indistinguishable from exact numerical result

[3] J. O. Andersen et al., Phys. Rev. D **63**, 105008 (2001)

Convergence properties

- Two-loop result:
  - Indistinguishable from exact numerical result[3].
  - Very fast convergence (g^4).
- Three-loop result:
  - Indistinguishable from exact numerical result
  - Fairly fast convergence (g^6).

[3] J. O. Andersen et al., Phys. Rev. D **63**, 105008 (2001)

Convergence properties

- Two-loop result:
  - Indistinguishable from exact numerical result[3].
  - Very fast convergence (g^4).
- Three-loop result:
  - Indistinguishable from exact numerical result
  - Fairly fast convergence (g^6).
- Four-loop result:
  - Exact numerical results not available.

[3] J. O. Andersen et al., Phys. Rev. D **63**, 105008 (2001)

Convergence properties

- Two-loop result:
  - Indistinguishable from exact numerical result[3].
  - Very fast convergence (g^4).
- Three-loop result:
  - Indistinguishable from exact numerical result
  - Fairly fast convergence (g^6).
- Four-loop result:
  - Exact numerical results not available.
  - Reasonable approximation?

[3] J. O. Andersen et al., Phys. Rev. D **63**, 105008 (2001)

Summary

- Have calculated the pressure of a massless ϕ^4 theory to four-loop order in screened perturbation theory.

Summary

- Have calculated the pressure of a massless ϕ^4 theory to four-loop order in screened perturbation theory.
- Used double power expansion in g^2 and m/T, truncating series at g^7.

Summary

- Have calculated the pressure of a massless ϕ^4 theory to four-loop order in screened perturbation theory.
- Used double power expansion in g^2 and m/T, truncating series at g^7.
- Resummation of selected diagrams from all loop orders in the massless theory gives better convergence properties.

Summary

- Have calculated the pressure of a massless ϕ^4 theory to four-loop order in screened perturbation theory.
- Used double power expansion in g^2 and m/T, truncating series at g^7.
- Resummation of selected diagrams from all loop orders in the massless theory gives better convergence properties.
- Agreement with earlier results, both weak-coupling and exact numerical results.

Outlook

- Find g^7 term in weak-coupling expansion.

Outlook

- Find g^7 term in weak-coupling expansion.
  - This requires the evaluation of 5-loop diagrams in 3 dimensions.

Outlook

- Find g^7 term in weak-coupling expansion.
  - This requires the evaluation of 5-loop diagrams in 3 dimensions.
  - Work in progress.

Outlook

- Find g^7 term in weak-coupling expansion.
  - This requires the evaluation of 5-loop diagrams in 3 dimensions.
  - Work in progress.
- g^8?

Outlook

- Find g^7 term in weak-coupling expansion.
  - This requires the evaluation of 5-loop diagrams in 3 dimensions.
  - Work in progress.
- g^8?
  - Necessary to confirm convergence of result at g^7.

That's all, folks!