

Center-symmetric dimensional reduction of hot Yang-Mills theory

Aleksi Kurkela,
Univ. of Helsinki

SEWM 2008

arXiv:0704.1416,
arXiv:0801.1566 with Philippe de Forcrand and Aleksi Vuorinen

Dimensional reduction

- At high T : For long distance properties ($\Delta x \gg 1/T$), the system looks 3d.



- Degrees of freedom are **static modes** $\phi_0(\mathbf{x})$
$$\phi(\mathbf{x}, \tau) = T \sum_{n=-\infty}^{\infty} \exp(i\omega_n \tau) \phi_n(\mathbf{x})$$
- Effective action: Integrate out non-static modes

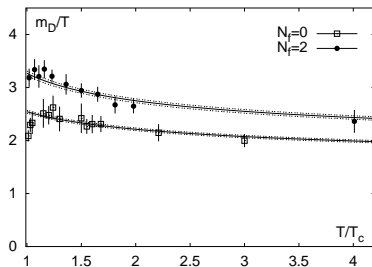
$$\begin{aligned} Z &= \int \mathcal{D}\phi_0 \mathcal{D}\phi_n \exp(-S_0(\phi_0) - S_n(\phi_0, \phi_n)) \\ &= \int \mathcal{D}\phi_0 \exp(-S_0(\phi_0) - S_{\text{eff}}(\phi_0)) \end{aligned}$$

- In practice: Need scale separation between **static** and **non-static modes**

Where Dimensional Reduction works?

Scales in hot Yang-Mills:

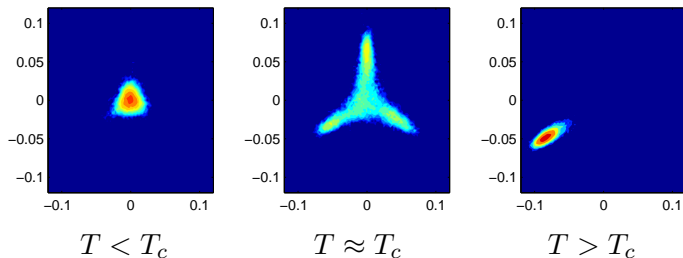
- Perturbatively ($g \sim 1/\log(T)$):
 - ▶ **Hard scale:** $2\pi T$ Typical thermal momentum, **non-static modes**
 - ▶ **Soft scale:** $m_D \sim gT$ Debye screening, **static modes**
⇒ Asymptotic dimensional reduction
- Non-perturbatively: $m(T_c) \sim 3T_c \stackrel{?}{\ll} 2\pi T_c$



Kaczmarek, Zantow

Perturbative dimensional reduction

Polyakov loop $\Omega = \text{Tr} [P \exp (ig \int d\tau A_0)]$, has N_c minima in the deconfined phase $\Rightarrow Z_N$ center symmetry.



Deep in deconfined phase: Expand fields around **one** minimum to get EQCD (= 3D Yang-Mills + adjoint Higgs):

$$S_{\text{EQCD}} = \int d^3x \left[\underbrace{\frac{1}{2} \text{Tr} F_{ij}^2}_{\text{spatial gluons}} + \underbrace{\text{Tr} (D_i A_0)^2}_{\text{adjoint kinetic}} + \underbrace{\frac{1}{2} m_E^2 \text{Tr} A_0^2 + \frac{1}{4} \lambda_E \text{Tr} A_0^4 + \dots}_{\text{interactions from integration out}} \right]$$

Center-symmetric effective theories

- Goal: Want to construct an effective theory that
 - ▶ Preserves the Z_N center symmetry
 - ▶ Reduces to EQCD at high T
 - ▶ Is superrenormalizable
- Effective theory of Wilson lines not (super)renormalizable

(Pisarski hep-ph/0608242)

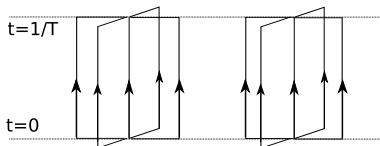
Center-symmetric effective theories

- Goal: Want to construct an effective theory that
 - ▶ Preserves the Z_N center symmetry
 - ▶ Reduces to EQCD at high T
 - ▶ Is superrenormalizable
- Effective theory of Wilson lines not (super)renormalizable
(Pisarski hep-ph/0608242)

Idea: Construct effective theory for *coarse grained* Wilson loop

(Yaffe+Vuorinen hep-ph/0604100)

$$\mathcal{Z}(\mathbf{x}) = \frac{T}{V_{\text{Block}}} \int_V d^3y U(\mathbf{x}, \mathbf{y}) W(\mathbf{y}) U(\mathbf{y}, \mathbf{x}), \notin SU(N_c)$$



Center-symmetric theory for $SU(2)$

- For $SU(2)$, sum of matrices proportional to $SU(2)$

$$\mathcal{Z} = \lambda \Omega, \quad \Omega \in SU(2), \quad \lambda > 0$$

$$\mathcal{Z} = \frac{1}{2} \left\{ \underbrace{\Sigma \mathbf{1}}_{\text{Singlet}} + i \underbrace{\Pi_a \sigma_a}_{\text{Adjoint scalar}} \right\} = \begin{pmatrix} \frac{1}{2}\Sigma + i\Pi_1 & i\Pi_2 - \Pi_3 \\ i\Pi_2 + \Pi_3 & \frac{1}{2}\Sigma - i\Pi_1 \end{pmatrix}$$

- Transforms exactly like Wilson line

$$\begin{aligned} \mathcal{Z} &\longrightarrow \lambda^{-1}(\mathbf{x}) \mathcal{Z} \lambda(\mathbf{x}) && \text{gauge} \\ \mathcal{Z} &\longrightarrow -\mathcal{Z} && \text{center } Z_2 \end{aligned}$$

Center-symmetric theory for $SU(2)$

- For $SU(2)$, sum of matrices proportional to $SU(2)$

$$\mathcal{Z} = \lambda \Omega, \quad \Omega \in SU(2), \quad \lambda > 0$$

$$\mathcal{Z} = \frac{1}{2} \left\{ \underbrace{\Sigma \mathbf{1}}_{\text{Singlet}} + i \underbrace{\Pi_a \sigma_a}_{\text{Adjoint scalar}} \right\} = \begin{pmatrix} \frac{1}{2}\Sigma + i\Pi_1 & i\Pi_2 - \Pi_3 \\ i\Pi_2 + \Pi_3 & \frac{1}{2}\Sigma - i\Pi_1 \end{pmatrix}$$

- Most general superrenormalizable Lagrangian with A_i and \mathcal{Z} :

$$\mathcal{L}_{Z(2)} = \underbrace{\frac{1}{2} \text{Tr} F_{ij}^2}_{\text{spatial gluons}} + \underbrace{\text{Tr} \left(D_i \mathcal{Z}^\dagger D_i \mathcal{Z} \right)}_{\text{Adjoint Kinetic}} + V(\mathcal{Z})$$

$$V(\mathcal{Z}) = \underbrace{b_1 \Sigma^2 + b_2 \Pi_a^2 + c_1 \Sigma^4 + c_2 (\Pi_a^2)^2 + c_3 \Sigma^2 \Pi_a^2}_{\text{interaction from integration out}}$$

- Higher order terms suppressed by scale difference m_D/T .

Matching at $T \rightarrow \infty$

Parameters can be (almost) matched in perturbation theory (series in $\frac{g^2}{16\pi^2}$!):

$$b_1 = -\frac{1}{4}r^2T^2,$$

$$b_2 = -\frac{1}{4}r^2T^2 + 0.441841g^2T^2,$$

$$c_1/g_3^2 = 0.0311994r^2 + 0.0135415g^2,$$

$$c_2/g_3^2 = 0.0311994r^2 + 0.008443432g^2,$$

$$c_3/g_3^2 = 0.0623987r^2,$$

$$g_3^2 = g^2T$$

- Parameters functions of full theory parameters (g, T) and r
 - ▶ rT : mass of fluctuation from $SU(2)$ manifold
- r needs to be matched non-perturbatively

On the lattice:

$$S_a = S_W + S_Z + V(\hat{\Sigma}, \hat{\Pi}),$$

$$\beta = \frac{4}{ag_3^2}$$

$$S_W = \beta \sum_{x, i < j} \left[1 - \frac{1}{2} \text{Tr} [U_{ij}] \right],$$

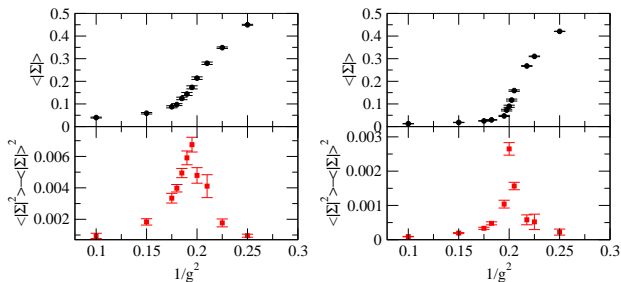
$$S_Z = 2 \left(\frac{4}{\beta} \right) \sum_{x, i} \text{Tr} \left[\hat{\Pi}^2 - \hat{\Pi}(x) U_i(x) \hat{\Pi}(x + \hat{i}) U_i^\dagger(x) \right] \\ + \left(\frac{4}{\beta} \right) \sum_{x, i} \left(\hat{\Sigma}^2(x) - \hat{\Sigma}(x) \hat{\Sigma}(x + \hat{i}) \right),$$

$$V = \left(\frac{4}{\beta} \right)^3 \sum_x \left[\hat{b}_1 \hat{\Sigma}^2 + \hat{b}_2 \hat{\Pi}_a^2 + \hat{c}_1 \hat{\Sigma}^4 + \hat{c}_2 \left(\hat{\Pi}_a^2 \right)^2 + \hat{c}_3 \hat{\Sigma}^2 \hat{\Pi}_a^2 \right],$$

$$\begin{aligned}
\hat{b}_1 &= b_1/g_3^4 - \frac{2.38193365}{4\pi}(2\hat{c}_1 + \hat{c}_3)\beta \\
&+ \frac{1}{16\pi^2} \left\{ (48\hat{c}_1^2 + 12\hat{c}_3^2 - 12\hat{c}_3) [\log 1.5\beta + 0.08849] - 6.9537 \hat{c}_3 \right\} \\
&+ \mathcal{O}(a), \\
\hat{b}_2 &= b_2/g_3^4 - \frac{0.7939779}{4\pi}(10\hat{c}_2 + \hat{c}_3 + 2)\beta \\
&+ \frac{1}{16\pi^2} \left\{ (80\hat{c}_2^2 + 4\hat{c}_3^2 - 40\hat{c}_2) [\log 1.5\beta + 0.08849] \right. \\
&\left. - 23.17895 \hat{c}_2 - 8.66687 \right\} + \mathcal{O}(a).
\end{aligned}$$

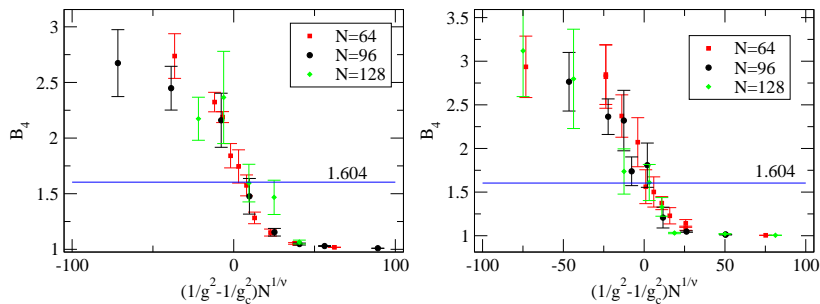
Results from simulations:

Z_2 -restoring phase transition



- left ($\beta = 12, n = 64, r^2 = 5$)
- right ($\beta = 6, n = 64, r^2 = 5$)

Results from simulations:

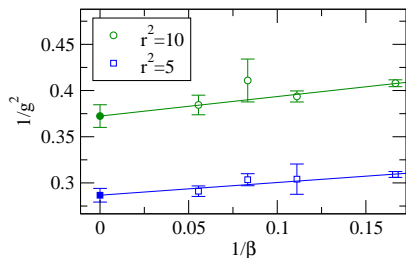


- left $r^2 = 5$, right $r^2 = 10$.
- 3d Ising universality class.

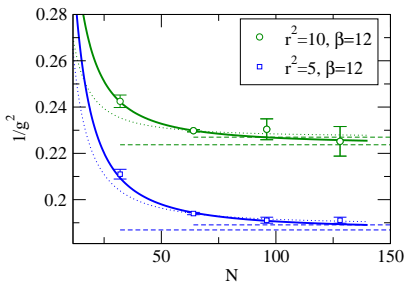
$$B_4 = \langle \bar{\Sigma}^4 \rangle / \langle \bar{\Sigma}^2 \rangle^2 = 1.604 \dots \text{ at criticality}$$

$$\nu = 0.63$$

Results from simulations:



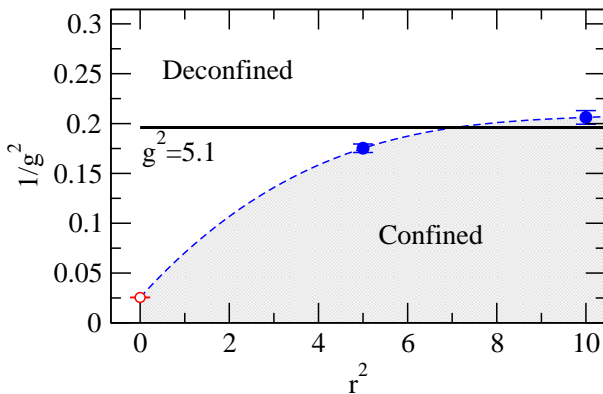
$a \rightarrow 0$



$V \rightarrow \infty$ ($A + BL^{-1/\nu}$)

- large $r \rightarrow$ short correlation length \rightarrow fine lattice
- small $r \rightarrow$ long correlation length \rightarrow large volume
- $r = 0$: Σ decouples $\rightarrow \lambda\phi^4$ already done, X.P. Sun hep-lat/0209144

Results from simulations:



- Phase diagram resembles the full theory (unlike in EQCD).
- Insensitive to $r > 1$
- Phase transition at correct $g!$

Outlook

Implementing center-symmetry to the effective theory gives correct phase transition in SU(2)

Lots of simulations to do:

- Check accuracy near T_c :
 - ▶ Domain wall tension
 - ▶ Spatial string tension
 - ▶ Screening masses
- Make predictions:
 - ▶ Heavy quarks: Z_N breaking terms
 - ▶ Finite chemical potential (Correct phase transitions?)
 - ▶ Extensions to large N_c :
 - ★ At $N_c > 4$ no more N_c -dependent (super-renormalizable) operators.
 - ★ Poster by Chris Korthals Altes
- Can the theory accommodate “fuzzy bag”?
 - ▶ $p(T) = B_{\text{MIT}} + B_{\text{fuzzy}}T^2 + f_{\text{pert}}T^4$