Chiral restoration of strong coupling QCD at finite baryon density

M.Fromm, Ph. de Forcrand
ETH, Zürich

SEWM'08, August 28th, 2008

Motivation

Strong coupling QCD (SCQCD) under investigation for $>$ 25 years...

Analytical (Mean Field $1/d$)

- Mass spectrum: Kawamoto and Smit '81, Kluberg-Stern, Morel, Petersson '82
- Phase diagram: Damgaard, Kawamoto, Shigemoto '84
- Phase diagram with $1/g^2$ corrections: Faldt and Petersson '86, Bilić et al.'92
- Latest: Nishida '04, Kawamoto et al. '05, Miura and Ohnishi '08

Numerical

- Karsch and Mütter '89: MDP-approach ($T \approx 0$, $\mu \approx \mu_c$)
- Boyd et al. '92: MDP at $T \approx T_C, \mu = 0$
- Azcoiti et al. '99: MDP under scrutiny
- de Forcrand and Kim '06: HMC, mass spectrum

Strong coupling QCD

Some Definitions:

$$Z = Z(m, \mu, \beta) = \int \mathcal{D}U \mathcal{D}\bar{\chi} \mathcal{D}\chi \, \mathrm{e}^{-S_{\mathrm{F}} - \beta S_{\mathrm{G}}},$$

μ chemical potential, m staggered quark mass, $\beta = \frac{6}{g_0^2}$ inverse gauge coupling

$$\begin{aligned} S_{\mathrm{G}} &= \sum_P \left(1 - \frac{1}{3} \mathrm{Re}\,\mathrm{tr}[U_P]\right) \\ S_{\mathrm{F}} &= \sum_{x,\nu} \bar{\chi}_x \left[\eta_{x\nu} U_{x\nu} \chi_{x+\nu} - \eta_{x\nu}^{-1} U_{x-\nu\nu}^{\dagger} \chi_{x-\nu}\right] + 2m \sum_x \bar{\chi}_x \chi_x \end{aligned}$$

$\eta_{x\nu} = \mathrm{e}^{\mu}$ $(\nu = 0)$ and $(-1)^{\sum_{\rho<\nu} x_\rho}$ otherwise.

Strong coupling QCD (SCQCD)

In Strong (infinite) coupling limit, $\beta = 0$ - can do integral in links $U_{x\nu}$ first [Rossi & Wolff]:

$$Z(m, \mu) = \int \mathcal{D}\bar{\chi}\mathcal{D}\chi \mathrm{e}^{2m \sum_x \bar{\chi}_x \chi_x} \prod_{\langle xy \rangle} F_{xy}$$

where $F_{xy} = \sum_{k=0}^{3} \alpha_k (M_x M_y)^k + \kappa \left[\bar{B}_x B_y - \bar{B}_y B_x \right]$ and

$$\kappa = \begin{cases} 0, & \text{for } U(3) \\ 1, & \text{for } SU(3) \end{cases}$$

New degrees of freedom are color singlet

Monomers $M_x = \sum_{a,x} \bar{\chi}_{ax} \chi_{ax}$, $(\bullet)$, monomers per site $n_x = 0, \ldots, 3$

Dimers $D_{k,xy} = \frac{1}{k!}(M_x M_y)^k$ $(-, =, \equiv)$, bond number $n_b = 0, \ldots, 3$

(Anti-)Baryons $B_x = \chi_{1x}\chi_{2x}\chi_{3x}$, $\bar{B}_x = \bar{\chi}_{3x}\bar{\chi}_{2x}\bar{\chi}_{1x}$, $- - -$

SCQCD loop gas

$$Z(m,\mu) = \sum_{\{n_x, n_b, \square\}} \prod_b \frac{(3-n_b)!}{3!\,n_b!} \prod_x \frac{3!}{n_x!}(2m)^{n_x} \prod_{\text{loops } C} \rho(C) \ ,$$

with a "close packing" constraint $n_x + \sum_{b_x} n_{b_x} = 3$ and two types of self-avoiding loops:

- Polymer loops $\tilde{C}$- sequences of D_1, D_2 dimers with weight $w(\tilde{C}) = 1$
- Baryon loops C of pairs $\bar{B}_x B_y$ with signed weights $\rho(C)$, where

$$\rho(C) = \begin{cases} \pm 2\cosh 3k\mu/T & \text{Polyakov-type loops with winding number } k \\ \pm 2, & \text{non-winding loops} \end{cases}$$

SCQCD loop gas

$$Z(m,\mu) = \sum_{\{n_x, n_b, \square\}} \prod_b \frac{(3-n_b)!}{3!\,n_b!} \prod_x \frac{3!}{n_x!}(2m)^{n_x} \prod_{\text{loops } C} \rho(C) \ ,$$

with a "close packing" constraint $n_x + \sum_{b_x} n_{b_x} = 3$ and two types of self-avoiding loops:

- Polymer loops $\tilde{C}$- sequences of D_1, D_2 dimers with weight $w(\tilde{C}) = 1$
- Baryon loops C of pairs $\bar{B}_x B_y$ with signed weights $\rho(C)$, where

$$\rho(C) = \begin{cases} \pm 2\cosh 3k\mu/T & \text{Polyakov-type loops with winding number } k \\ \pm 2, & \text{non-winding loops} \end{cases}$$

Treating the sign

MDP prescription [Karsch & Mütter, '89]: Group together polymer and baryon-loop configuration of same geometry:

$$Z = \sum_{\{n_x, n_b, C\}} (\dots) \prod_{\text{loops } C} \rho(C) = \sum_{\{n_x, n_b, \tilde{C}\}} (\dots) \prod_{\text{loops } \tilde{C}} \underbrace{\left(w(\tilde{C}) + \frac{1}{2}\rho(C) \right)}_{w_{\text{polymer}}} .$$

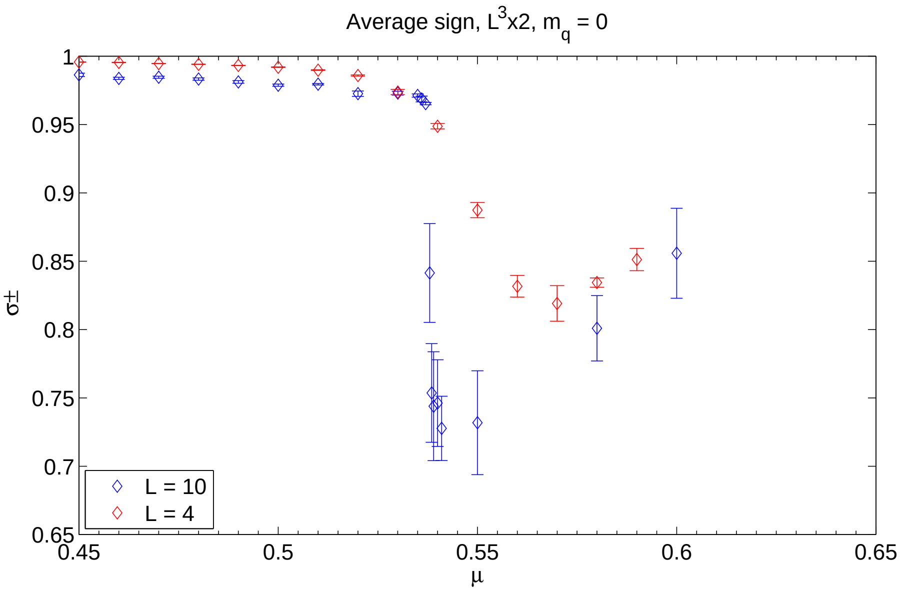
Corresponds to map

$$\rho_B(C) \rightarrow w_{\text{polymer}}(C)$$

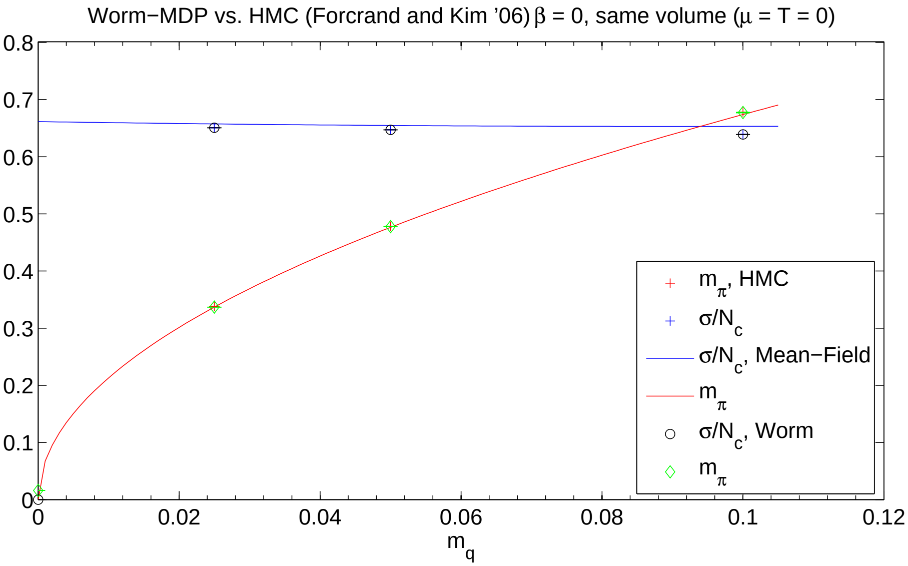
$$\begin{cases} \pm 2 \cosh 3k\mu/T \\ \pm 2 \end{cases} \rightarrow \begin{cases} 1 \pm \cosh 3k\mu/T \\ 0, 2 \end{cases}$$

which *softens* the sign problem.

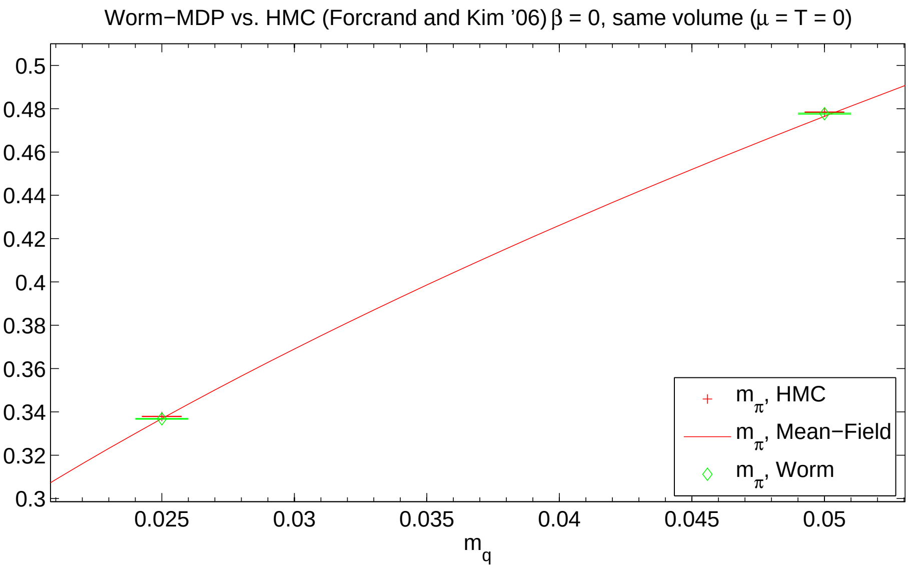
Average sign sample



Consistency check with HMC

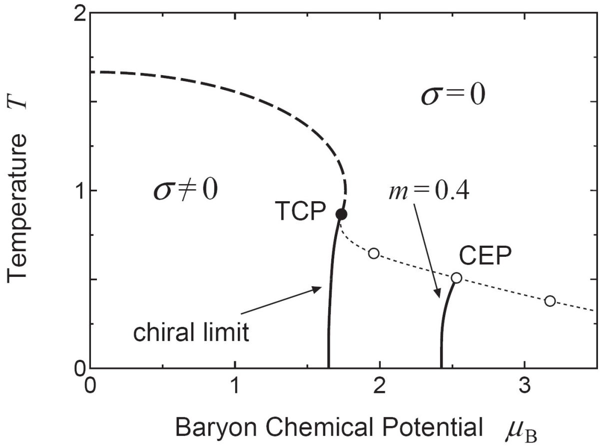


Consistency check with HMC



SCQCD chiral restoration transition

Phase diagram as obtained by *mean-field* calculations [Nishida '04]



Predictions: $T_c(\mu = 0) = \frac{5}{3}$, $\mu_c(T = 0) = \mu_B/3 = 0.55$. $(\sigma \propto \langle \bar{\chi}_a \chi_a \rangle)$

SCQCD chiral restoration transition

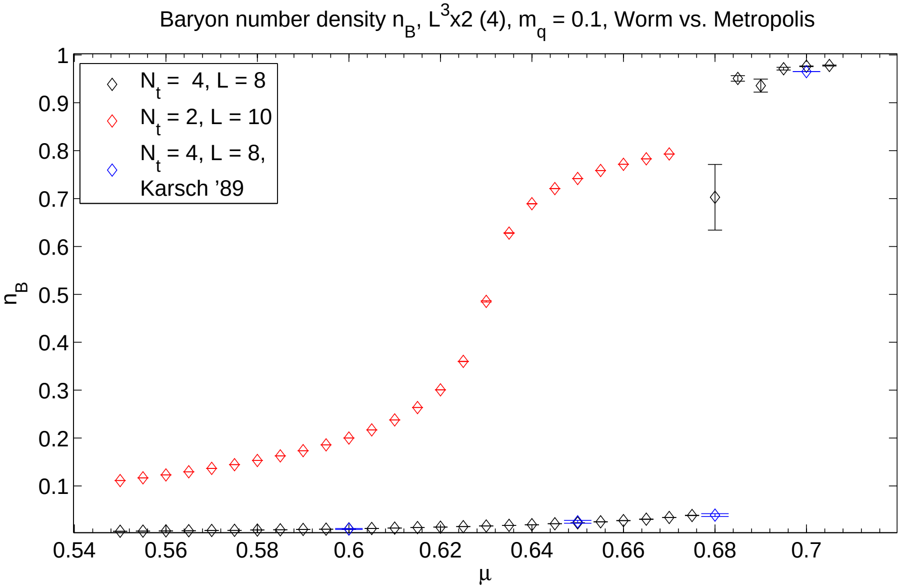
Puzzle

- Strong coupling MC-simulations [Karsch & Mütter 1989] at *finite* quark mass and $T = 1/4$ confirm 1st order finite μ transition and extrapolate to $\mu_c(T \approx 0, m = 0) = 0.63$ (in agreement with mean-field)
- However: Expect ($T = 0$)-phase transition when

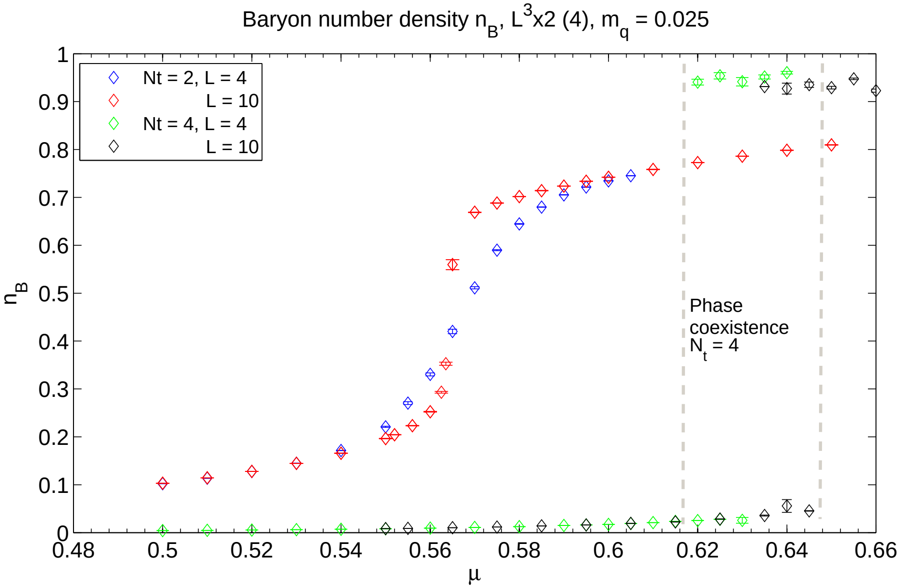
$$3\mu \geq F_B \approx M_{\mathrm{Nucleon}} \approx 3\,, \text{i.e.} \quad \mu_c \approx 1$$

- Nuclear attraction strong, $\mathcal{O}(300\,\mathrm{MeV})$?
- Or: Finite T effects (MC), extrapolation in m (MC) or mean field approach inaccurate ?
  $\rightarrow$ Check with worm-MC in the chiral limit, $T \approx 0$.
- Note: Mean field calculations with $1/g^2$ corrections [Bilić et al. 1992] show that $\mu_c \rightarrow M_N/3$

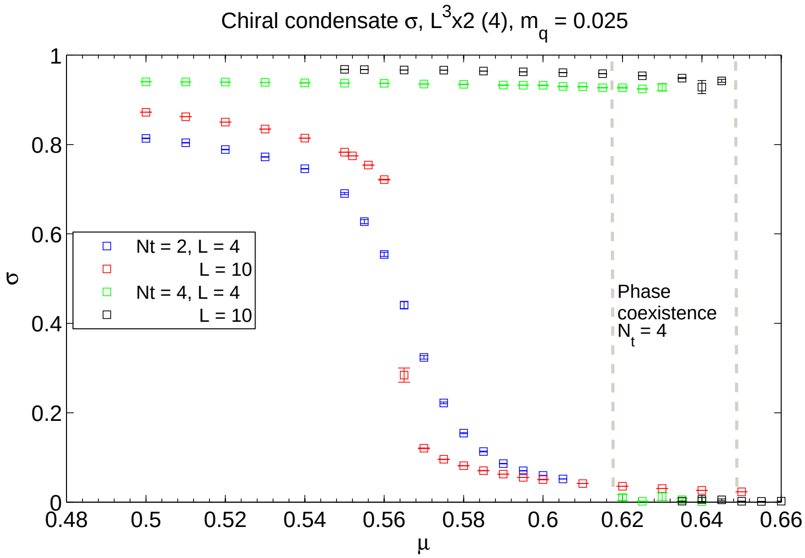
Preliminary Results



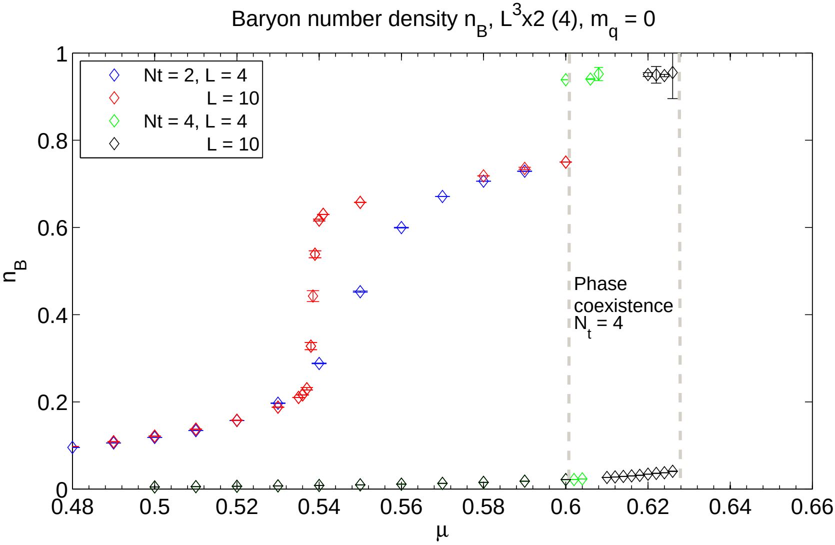
Preliminary Results



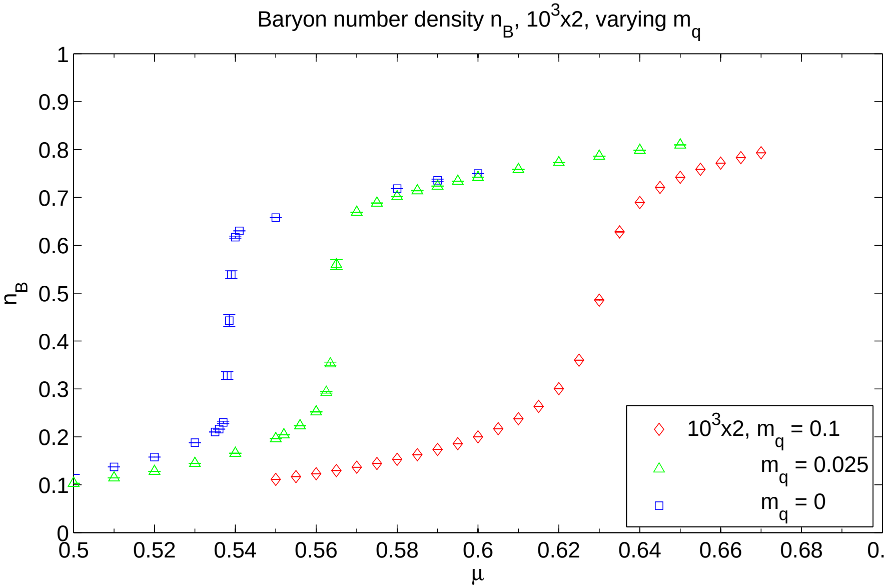
Preliminary Results



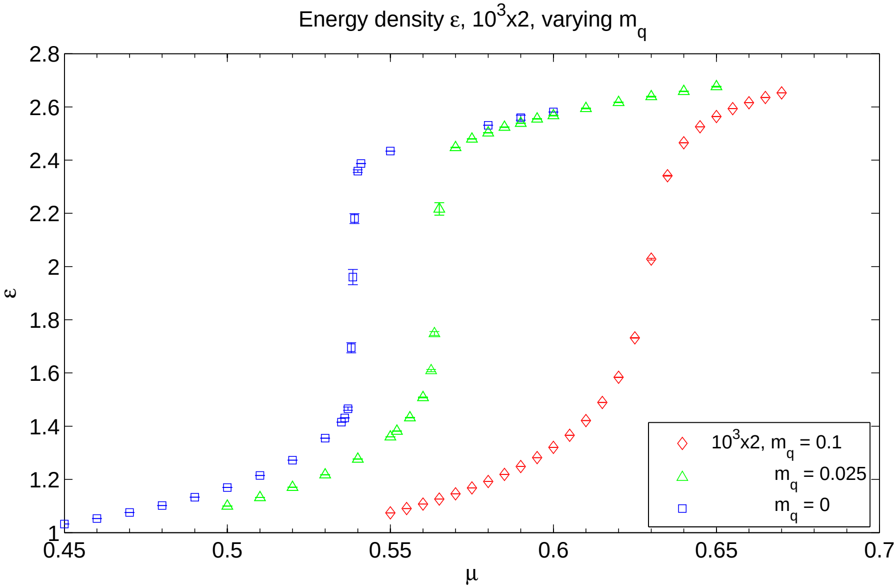
Preliminary Results



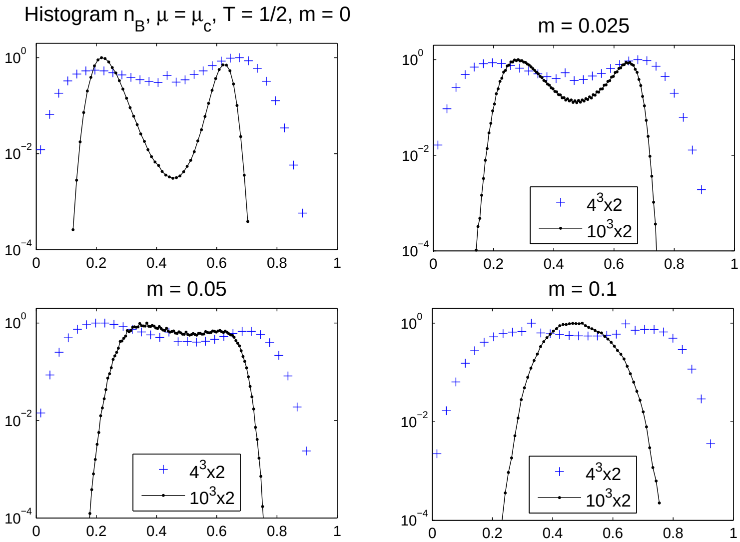
Preliminary Results



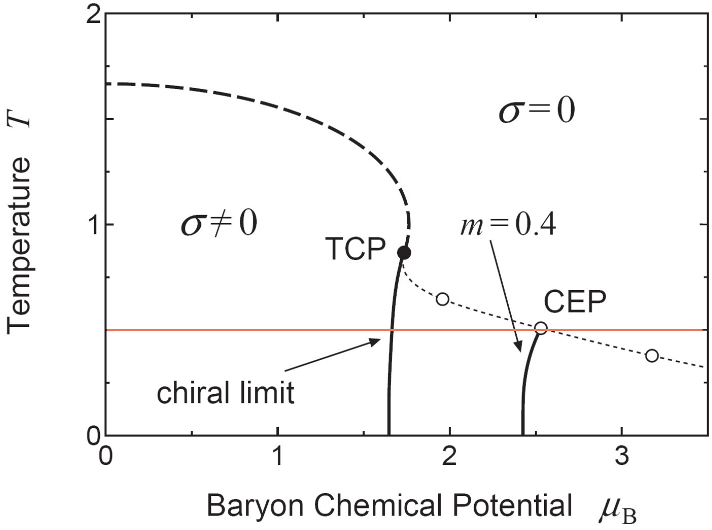
Preliminary Results



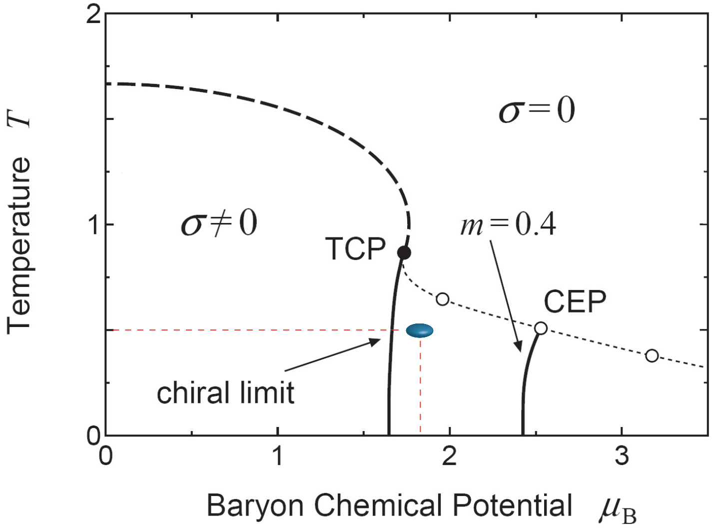
n_B histograms, $T = 1/2, \mu = \mu_c$ for increasing mass



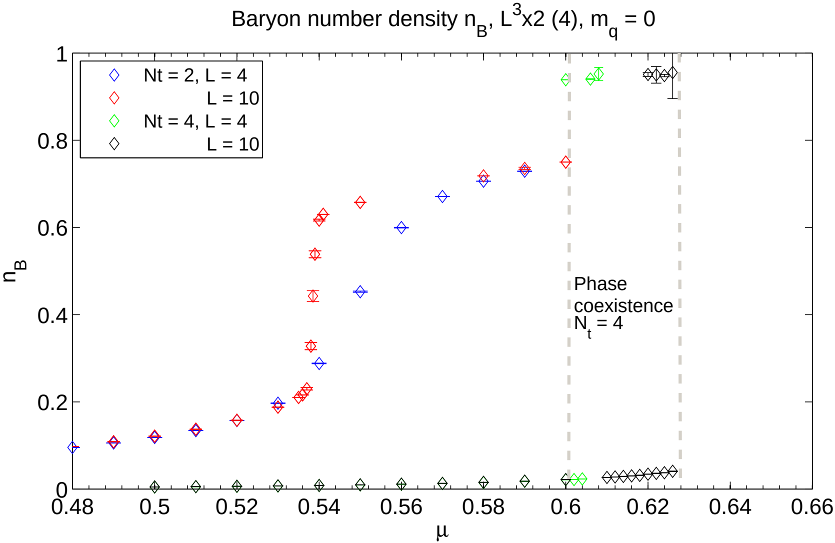
c.f. Mean-Field phase diagram, Nishida '04: qualitative agreement



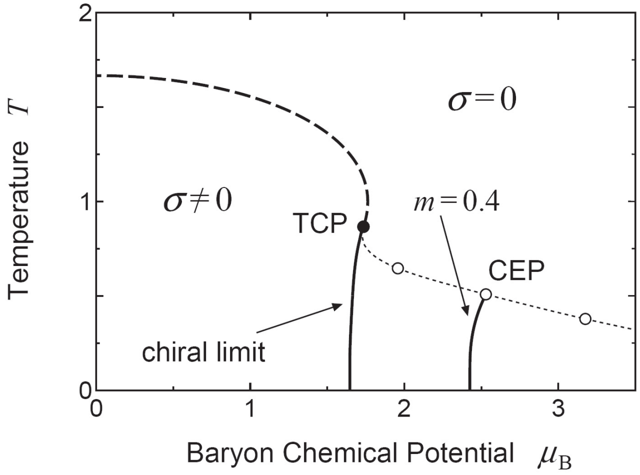
c.f. Mean-Field phase diagram, Nishida '04: qualitative agreement



Preliminary Results, $m = 0$

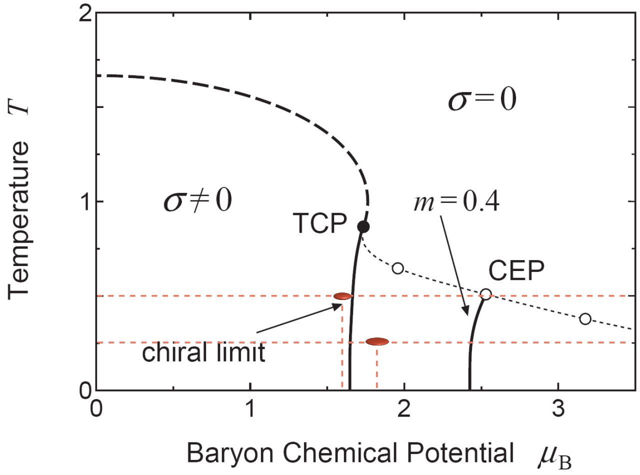


Preliminary Results, $m = 0$



Take $T_c = 5/3$, c.f. MC $T_c \approx 1.4$ [Boyd et al.'92]

Preliminary Results, $m = 0$



Take $T_c = 5/3$, c.f. MC $T_c \approx 1.4$ [Boyd et al.'92]

Summary & Outlook

- Can locate $T = 1/4$ transition with $\mu_c \approx 0.62$ in the chiral limit ($< m_B/3$), $T = 1/2$, $\mu_c \approx 0.54$
- Observe smoothening of finite T transition with increasing mass - in accord with mean-field

Assignment

- Extrapolation $T \to 0$ remains open,
  - Study 1st order PT with multicanonical algorithm
- SCQCD Phase diagram
  - In the chiral limit: Locate TCP
  - CEP for varying quark mass
  - Flavor dependence of phase diagram
- Consistency check $\mu \to i\mu$

T = 1/4, first order coexistence