

TRANSPORT COEFFICIENTS AND n PI METHODS

to measure efficiency with which a conserved quantity is transported over ‘long’ distances
(long compared to microscopic relaxation scales)

effective kinetic theory:

small deviations from thermal equilibrium

weak coupling

→ using equilibrium FT tools

will show:

3PI effective theory → same result (σ_{qed})

motivation:

[1] in principle can use n PI far from equilib

[2] possibility to go beyond leading order (?)

n PI METHODS:

motivation:

sometimes standard pt is poorly convergent

try to improve convergence with non-pert techniques

ex: HTL effective theory (high T gauge theories)

n PI Γ : self-consistent in terms of dressed n -pt fcns
far-from-equib dynamics

2PI QED:

$$Z[J, \eta, \bar{\eta}, C, B]$$

$$= \int D[\mathcal{A}\Psi\bar{\Psi}] \text{Exp} [i(S_{cl} + J_1\mathcal{A}_1 + \bar{\eta}_1\Psi_1 + \bar{\Psi}_1\eta_1 + \frac{1}{2}C_{12}\mathcal{A}_1\mathcal{A}_2 + B_{12}\Psi_1\bar{\Psi}_2)]$$

legendre transform

$$\begin{aligned}
\Gamma[\psi, \bar{\psi}, A, S, D] &= S_{cl}[\psi, \bar{\psi}, A] \\
&+ \frac{i}{2} \text{Tr} \text{Ln} D_{12}^{-1} + \frac{i}{2} \text{Tr} \left[(D_{12}^0)^{-1} (D_{21} - D_{21}^0) \right] \\
&- i \text{Tr} \text{Ln} S_{12}^{-1} - i \text{Tr} \left[(S_{12}^0)^{-1} (S_{21} - S_{21}^0) \right] \\
&+ \Phi[S, D]
\end{aligned}$$

$S_{cl}[\psi, \bar{\psi}, A]$ is the classical action

S_0 and D_0 are the free propagators

$\Phi[S, D]$ is the sum of all 2PI diagrams

EoM obtained from the stationarity of the action:

$$\begin{aligned} \frac{\delta\Gamma[\psi, \bar{\psi}, A, S, D]}{\delta A} &= 0; & \frac{\delta\Gamma[\psi, \bar{\psi}, A, S, D]}{\delta D} &= 0 \\ \frac{\delta\Gamma[\psi, \bar{\psi}, A, S, D]}{\delta\psi} &= 0; & \frac{\delta\Gamma[\psi, \bar{\psi}, A, S, D]}{\delta\bar{\psi}} &= 0 \\ \frac{\delta\Gamma[\psi, \bar{\psi}, A, S, D]}{\delta S} &= 0 \end{aligned}$$

systematic non-pert approx by:

expand Φ (ex: loop or $1/N$ expansion)

solve EoM w/o further approx

consrv laws corresponding to global symmetries are respected (at any approx order)

PROBLEM:

truncations \Rightarrow gauge dependence

wi depend on cancellations btwn different topologies
(vertex corrections and self energy corrections)

2PI effective theory

\rightarrow corrected propagators but not corrected vertices

\rightarrow expect the ward identities are not satisfied

STRATEGY

introduce the resummed effective action

define w.r.t self-consistent solns of the propagators

$$\frac{\delta\Gamma[\psi, \bar{\psi}, A, S, D]}{\delta S} \Big|_{\{S=\tilde{S}[\psi, \bar{\psi}, A], D=\tilde{D}[\psi, \bar{\psi}, A]\}} = 0$$

$$\frac{\delta\Gamma[\psi, \bar{\psi}, A, S, D]}{\delta D} \Big|_{\{S=\tilde{S}[\psi, \bar{\psi}, A], D=\tilde{D}[\psi, \bar{\psi}, A]\}} = 0$$

substituting the self consistent solutions we obtain the resummed action:

$$\tilde{\Gamma}[\psi, \bar{\psi}, A] = \Gamma[\psi, \bar{\psi}, A, \tilde{S}[\psi, \bar{\psi}, A], \tilde{D}[\psi, \bar{\psi}, A]]$$

MOTIVATION FOR RESUMMED ACTION

[A] fcnal derivs of resummed action $\tilde{\Gamma}[\psi, \bar{\psi}, A]$
→ n -point functions for the ‘external’ fields
satisfy standard ward identities

[B] renormalizable

finite # of local medium independent counter-trms
J. Berges, S. Borsanyi, U. Reinosa, J. Serreau, Annals Phys. **320**, 344
(2005); *U. Reinosa, J. Serreau, JHEP* **0607**, 028 (2006).

Different possible defns of n -point functions:

(*U. Reinosa and J. Serreau, JHEP 0711, 097 (2007)*)

$$V_{1,2,\dots,n}^{(n)} \sim \frac{\delta^n \tilde{\Gamma}(A)}{\delta A_1 \delta A_2 \cdots \delta A_n} \Big|_{A=\tilde{A}}$$

$$\hat{V}_{1,2,\dots,n}^{(n)} \sim \frac{\delta}{\delta A_1 \delta A_2 \cdots \delta A_{n-2}} \left(\frac{\delta \Phi(A, D)}{\delta D_{n-1,n}} \Big|_{D=\tilde{D}(A)} \right) \Big|_{A=\tilde{A}}$$

- other mixed definitions possible
- all definitions equivalent at exact level
- there are relations between the definitions
(*chain rule*)

general idea:

integral eqns for the n -point fcns of external fields
 kernels of integral eqns from fcnal derivs of Φ

define 'external' propagators:

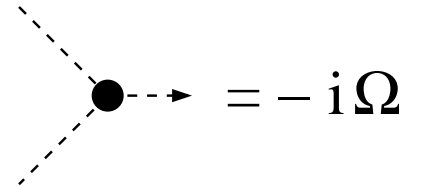
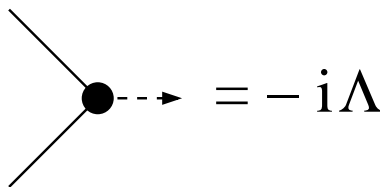
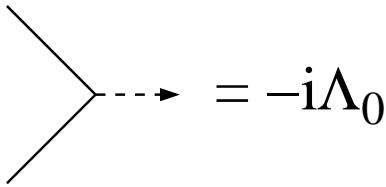
$$(D_{12}^{\text{ext}})^{-1} = \frac{\delta^2}{\delta A_2 \delta A_1} \tilde{\Gamma}; \quad (S_{12}^{\text{ext}})^{-1} = \frac{\delta^2}{\delta \psi_2 \delta \bar{\psi}_1} \tilde{\Gamma}$$

define vertices

$$\Lambda_{132}^0 = -\frac{\delta^3 S_{cl}}{\delta A_3 \delta \psi_2 \delta \bar{\psi}_1} = -\frac{\delta(S_{12}^0)^{-1}}{\delta A_3}$$

$$\Lambda_{132} = -i \frac{\delta^2 \Phi}{\delta A_3 \delta S_{21}} = -\frac{\delta \tilde{S}_{12}^{-1}}{\delta A_3}$$

$$\Omega_{132} = i \frac{\delta \Phi}{\delta A_3 \delta D_{21}} = -\frac{1}{2} \frac{\delta \tilde{D}_{12}^{-1}}{\delta A_3}$$



4-point functions:

$$M_{54;21}^{SS} = -\frac{\delta^2\Phi[\tilde{S}, \tilde{D}]}{\delta\tilde{S}_{12}\delta\tilde{S}_{45}}; \quad M_{54;21}^{SD} = -2\frac{\delta^2\Phi[\tilde{S}, \tilde{D}]}{\delta\tilde{D}_{12}\delta\tilde{S}_{45}}$$

$$M_{54;21}^{DS} = -2\frac{\delta^2\Phi[\tilde{S}, \tilde{D}]}{\delta\tilde{S}_{12}\delta\tilde{D}_{45}}; \quad M_{54;21}^{DD} = 4\frac{\delta^2\Phi[\tilde{S}, \tilde{D}]}{\delta\tilde{D}_{12}\delta\tilde{D}_{45}}$$



$$= -iM_{54;21}^{SS}$$



$$= -iM_{54;21}^{SD}$$

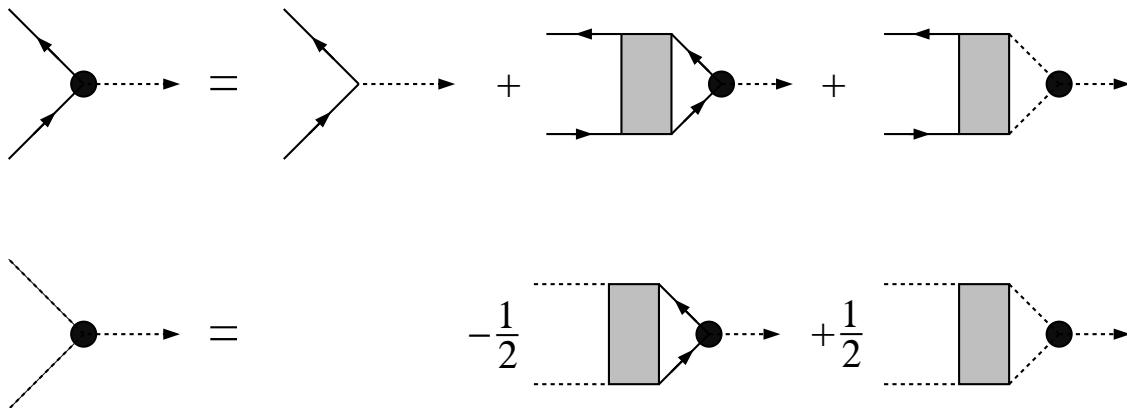
BETHE-SALPETER EQNS

Bethe-Salpeter type equations for the vertices:
from fnal derivatives of the dyson equations wrt A

$$\tilde{S}^{-1} = (S^0)^{-1} + \underbrace{i \frac{\delta \Phi}{\delta S}(S, D)}_{-\Sigma} \Big|_{\tilde{S}, \tilde{D}}$$

$$\tilde{D}^{-1} = (D^0)^{-1} - \underbrace{2i \frac{\delta \Phi}{\delta D}}_{\Pi} \Big|_{\tilde{S}, \tilde{D}}$$

graphically:



Conductivity:

kubo formula:

$$\sigma = -\frac{1}{6e^2} \left(\frac{\partial}{\partial q_0} 2 \operatorname{Im} \Pi_{ret}^{ii}(q_0, 0) \right) \Big|_{q_0 \rightarrow 0}$$

∞ # terms contribute at the same order
(from low frequency limit in the kubo formula)

pairs of ret/adv propagators with same momenta
integrating a term $\int dp_0 G^{ret}(P) G^{adv}(P)$
→ a divergence called a ‘pinch singularity’

regulate using resummed propagators
(finite width of thermal excitations)

→ extra factors of the coupling in the denominators
→ infinite set of graphs which contain products of
pinching pairs that all need to be resummed

RESUMMATION PINCH SINGULARITIES

we use:

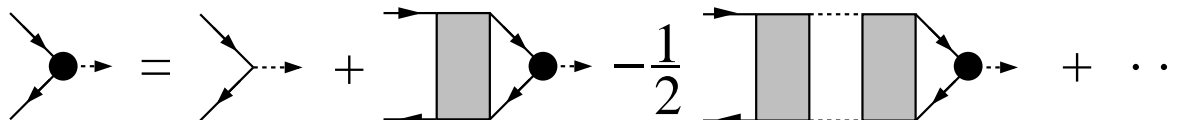
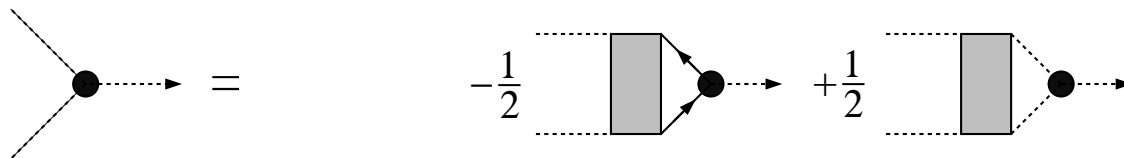
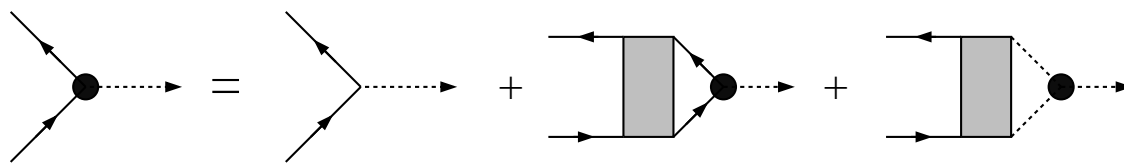
$$\Pi = -i\Lambda^0 S \Lambda S$$

there is an int eqn for Λ that resums pinch terms

kernel is square of the matrix elements that correspond to the $2 \rightarrow 2$ scattering and production processes (AMY)

show this int eqn produced by the 2PI formalism

iterate BS eqns:



calculate M's from derivatives of Φ

$$\Phi(S,D) = \mathbf{i} / 2 \text{ (circle with horizontal dashed line) } + \mathbf{i} / 4 \text{ (circle with vertical dashed line and two solid lines forming a V-shape) }$$

results:

$$-\mathbf{i}M^{SS} = \text{(diagram 1)} + \text{(diagram 2)} + \text{(diagram 3)} + \text{(diagram 4)}$$

The diagrams for $-\mathbf{i}M^{SS}$ are:

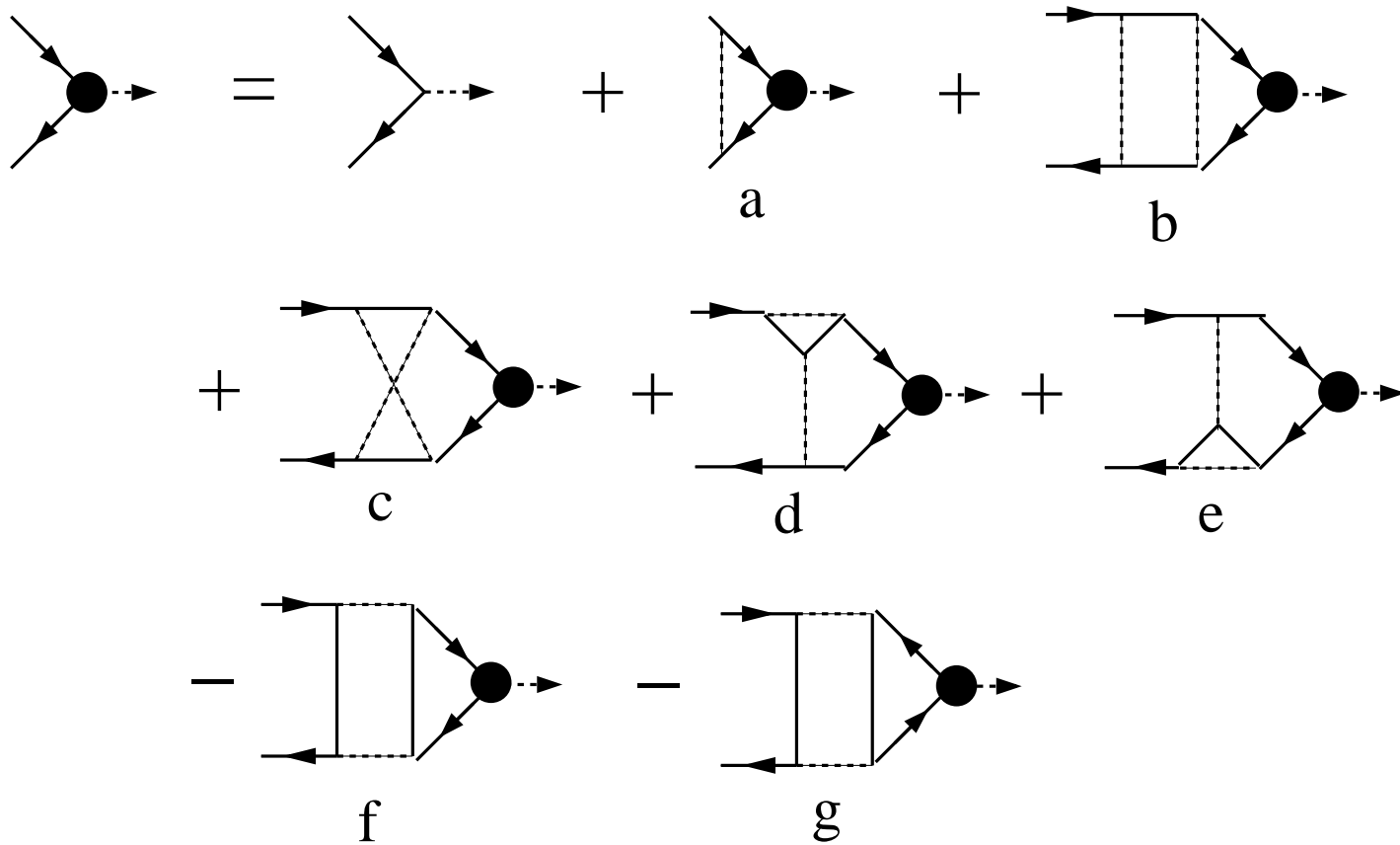
- Diagram 1: Two horizontal lines with arrows pointing right. A vertical dashed line connects them.
- Diagram 2: Two horizontal lines with arrows pointing right. Two diagonal dashed lines cross each other between the lines.
- Diagram 3: Two horizontal lines with arrows pointing right. A vertical dashed line connects them, and a solid line forms a V-shape pointing down from the top line.
- Diagram 4: Two horizontal lines with arrows pointing right. A vertical dashed line connects them, and a solid line forms an inverted V-shape pointing up from the bottom line.

$$-\mathbf{i}M^{SD} = \text{(diagram 5)} + \text{(diagram 6)}$$

The diagrams for $-\mathbf{i}M^{SD}$ are:

- Diagram 5: Two horizontal lines with arrows pointing right. A vertical solid line connects them, with a dashed line above and below.
- Diagram 6: Two horizontal lines with arrows pointing right. A vertical solid line connects them, with a dashed line above and below. The arrow on the bottom line points left.

integral eqn for Λ



integral eqn:

$$\text{Re}\hat{\Lambda}^i(3, P) = \text{Re}\hat{\Lambda}_0^i(3, P) + \sum_{j \in \{a, b, c, d, e, f, g\}} \frac{1}{2} \int dK \text{Re} [\hat{M}^{(j)}(P, K)] \frac{\rho(K)}{2\text{Im}\hat{\Sigma}(K)} \text{Re}\hat{\Lambda}^i(3, K)$$

$$S_{ret}(P) = \not{P} G_{ret}(P), \quad G_{ret}(P) G_{adv}(P) = -\frac{\rho(P)}{\text{Im}\hat{\Sigma}(P)}$$

$$\hat{\Sigma}(K) = \text{Tr}(\not{K} \Sigma_{ret}(K))$$

$$\hat{\Lambda}^i(3, K) = \text{Tr}(\not{K} \Lambda_{rar}(K))$$

$$\hat{M}(P, K) = \text{Tr}(\not{P} [-i\mathbf{M}(P, K)] \not{K})$$

$$\text{kernel } \text{Re} [\hat{M}^{(j)}(P, K)] \rightarrow |ME|^2$$

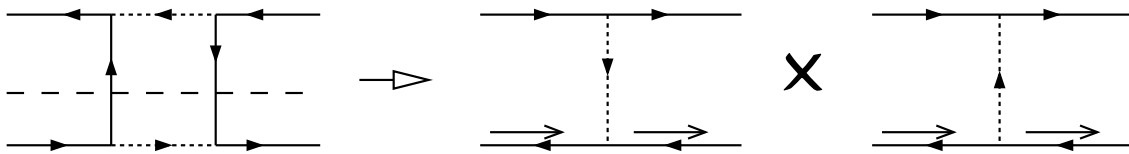
$\Phi(2 - \text{loop}) \rightarrow$ the square of the s-channel
complete to leading log order

G. Aarts and J. Martinez-Resco, JHEP 03, 074 (2005).

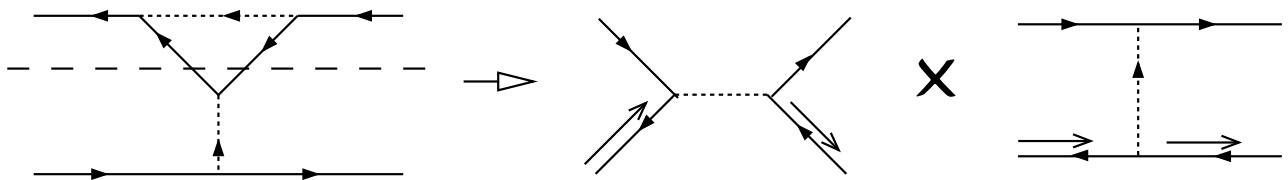
$\Phi(3 - \text{loop}) \rightarrow t$ - and u -channels
part of full lo contribution

MEC and E. Kovalchuk, Phys. Rev. D 76, 045019 (2007).

$$\text{Re } \hat{M}(f) \rightarrow |m_{e^+e^- \rightarrow e^+e^-}^t|^2$$



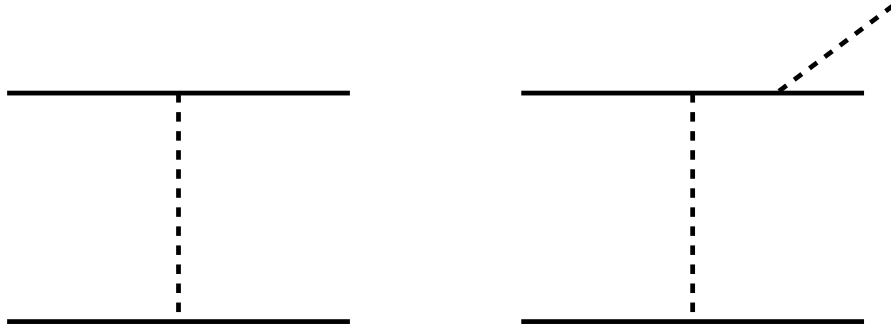
$$\text{Re } M^{(d)} \rightarrow m_{e^+e^- \rightarrow e^+e^-}^{t\dagger} \cdot m_{e^+e^- \rightarrow e^+e^-}^s$$



RESUM PINCHES \rightarrow FULL LEADING ORDER

basic idea:

compare $2 \rightarrow 2$ and $2 \rightarrow 3$



2nd is formally higher order

BUT collinear singularity \rightarrow enhancement

∞ series of collinear singularities must be resummed
(LPM effect)

\Rightarrow need 2 coupled integral equations that resum
pinching and collinear singularities

METHOD: 3PI EFFECTIVE ACTION

motivation:

need 3-loop diagram to get t - and u - channels in ME

heirarchy: $(n \rightarrow \infty)PI|_{3\text{-loop}} = 3PI|_{3\text{-loop}}$

J. Berges, Phys. Rev. D **70**, 105010 (2004).

result:

3PI $\Gamma \rightarrow 2$ int eqns: pinch and collinear singularities

MEC and E. Kovalchuk, Phys. Rev. D **77**, 025015 (2008).

3PI Γ :

$$\begin{aligned}
 \Gamma[\psi, \bar{\psi}, A, S, D, V, U] &= S_{cl}[\psi, \bar{\psi}, A] \\
 &+ \frac{i}{2} \text{Tr} \text{Ln} D_{12}^{-1} + \frac{i}{2} \text{Tr} \left[(D_{12}^0)^{-1} (D_{21} - D_{21}^0) \right] \\
 &- i \text{Tr} \text{Ln} S_{12}^{-1} - i \text{Tr} \left[(S_{12}^0)^{-1} (S_{21} - S_{21}^0) \right] \\
 &+ \Gamma_2^0[S, D, V, U] + \Gamma_2^{\text{int}}[S, D, V, U]
 \end{aligned}$$

$$\Gamma_2^0 = i \text{ } \langle \text{Diagram: a circle with a dashed horizontal line from the left to a black dot on the right} \rangle$$

$$\Gamma_2^{\text{int}} = -i/2 \text{ } \langle \text{Diagram: a circle with a dashed horizontal line between two black dots on the left and right} \rangle + i/12 \text{ } \langle \text{Diagram: a dashed circle with a dashed horizontal line between two black dots on the left and right} \rangle + i/3 \text{ } \langle \text{Diagram: a circle with a central black dot and three black dots on the perimeter, connected by dashed lines} \rangle + i/4 \text{ } \langle \text{Diagram: a circle with a central black dot and three black dots on the perimeter, connected by solid lines} \rangle - i/24 \text{ } \langle \text{Diagram: a dashed circle with a central black dot and three black dots on the perimeter, connected by dashed lines} \rangle$$

RESUMMED ACTION:

7 EoM:

functional derivatives wrt $\{A, \psi, \bar{\psi}, S, D, V, U\}$

solve last 4 simultaneously for the sc solns:

$$\tilde{S}[\psi, \bar{\psi}, A], \quad \tilde{D}[\psi, \bar{\psi}, A], \quad \tilde{V}[\psi, \bar{\psi}, A], \quad \tilde{U}[\psi, \bar{\psi}, A]$$

resummed action:

$$\tilde{\Gamma}[\psi, \bar{\psi}, A] =$$

$$\Gamma[\psi, \bar{\psi}, A, \tilde{S}[\psi, \bar{\psi}, A], \tilde{D}[\psi, \bar{\psi}, A], \tilde{V}[\psi, \bar{\psi}, A], \tilde{U}[\psi, \bar{\psi}, A]]$$

define external vertices same as before



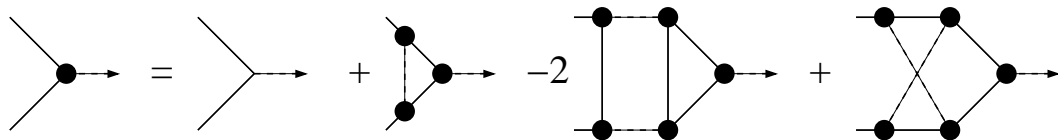
INTEGRAL EQUATIONS:

2 EoM from fcn derivs of Γ wrt S and D (SD eqns)

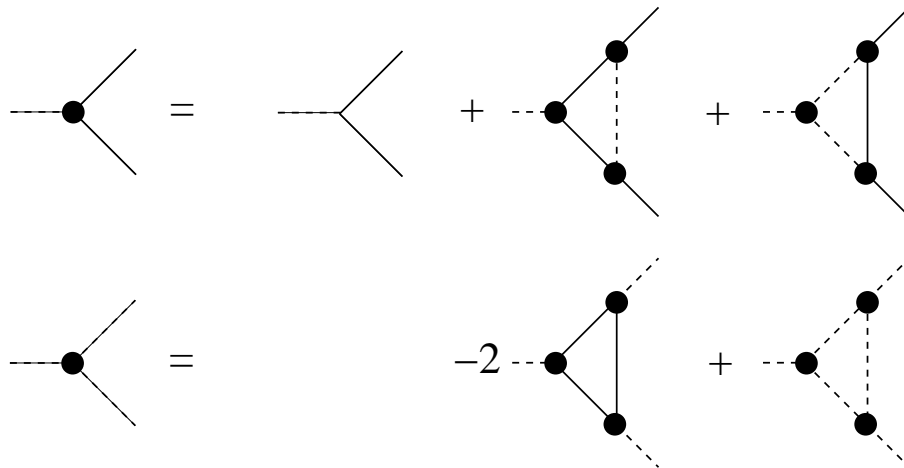
differentiate wrt A

→ BS eqn for Λ and Ω (many cancellations)

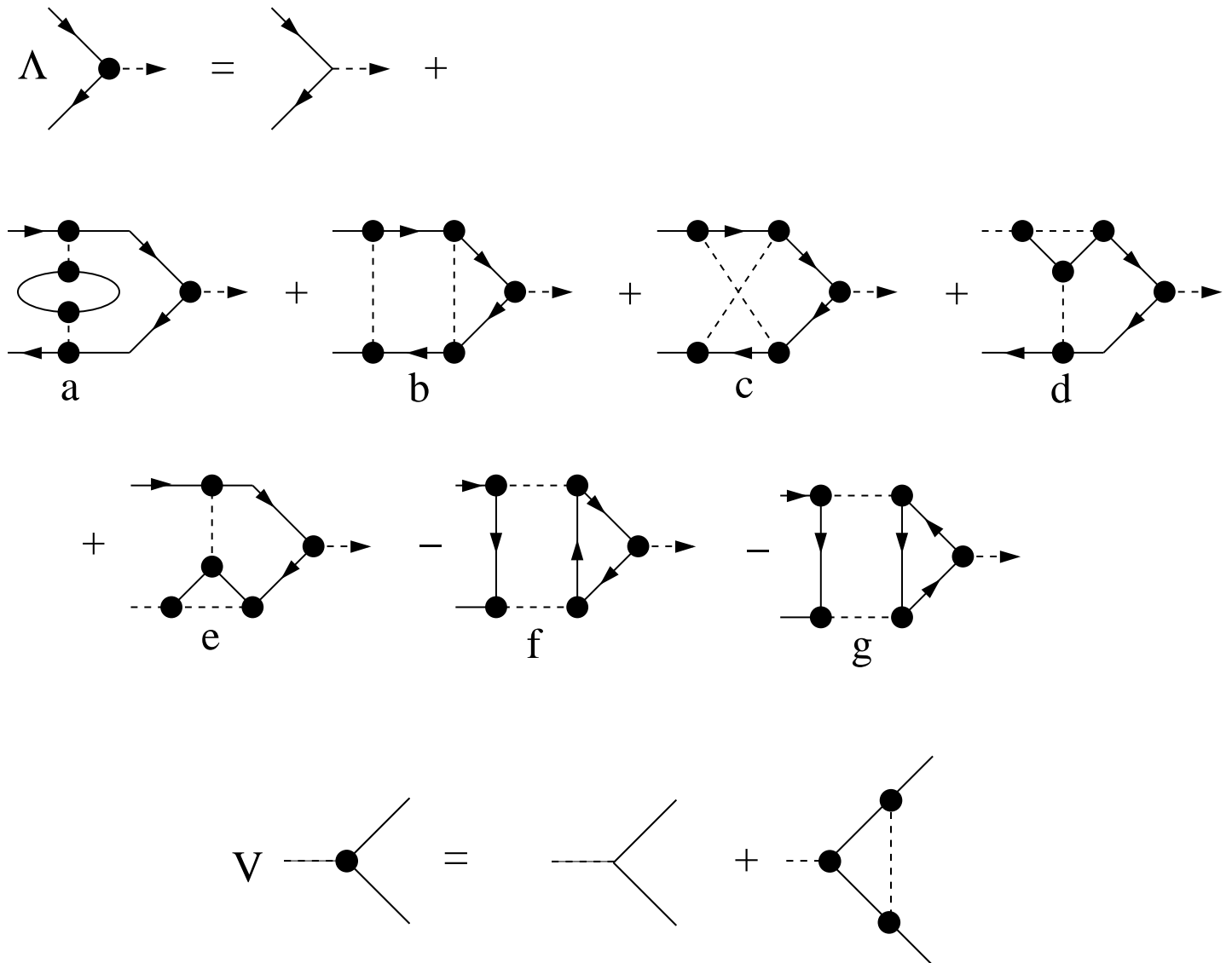
sub Ω eqn into Λ eqn and keep up to 2-loop order:



also: 2 EoM from fcn derivs of Γ wrt V and U



→ substitute again and keep up to 2-loop order:



1st is int eqn for $\Lambda \sim$ same form as 2PI
 2nd is sc int eqn for V

same eqns found using kinetic theory (AMY)
 also found using a diagrammatic approach
J-S Gagnon and S. Jeon, Phys. Rev. D75, 025014 (2007)

PROSPECTS

method should be generalizable to:

[1] other transport coefficients (shear viscosity)

[2] other theories (QCD)

[3] nlo (?)

KEY:

ALL LEADING ORDER TERMS APPEAR
NATURALLY W/O ANY POWER COUNTING

PROBLEMS

external n -point functions satisfy w
 may get gauge dependence from sc (internal props)

gauge invariance of the effective action:

calculate 2PI Γ to L -loop order (g^{2L-2})

→ gauge dependent terms appear at order g^{2L}

[1] from behaviour of Γ under BRS transformations
A. Arrizabalaga, J. Smit, Phys. Rev. D66, 065014 (2002).

[2] from 2PI Nielsen identities

explicit gauge dependence of Γ compensates gauge
 dependence of the vev

MEC, G. Kunstatter, H. Zaraket, Eur.Phys.J. C42, 253 (2005).

sc propagators are determined numerically from Γ
 \rightarrow expect also gauge indep up to order of truncation

certainly should be okay for thermodynamic observ.

checked for 2-loop qed pressure from 2PI Γ

S. Borsanyi, U. Reinosa - arXiv:0709.2316.