Bose-Einstein condensation in dense quark matter

Jens O. Andersen [1]

Department of Physics
Norwegian University of Science and Technology

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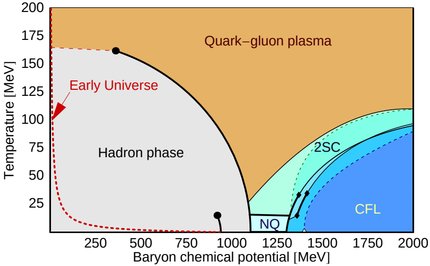
[1] In collaboration with Tomas Brauner and Lars Leganger.

Introduction and Motivation

- Condensation of bosons occur in different context in QCD
  - Pion condensation at finite isospin chemical potential
  - Kaon condensation in the CFL phase of dense quark matter



Pion condensation

- NJL models [2]
- Lattice QCD [3]
- Chiral perturbation theory [4]
- Effective theory: $O(N)$ linear sigma model.
  $SU(2)_L \times SU(2)_R \sim O(4)$ [5]

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[2] Barducci *et al*, Phys. Rev. D **69**, 096004 (2004). Ebert and Klimenko, J. Phys. **G32**, 599 (2006).

[3] J. B. Kogut and D. K. Sinclair, Phys. Rev. D **64** 014508 (2002); ibid D **66** 34505 (2002).

[4] K. Splittorff, D. T. Son and M. Stephanov, Phys. Rev. D **64** 016003 (2001). J. B. Kogut and D. Toublan, Phys. Rev. D **64** 034007 (2001).

[5] JOA, Phys. Rev. D **75**, 065011 (2007), JOA and T. Brauner, Phys. Rev. D **75**, 065011 (2008).

- Lagrangian

$$\mathcal{L} = (\partial_\mu \Phi_i^\dagger)(\partial_\mu \Phi_i) - \frac{H}{\sqrt{2}}\left(\Phi_1 + \Phi_1^\dagger\right) + m^2 \Phi_i^\dagger \Phi_i + \frac{\lambda}{2N}\left(\Phi_i^\dagger \Phi_i\right)^2$$

- Conserved charge

$$\partial_0 \Phi_i \rightarrow (\partial_0 - \mu_i)\,\Phi_i$$

- Chiral and pion condensates

$$\Phi_1 = \frac{1}{\sqrt{2}}(\phi_0 + i\rho_0 + \phi_1 + i\phi_2)$$

$$D_0^{-1} = \begin{pmatrix} \omega_n^2 + p^2 + m_1^2 & \frac{\lambda}{N}\phi_0\rho_0 & 0 & 0 \\ \frac{\lambda}{N}\phi_0\rho_0 & \omega_n^2 + p^2 + m_2^2 & -2\mu_I\omega_n & 0. \\ 0 & 2\mu_I\omega_n & \omega_n^2 + p^2 + m_3^2 & \\ 0 & 0 & 0 & \omega_n^2 + p^2 + m_4^2 \end{pmatrix}$$

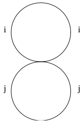
$$\begin{aligned} m_1^2 &= m^2 + \frac{3\lambda}{2N}\phi_0^2 + \frac{\lambda}{2N}\rho_0^2 \,, \\ m_2^2 &= -\mu_I^2 + m^2 + \frac{\lambda}{2N}\phi_0^2 + \frac{3\lambda}{2N}\rho_0^2 \,, \\ m_3^2 &= -\mu_I^2 + m^2 + \frac{\lambda}{2N}\phi_0^2 + \frac{\lambda}{2N}\rho_0^2 \,, \\ m_4^2 &= m^2 + \frac{\lambda}{2N}\phi_0^2 + \frac{\lambda}{2N}\rho_0^2 \,. \end{aligned}$$

2PI$-1/N$-expansion

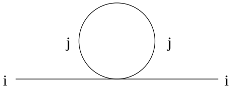
- Respects Goldstone's theorem order by order
- Selective resummation from all loop orders
- Effective action

$$\begin{aligned}\Gamma[\phi_0,\rho_0,D] &= \frac{1}{2}m^2\left(\phi_0^2+\rho_0^2\right)+\frac{\lambda}{8N}\left(\phi_0^2+\rho_0^2\right)^2-\frac{1}{2}\mu_I^2\rho_0^2\\ &-H\phi_0+\frac{1}{2}\mathrm{Tr}\ln D^{-1}+\frac{1}{2}\mathrm{Tr}D_0^{-1}D+\Phi[D]\ ,\end{aligned}$$

Vacuum diagrams and gap equations

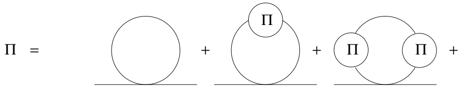


$$\frac{\delta\Gamma[\phi_0,\rho_0,D]}{\delta\phi_0}=0\ ,\qquad \frac{\delta\Gamma[\phi_0,\rho_0,D]}{\delta\rho_0}=0\ ,\qquad \frac{\delta\Gamma[\phi_0,\rho_0,D]}{\delta D}=0$$



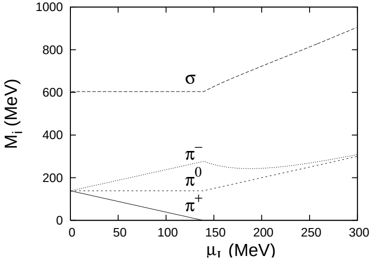
Diagrammatics of gap equation

$$
\begin{aligned}
D^{-1} &= P^2 + \Pi^2 \\
\Pi^2 &= \sum_{\omega_n = 2\pi n T} \int \frac{d^3p}{(2\pi)^3} \frac{1}{P^2 + \Pi^2}
\end{aligned}
$$

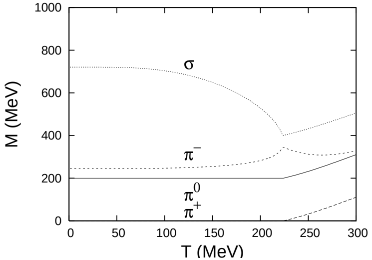


- Summation of daisy and superdaisy diagrams

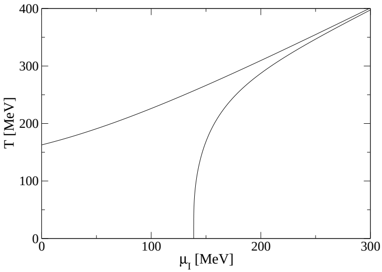
Masses at $T = 0$ at the physical point



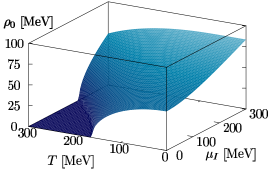
Temperature-dependent masses ($\mu_I = 200$ MeV)



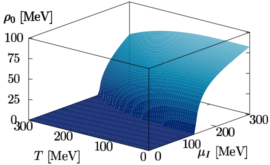
Phase diagram



Chiral limit



Physical point



- Summary
  - Using 2PI-$1/N$-expansion to determine phase diagram and medium-dependent masses of sigma and pions.
  - Goldstone theorem satisfied.
- Outlook
  - Still need to include electric charge neutrality:

$$\frac{\partial \mathcal{F}}{\partial \mu_Q} = 0$$

    Apply Hartree approximation [6].
  - Going beyond mean field, e.g including $1/N$ corrections.

[6] M. G. Alford, M. Braby, and A. Schmitt, J. Phys. **G**: nucl. Part. Phys. **35**, 025002 (2008); G. Fejos, A. Patkos, and Zs. Szep, Nucl. Phys.**A803**, 115 (2008) and this conference (Jakovac and Patkos).