IAC2009: Gamma-Ray Bursts — Problem set week 2

Problem 1: Superluminal motion

As you know, when a source ejects a very fast-moving blob of radiating fluid, we may appear to see source and blob separate at a speed greater than c. (For this, we need to assume that we know the distance to the source, so that we know how to translate angular velocity into space velocity.)

(a) For a blob ejected at an angle θ to our line of sight, with constant speed $v = \beta c$, derive the equation for the apparent transverse and radial velocities of the blob.

(b) Find the angle at a given β for which each of the velocities is maximal, and the maximum value.

Problem 2: Shape of an emitting shell

Assume that a shell of material is expanding spherically with speed $v = \beta c$, emitting radiation all the time. Due to the differences in light travel time, the radiation we receive at some observer time t does not seem to come from a spherical surface. What is the shape and size of that surface as a function of observer time? (Hint: you can first calculate what the arrival time is of radiation emitted at radius r from the centre by matter moving at angle θ relative to our line of sight, and then look at the surface $r(\theta)$ defined by $t_{obs} = cst$.)

Problem 3: Light curve of a flashing shell

Take a spherical shell of radius r at distance $D \gg r$ from us. At some point, the entire shell emits a flash of light that is a delta function of time. Each part of the shell emits the same amount and emits isotropically. Compute the shape of the light curve that we see, and comment on the often-used rule of thumb that if we see a source vary its flux significantly on a time scale δt, the source must be smaller than $c.\delta t$.